The Synchronous Fundamental dq Frame Theory Application for the Active Filtering

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Abstract— One of the methods that have been used for the analysis of three phase systems is the rotating fundamental dq frame. Although it was especially used for the analysis of three phase machines, because it is a time domain method is also suitable for the compensating current computation in the active filtering. In fact, it was already been used for the computation of the current fundamental, and if desired, for the computation of some of the upper harmonics. This way the compensator could eliminate from the power grid current all or some of the harmonic content. But this is also, the limitation of this method, because it cannot intrinsically obtain the compensation of reactive power. This is because this method refers to currents and not powers so the reactive power is not defined. At the same time and for the same reasons, it could not obtain the compensated current to have the same shape as the grid voltage. If the compensation goal includes the reactive current compensation, this method must be adapted to compute beside the load current fundamental, the active component of the load current fundamental. Another adaptation of this method is the unity power factor compensation, which implies that the compensated current has the same shape and phase with the grid voltage, so the active power is transported not only on the voltage fundamental, but also on the voltage harmonics.

I. INTRODUCTION

The active compensator performance is dependent on the compensating current computation, and a method which can be used in this respect is the synchronous fundamental dq frame. Although it’s having some disadvantages and limitations, this method can give good results, so the implementation and adaptation of this method for shunt active power filters is the aim of this paper.

After the introduction, the synchronous fundamental dq frame theory is presented in detail in the 2nd section. The 3rd section describes the implementation of each compensation goal and in the 4th section the virtual model of the filtering system and the simulation results are presented. In the 5th section the experimental results are briefly presented to validate the simulation results.

II. THE SYNCHRONOUS FUNDAMENTAL DQ FRAME THEORY

According to the synchronous fundamental dq frame theory, the Park transform is used to make the transition from the three-phase coordinate system of the current obtained from the current transducers, to a two-phase orthogonal coordinate system rotating at an imposed speed [1][2].

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos \theta & \cos \left( \theta - \frac{2\pi}{3} \right) & \cos \left( \theta - \frac{4\pi}{3} \right) \\ -\sin \theta & -\sin \left( \theta - \frac{2\pi}{3} \right) & -\sin \left( \theta - \frac{4\pi}{3} \right) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

Fig. 1. The current phasor projections on the rotating synchronous dq referential axis

Following this transformation, the current projections on the two axes will be [2][3]:

- a DC component due to the current harmonic with the angular frequency equal to the dq frame rotating speed;
- an AC component, due to the current harmonics of angular frequencies different from the dq frame rotating speed.

For the extraction of the DC component, low-pass filters are commonly used, and for the extraction of the AC components, high-pass filters are used [2][4].

One disadvantage of this method is the necessity to obtain the dq frame angular position (angle $\theta$), which involves the use of a PLL for determining the mains voltage phase [2].

Another disadvantage is that the DC component is extracted using numeric low pass filters. In this case, the lower is the cutting frequency the higher is the filter response time. This can be avoided using the running window averaging technique to compute the mean value of the phasor projection for the selected harmonic over the specified period. In this case, the response time is equal to the harmonic period. Assuming that the current fundamental is to be extracted, so the dq frame is rotating with the speed of $100\pi$ rad/sec, the current phasor projections on the dq frame axis are:
The mean value of the d axis phasor projection calculated for 50 Hz is null, while the q axis phasor projection mean value is just equal to the phasor amplitude which is required to be obtained. This is true only if the current system has the same succession as the voltage system. If the current succession is reversed, so the dq frame rotating direction is reversed to the current phasor direction, then the mean values of both projections are null. This is because when the rotating directions are opposite, the projections on the dq frame of the current phasor are not DC components but AC components of twice the angular frequency (the angular frequency of the phasor summed with the angular frequency of the dq frame).

In this case, the projections on the d and q axis of the rotating dq frame are:

\[ i_d = \sqrt{2} \cdot I_{RMS} \cdot \cos(2\omega t) \]  
\[ i_q = -\sqrt{2} \cdot I_{RMS} \]  

Whose mean values over one period of the desired harmonic are, obviously null.

According to the compensation goals, there may be different approaches. This is because some authors consider that the optimum power transfer is obtained when the current follows the power grid voltage shape, so the power is transferred on all the voltage harmonics, not only on the fundamental. But, other authors consider that when the grid current is sinusoidal then no voltage distortion appears and the voltage is also sinusoidal. In this case, the active power is transferred only on the voltage fundamental but there is no voltage distortion to affect the operation of the grid connected loads.

Thus, the compensation goals can be defined as follows:

- **sinusoidal grid current:**
  - partial compensation – only the distortion component of the current is compensated, so the current is sinusoidal keeping its phase delay;
  - total compensation – the distortion and the reactive components of the grid current are compensated, so the current is sinusoidal and in phase with the voltage.

- **unity power factor** – the power absorbed from the grid is only the active power, so the compensated current is in phase with the voltage, but for distorted grid voltage, the grid current will be also distorted.

