Evolution of the Electric Power Components Definitions

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Abstract **– This paper contains a review of the scientific literature published till date in the field of power theory for systems with periodic non-sinusoidal waveforms. Dynamic increase in the number of installed nonlinear loads, that are the sources of higher harmonics in current and voltage waveforms, results in deterioration of electrical energy parameters. Higher harmonics lead to corrupted current and voltage waveforms, hence a much worsened energy quality. The number of power theories and papers concerning these issues give evidence about the importance of the problems of working condition optimisation in power systems.**

Keywords: *powers in nonsinusoidal situations, measurement of electric power quantities under sinusoidal, nonsinusoidal, balanced, or unbalanced conditions, harmonic pollution of power systems.*

I. INTRODUCTION. MILE STONES

1888. W. Stanley [1] was the first engineer who installed a high voltage transmission line using transformers. He also was among the first researchers who understood the correlation between the voltage and current that flows through an inductor.

1894. **E. Houston** and **Arthur E. Kennelly** [2] explained the characteristics of circuits contaminated with harmonics. It is the first English language paper that provides the basic explanations of phenomena caused by current and voltage harmonics. Word "harmonic" was printed and used in the context of Fourier series applied to electrical systems.

1922. F. Buchholtz [3] introduces the concept of Collective Voltage and Current:

$$E_i = \sqrt{E_1^2 + E_2^2 + E_3^2} = \sqrt{\frac{1}{3}\left(E_{12}^2 + E_{23}^2 + E_{31}^2\right)} \quad (1)$$

$$I_a = \sqrt{I_1^2 + I_2^2 + I_3^2} \quad (2)$$

and the effective power

$$N = E_i \cdot I_a \text{, (VA)} \quad (3)$$

1927. C. Budeanu [4] was the first engineer and researcher who realized that in the resolution of apparent power for nonsinusoidal conditions there are additional components we call today "distortion powers".

C. Budeanu was the first scientist who understand the fact that the apparent power in nonsinusoidal systems has more than two components and can be represented in a three-dimensional system.

1932. S. Fryze [5] introduces the concept of active and non-active current.

1933. A. Knowlton [6]. The resolution of apparent power for nonsinusoidal conditions was from the beginning a controversial topic that caused, and is still causing today, passionate discussions. In 1933 A. E. Knowlton chaired the famous AIEE Schenectady meeting that turned into one of the most heated debates in the history of electrical engineering. The discussions were fueled by a set of papers presented by the AIEE elite: C. L. Fortescue, V. G. Smith, W. V. Lyon and W. H. Pratt.

C. L. Fortescue [7]**.** In the 1920s and 1930s the pillars of the electrical engineering community, charmed by the elegance of Fortescue's symmetrical components theory, which fitted hand-in-glove with the vector apparent power definition, dismissed Lyon's recommendation. Symmetrical components help to gain insight into the structure of the effective current and voltage and play a significant role when different apparent power definitions are evaluated, one against the other.

However, symmetrical components alone, without a correct interpretation of the physical mechanisms of energy transmission and conversion, cannot lead to a correct apparent power definition.

Waldo V. Lyon. [8] *For single-phase systems, operating under sinusoidal conditions, the apparent power of a load or a cluster of loads supplied by a feeder is the maximum active power that can be transmitted through the feeder, while keeping the receiving end rms (root mean square) voltage and the feeder variable losses constant.* This definition can also be extended to a source: *The apparent power of a source is the maximum active power that can be supplied, or generated by, the source, while keeping its output voltage and the internal variable power losses constant.*

The above definition was introduced in a modified form by W. V. Lyon in 1920, promoted by A. Lienard in 1926, and later advocated by H. L. Curtis and F. B. Silsbee.

1935. H. Curtis and **F. Silsbee** [8] extend Budeanu's single-phase theory to three-phase systems.

1962. M. Depenbrock [9] introduced the new concept of "powerless" currents, which do not contribute to the collective instantaneous power of a load.

1984 L. Czarnecki [10] explains the limitations of Budeanu's reactive power definition. As a consequence of the use of active current he defines the scattered power, an artificial component without practical consequences.

1986 A. Țugulea [11] [12] [19] [20] demonstrates that the zero- and the negative-sequence components are generated by unbalanced loads that convert positive-sequence energy in negative- and zero-sequence energy.

1993. "The FBD" (Fryze, Buchholtz, Depenbrock) method is a generally applicable tool for analyzing power relations. Using the concept of active current the load current is divided in three components: active; orthogonal; and proportional asymmetrical. The paper [13] is the first scientific work that provides a complete detailed model of three-phase nonsinusoidal unbalanced.

1995 A. E. Emanuel [14] [15] introduce a practical resolution of apparent power. A. E. Emanuel, coordinator of the "Working Group" formed by the IEEE since the early 90s, it is co-author of IEEE Std 1459-2000 [16], and several studies involving new definitions related to the amounts of power under non-sinusoidal conditions, which are based on the proposal of Blondel [17] suggesting that tensions should be measured in relation to a system of conductors [18].

II. Apparent Power. Fundamental Concept

The authors of this study believe that the key to a practical resolution of the apparent power S it is a direct function of its very definition. For single-phase sinusoidal conditions the apparent power has the universally accepted expression:

$$S = V\,I \text{ (VA)}, \tag{4}$$

that according to Andre Lienard can be interpreted as the maximum active power that can be transmitted through a hypothetical feeder to a load in such a manner that the line power loss, $r_S \cdot I^2$, and the load RMS voltage V remain unchanged. This means that the energy conversion at the user's end remains unchanged.

This definition led to the concept of power factor (PF)

$$PF = \frac{\int_0^\tau P\,dt}{\int_0^\tau S\,dt} = \frac{<P>}{<S>} = \frac{W_P}{W_S} \tag{5}$$

The PF is a coefficient that sheds light over the feeder utilization. An excellent interpretation of PF results from the following equation

$$PF = \frac{P}{S} = \frac{VI\cos\theta}{VI} = \sqrt{\frac{r_S\left[I\cos(\theta)\right]^2}{r_S\,I^2}} = \sqrt{\frac{\Delta P_C}{\Delta P}} \tag{6}$$

where ΔP_C feeder power loss after PF compensation to unity and ΔP feeder power loss before PF compensation.

Thus $\frac{\Delta P_C}{\Delta P} = \frac{P^2}{S^2} = (PF)^2$ that leads to what may be a most significant apparent power definition $S = \frac{P}{\Delta P_C / \Delta P}$.

III. Nonsinusoidal Single-Phase Conditions. (IEEE Standard 1459- 2010)

Assuming a nonlinear load supplied with nonsinusoidal voltage and current

$$v = \sum \hat{V}_h \cos(h\omega t + \alpha_h) \tag{7}$$

$$i = \sum \hat{I}_h \cos(h\omega t + \beta_h) \tag{8}$$

with the rms values

$$V = \sqrt{V_1^2 + \sum_{h\neq 1} V_h^2} = \sqrt{V_1^2 + V_H^2} \tag{9}$$

$$I = \sqrt{I_1^2 + \sum_{h\neq 1} I_h^2} = \sqrt{I_1^2 + I_H^2} \tag{10}$$

and using Lienard's approach we obtain for the apparent power

$$S = \sqrt{V^2 I^2} = \sqrt{(V_1^2 + V_H^2)(I_1^2 + I_H^2)} \tag{11}$$

$$S^2 = P_1^2 + Q_1^2 + D_I^2 + D_V^2 + S_H^2 \tag{12}$$

(see Appendix I) where:

$P_1 = V_1 I_1 \cos(\vartheta_1)$ is the 60/50 Hz active power (W),

$Q_1 = V_1 I_1 \sin(\vartheta_1)$; with $\vartheta_1 = \alpha_1 - \beta_1$ is the 60/50 Hz reactive power (VAR),

$D_I = V_1 I_H = S_1\,THD_I$ is the current distortion power (VAR) and

THD_I is the total harmonic distortion of the current and S_1 is the 60/50 Hz apparent power,

$D_V = V_1 I_H = S_1\,THD_V$ is the voltage distortion power (VAR) and

THD_V is the total harmonic distortion of the voltage

$S_H = V_H I_H = S_1\,THD_V\,THD_I = \sqrt{P_H^2 + D_H^2}$ is the total harmonic apparent power,

D_H is the harmonic distortion power and

$P_H = \sum_{h\neq 1} V_h I_h \cos(\vartheta_h)$ is the total harmonic active power.

The apparent power S contains four nonactive terms, however, only the 60/50 Hz, Q_1, can be considered reactive power.

This observation stems from the fact that typical loads are generating or sinking reactive power in form of electromagnetic energy that oscillates between the respective loads (motors or transformers) and one or more supply sources, or even between capacitive and inductive loads. These oscillations do not contribute to net transfer of energy and the oscillations take place at 120/100 Hz. Loads and equipment that depend on a given magnetic flux require to be supplied with the right amount of 60/50 Hz reactive power. The operation of equipment that contains any type of inductors has magnetizing currents

90^{o} out of phase with the supplied voltage, currents sustained by the reactive power that correlates with the magnetic flux. In a transformer the primary and the secondary windings are linked by the magnetic flux quantified by magnetizing currents that together with the supplied voltage define the instantaneous 60/50 Hz reactive power. In a synchronous or induction machine the existence of the rotating magnetic field is due to the three-phase magnetizing currents that are "supported" by the well defined reactive power whose physical mechanism has its roots on the ***Poynting*** Vector theory. In such rotating machines it is the positive sequence rotating field, that is supported by the 60/50 Hz reactive power that sustains the dominant (useful) torque.