### III. THE IMPLEMENTATION OF THE SYNCHRONOUS ROTATING DQ FRAME THEORY

The implementation of this compensating current computation method was done using the Matlab Simulink environment, because of two reasons:

- the virtual filtering system is a Matlab Simulink model in which the active compensator computation section is a Simulink block;
- the experimental filtering system is using an active compensator whose computing hardware is the dSpace DS1103 control board which is programmed starting from the Simulink environment. Consequently, the compensating current computation can be implemented for the experimental equipment using the same Simulink block as in the virtual system.

The compensation current computation block is constructed starting from (1) and implements all the compensation goals.

#### A. Sinusoidal grid current

For the partial compensation, which is the intrinsic result of the synchronous rotating dq frame theory, the compensating current is simply obtained by subtracting the fundamental computed with (1) from the load current:

\[
\begin{bmatrix}
    i_{Fa}^* \\
    i_{Fb}^* \\
    i_{Fc}^*
\end{bmatrix}
= \begin{bmatrix}
    i_{La} \\
    i_{Lb} \\
    i_{Lc}
\end{bmatrix} - \begin{bmatrix}
    i_{1a} \\
    i_{1b} \\
    i_{1c}
\end{bmatrix}
\]  

The Simulink block for this part is illustrated in Fig. 2. It can be seen that the dq frame instantaneous angular position is the angular position of the grid voltage fundamental obtained at a PLL output (Fig. 3).

The total compensation implies the injection in the PCC by the active filter of both the distortion and reactive currents. But the latter is not defined by the synchronous rotating dq frame theory. So this goal is reached in two steps: at first, the load current fundamental is computed (Fig. 2), and later, the fundamental is brought in phase with the voltage and corrected. The phase shift is obtained using the model in Fig. 3. In fact, the load current fundamental is brought in phase with the grid voltage fundamental by using a Park transform of the first with the dq frame angular position of the load current fundamental (initial theta) and a reverse Park transform with the dq frame angular position of the grid voltage fundamental (final theta).
As a result the current phasor will be rotated with the subtraction of the two angular positions, which is the phase shift between the voltage and current. But, just the phase shifting of the current phasor is not enough, because the RMS value of the current remains unchanged and equal to the value which gives the apparent power of the load, not the active power.

At the same time, because the compensated current is sinusoidal despite the non-sinusoidal grid voltage, the active power is transmitted only on the grid voltage fundamental, the active power corresponding to the higher voltage harmonics being equal to zero (the current harmonics are null). In this case the apparent power drawn from the grid after the compensation is equal to the active power of the load. It follows that the compensated current RMS value (when the reactive power is compensated) is \( \cos \theta \) times smaller compared to the case when the reactive power is not compensated. So, when the reactive power is to be compensated, the current phasor will be corrected after its phase shifting with a correction coefficient equal to \( \cos \theta \) (Fig. 3). Finally, the compensating current is (Fig. 4):

\[
\begin{bmatrix}
    i^*_F a \\
    i^*_F b \\
    i^*_F c
\end{bmatrix} = \begin{bmatrix}
    i_{La} \\
    i_{Lb} \\
    i_{Lc}
\end{bmatrix} - \begin{bmatrix}
    i_{a1a} \\
    i_{a1b} \\
    i_{a1c}
\end{bmatrix}
\]  

(7)

Where \( i_{a1a}, i_{a1b}, i_{a1c} \) are the desired active currents computed with (2), shifted and corrected.

**B. Unity power factor**

This compensation goal implies that the compensated current has the same waveform as the grid voltage. This also implies that the compensated current is in phase with the grid voltage, so the partial compensation cannot be chosen. This is because each current harmonic would be corrected as described above with an unnecessary complication of the model followed by the corresponding increase in the computation time.

Because in this case, the compensated current harmonic spectrum is similar to the grid voltage harmonic spectrum, the active power will be drawn from the power grid not only on the voltage fundamental, but also on each voltage harmonic. The RMS value of the compensated current will be the same to the sinusoidal compensated current, the active power absorbed by the load being the same.

On the other hand, the RMS value of the compensated current fundamental will be smaller than the corresponding value of the sinusoidal compensated current.

In order to obtain the compensating current, the equivalent conductance of the nonlinear load was used. This was calculated starting from the premise that if the compensated current is sinusoidal, the active power is drawn on the voltage fundamental, resulting in a linear equivalent load, which is having the conductance:

\[
G_{ech} = \frac{I_{s1}}{U_{s1}}
\]  

(8)

So the compensating current is obtained by subtracting the active current of the equivalent load from the real load current:

\[
\begin{bmatrix}
    i^*_F a \\
    i^*_F b \\
    i^*_F c
\end{bmatrix} = \begin{bmatrix}
    i_{La} \\
    i_{Lb} \\
    i_{Lc}
\end{bmatrix} - G_{ech} \begin{bmatrix}
    u_a \\
    u_b \\
    u_c
\end{bmatrix}
\]  

(9)

**IV. VIRTUAL IMPLEMENTATION OF THE ACTIVE FILTERING SYSTEM**

The virtual implementation of the active filtering system is a Simulink model which contains all the system components built with SymPowerSystems or Simulink library blocks, as necessary (Fig. 5):

- the power inverter (with 1200V and 100A IGBT blocks);
- the interface filter (first order inductive filter);
- the compensating capacitor;
- the nonlinear load (three phase thyristor AC voltage converter);
- the active filter control block (which contains the compensating current computation block (Fig. 4), the output current and DC-Link voltage regulating loops [5], auxiliary blocks for the active filter start-up process control);
- auxiliary measurement and computation blocks.
The current absorbed by the nonlinear load is about 15A, and its waveform and harmonic spectra is illustrated in Fig. 6. For clarity only the first 31 harmonics are illustrated.