All four nonactive powers, $Q_1, D_{I,} D_V$ and D_H cause power loss in the conductors that help supply the loads (Joule, hysteresis, eddy and proximity currents), but while Q_1 is a necessary component of S the others three components are share electromagnetic pollution.

IV. Three-Phase Systems

A three-phase unbalanced load that operates with nonsinusoidal waveforms

$$\begin{aligned} v_a &= \sqrt{2}V_{ah}\sin(h\omega t + \alpha_{ah}) \\ v_b &= \sqrt{2}V_{bh}\sin(h\omega t + \alpha_{bh} - 2\pi/3) \\ vc &= \sqrt{2}V_{ch}\sin(h\omega t + \alpha_{ch} + 2\pi/3) \end{aligned} \tag{13}$$

$$\begin{aligned} i_a &= \sqrt{2}I_{ah}\sin(h\omega t + \beta_{ah}) \\ i_b &= \sqrt{2}I_{bh}\sin(h\omega t + \beta_{bh} - 2\pi/3) \\ i_c &= \sqrt{2}I_{ch}\sin(h\omega t + \beta_{ch} + 2\pi/3) \end{aligned} \tag{14}$$

This section presents the concept of equivalent current and voltage, The equivalent current is a positive-sequence current that supplies the three-phase load with the same energy as the original system.

If the skin effects are neglected the rms effective current I_e, is computed using the power equivalence expression $3r_S I_e^2 = r_S \sum_h (I_{ah}^2 + I_{bh}^2 + I_{bh}^2 + \rho I_{nh}^2)$

$$\begin{aligned} I_e &= \sqrt{\frac{1}{3}\sum_h (I_{ah}^2 + I_{bh}^2 + I_{ch}^2 + \rho I_{nh}^2)} \\ I_e &= \sqrt{I_{e1}^2 + I_{eH}^2} \end{aligned} \tag{15}$$

where I_{nh} is the neutral harmonic current of order h and ρ is the ratio of neutral current path resistance, r_n, over the line current resistance r_S.

$$\begin{aligned} I_{e1} &= \sqrt{\frac{1}{3}\sum_h (I_{a1}^2 + I_{b1}^2 + I_{c1}^2 + \rho I_{n1}^2)} \\ I_{e1} &= \sqrt{(I_1^+)^2 + (I_1^-)^2 + (1+3\rho)(I_1^0)^2} \end{aligned} \tag{16}$$

It is necessary to determine ρ (see Appendix II).

$$I_{eH} = \sqrt{\frac{1}{3}\sum_{h\neq 1} (I_{ah}^2 + I_{bh}^2 + I_{ch}^2 + \rho I_{nh}^2)} \tag{17}$$

The effective voltage is approached in a similar manner. The observed load is assumed to have ungrounded and Δ connected loads dissipating the active power P_Δ; grounded Y loads dissipating the active power P_Y. The power equivalence equation is

$$\begin{aligned} &\frac{3(V_a^2 + V_b^2 + V_c^2)}{R_Y} + \frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{R_\Delta} = \\ &= \frac{3V_e^2}{R_Y} + \frac{9V_e^2}{R_\Delta} \end{aligned} \tag{18}$$

and yields

$$\begin{aligned} V_e &= \sqrt{\frac{3(V_a^2 + V_b^2 + V_c^2) + \xi\left(V_{ab}^2 + V_{bc}^2 + V_{ca}^2\right)}{9(1+\xi)}} \\ V_e &= \sqrt{\left(V^+\right)^2 + \left(V^-\right)^2 + \frac{\left(V^0\right)^2}{1+\xi}} \end{aligned} \tag{19}$$

(see Appendix III) where V^+, V^- and V^0 are the equivalent's voltage symmetrical components. ξ have to be ignored.

The two groups of loads are characterized by the ratio

$$\xi = \frac{P_\Delta}{P_Y} = \frac{9V_e^2 R_Y}{3R_\Delta V_e^2} = \frac{3R_Y}{R_\Delta} \tag{20}$$

The effective power $S_e = 3V_e I_e$ has components similar to single-phase case:

$$S_e^2 = S_{e1}^2 + S_{eN}^2 \tag{21}$$

where:

$S_{e1} = 3V_{e1} I_{e1}$ is the fundamental effective apparent power,

$S_{eN} = \sqrt{S_e^2 - S_{e1}^2} = \sqrt{D_{eI}^2 + D_{eV}^2 + S_{eH}^2}$, and $D_{e1} = 3V_{e1} I_{eH}$ is the current distortion power, $D_{eV} = 3V_{eH} I_{e1}$ is the voltage distortion power, $S_{eH} = 3V_{eH} I_{eH} = \sqrt{P_H^2 + D_{eH}^2}$ is the harmonics apparent power and covers the harmonic active power, P_H, and the harmonic distortion power D_{eH}.