A. Sinusoidal compensated current – partial compensation

The first compensation goal corresponds to the partial compensation and sinusoidal grid current. The grid current waveform and harmonic spectra are illustrated in Fig. 7.

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Fig. 5. The active filtering system complete model

Fig. 6. The grid voltage and nonlinear load current:
(a) waveforms, (b) harmonic spectra

Fig. 7. The grid voltage and current after the compensation for the partial sinusoidal compensation:
(a) waveforms, (b) harmonic spectra
In this case, the compensated current should be sinusoidal keeping its phase shift, which is confirmed both by the current waveform and by the harmonic spectra. It can be seen that the current harmonics corresponding to the voltage important harmonics (5th and 7th) are greatly reduced, as well as the other current harmonics. At the same time the current fundamental RMS value relative to the global RMS value is nearly equal to one, which emphasizes even more the fact that the filtering goal is achieved.

Regarding the numerical results, the compensated current total harmonic distortion factor is 2 % considering the load current THD of 77.75%, which gives a filtering efficiency of 38.78. It must be noticed that the calculated hysteresis band is 2.33 A, calculated for the given interface filter in order to obtain the maximum switching frequency of 7500 Hz. In fact, the measured hysteresis band after the simulation is 0.25 A, with the consequence of increased harmonic distortion. For example, the partial harmonic distortion factor calculated for the first 51 harmonics is 1.88% which means that some of the compensated current harmonic distortion due to the active filter switching operation can be reduced by reducing the current overshoot.

B. Sinusoidal compensated current – total compensation

In the case of total compensation, only the fundamental of the active component of the load current remains to be absorbed from the power grid. Because the considered nonlinear load is a special load designed especially to test the functionality and performance of active filters, it absorbs from the power grid only reactive and distortion power. Because it absorbs no active power, after the compensation the current drawn from the grid must be null. In fact, a small active current is absorbed from the grid to cover the losses of both the nonlinear load and active filter. So, for the considered case, after the compensation, the current RMS value is reduced to 1.21 A. The small value of the grid current it can be observed also by analyzing the waveforms illustrated in Fig. 8.

C. Unity power factor compensation

For the case of unity power factor, the current must have the same shape as the voltage but because almost no active power is drawn by the load, this current has also a very small value (of 1.22 A).

![Fig. 8. The grid voltage and current waveforms after the compensation for the total sinusoidal compensation](image)

![Fig. 9. The grid voltage and current waveforms after the compensation for unity power factor compensation](image)

Regarding the numeric values for the two previous cases it can be seen that they are similar, the RMS values being 1.21A and 1.22 A respectively. This is because they are only due to the load and active filter losses, and not related at all to the compensating current computation.

Nevertheless, the obtained results are valid because as it was mentioned the analyzed load is dedicated to test the active filters operation and performances. So, for the considered cases this is the desired result proving the accuracy of the compensating current computation method and the good operation of the entire filtering system.

V. EXPERIMENTAL RESULTS

The experimental waveforms of the compensated current for the three stated compensation goals are illustrated in Fig. 10 and the numerical results are synthesized in the second section of Table I.

It can be observed that the experimental waveforms are similar to the simulation waveforms for all cases. This is also true for the power quality indicators in Table I, whose similar values proves the validity of the filtering system implementation.

<table>
<thead>
<tr>
<th>Case</th>
<th>THDu [%]</th>
<th>THDi [%]</th>
<th>IRMS [A]</th>
<th>FE</th>
</tr>
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<tbody>
<tr>
<td>Partial sin</td>
<td>3.17</td>
<td>77.75</td>
<td>2.00</td>
<td>16.43</td>
</tr>
<tr>
<td>Total sin</td>
<td>3.17</td>
<td>77.75</td>
<td>19.69</td>
<td>16.43</td>
</tr>
<tr>
<td>Unity PF</td>
<td>3.17</td>
<td>77.75</td>
<td>20.17</td>
<td>16.43</td>
</tr>
</tbody>
</table>

TABLE I. NUMERICAL RESULTS

![Annals of the University of Craiova, Electrical Engineering series, No. 38, 2014; ISSN 1842-4805](image)
CONCLUSIONS

Although the performance of the active filtering system based on the proposed implementation of the synchronous fundamental dq frame was proved, some differences between the simulation and the experimental results are visible. This is because the DS1103 control board time step cannot be too low because of the necessary time of the ADCs to convert all the needed signals. If the simulation step would have been adapted to the experimental time step the results would be similar, but the first was kept as low as possible in order to verify the maximum performance which can be obtained by this computation method.

REFERENCES