V. Conclusions and Observations

1. It was shown that both the effective voltage V_e and the effective current I_e in the presence of zero sequence components are affected by the coefficients ρ and ξ. However ρ depends on the moisture and temperature of the return path of the neutral current, depends on the skin effect factor and proximity effects. These factors change

with the weather and loading conditions. It is practically almost impossible to track the values of P_Δ and P_Y. Such condition lead to arbitrary values for ρ and ξ, and acceptance of a minor measurement error due to presence of zero-sequence components.

The authors present for discussion and constructive critique two new methods for the computation of the effective apparent power.

If the zero-sequence voltage $V_0 << V^+$ is neglected, then, according to (19) the effect of the parameter ξ can also be ignored. From (19) it will be concluded that the maximum error in computation of the effective voltage V_e takes place when $V^- = 0$ and $\xi = 0$. Thus the maximum relative error is

$$\% \frac{\Delta V_e}{V^+} \le \left[\sqrt{(V^+)^2 + (V^0)^2} - V^+ \right] \cdot 100$$

$$\% \frac{\Delta V_e}{V^+} \le 100 \cdot V^+ \left[\sqrt{1 + \left(\frac{V^0}{V^+} \right)^2} - 1 \right] \quad (22)$$

In actual distribution and transmission systems $V_0 / V^+ < 0.05$.

In Table I are summarized the maximum error $\% \frac{\Delta V_e}{V^+}$ in function of V_0 / V^+:

TABLE I.
MAXIMUM ERROR VALUES

V_0 / V^+	0.01	0.02	0.03	0.04	0.05
$\% \frac{\Delta V_e}{V^+}$	0.005	0.02	0.045	0.08	0.125

This minute error probably can be neglected. This result will be submitted for evaluation and voting by a committee of experts and, if approved, included in the next version of the IEEE Standard 1459.

As concerns the ratio $\rho = r_n / r_s$ it can be measured based on the expression $r_n = \Re\{V_n / I_n\}$ using the very instrument that monitors V^+, V^-, V^0 …

Another possibility starts with the equation:

$$S = \sqrt{\frac{\Delta P}{\Delta P_C}} P \quad (23)$$

that can be expanded to three-phase systems. The powers lost in a feeder can be measured or predicted

$$\Delta P = r_S (I_a^2 + I_b^2 + I_c^2) \quad (24)$$

and

$$\Delta P_C = 3 r_S (I_1^+)^2 < \Delta P \quad (25)$$

2. Fryze's method as well as Czarnecki's and Depenbrock's are using an active current. This is a current with a waveform that is a replica of the voltage wave form. The frequency spectrum of the active current is different than the actual spectrum, the total power associated with he active power has a correct value but the harmonics are fictitious. Is the authors' opinion that the active current approach must be reconsidered.

APPENDIX I

[15] The apparent power is separated into three terms $S^2 = P^2 + Q_B^2 + D_B^2$.

The resolution of $S_e = 3 V_e I_e$ is: the effective power which is separated into two major terms:

$$S_e^2 = S_{e1}^2 + S_{eN}^2 \quad (26)$$

where $S_{e1} = 3 V_{e1} I_{e1}$ is the fundamental, or 60/50 Hz, effective apparent power and the term S_{eN} is the nonfundamental effective apparent power.

In turn S_{eN} has three components

$$S_{eN}^2 = S_e^2 - S_{e1}^2 = D_{eI}^2 + D_{eV}^2 + S_{eH}^2 \quad (27)$$

where:

$D_{eI} = 3 V_{eI} I_{eH}$ is the current distortion power, usually the largest component of S_{eN},

$D_{eV} = 3 V_{eH} I_{e1}$ is the voltage distortion power and

$S_{eV} = 3 V_{eH} I_{eH}$ is the effective harmonic apparent power.

Two components characterize S_{eH},

$$S_{eH}^2 = P_H^2 + D_{eH}^2 . \quad (28)$$

Here

$$P_H = \sum_{h \ne 1} \{ V_{ah} I_{ah} \cos(\theta_{ah}) + V_{bh} I_{bh} \cos(\theta_{bh}) + V_{ch} I_{ch} \cos(\theta_{ah}) \}$$

is the total harmonic active power and

$D_{eH} = \sqrt{S_{eH}^2 - P_H^2}$ is the harmonic distortion power.

The components of S_{eN} can be expressed in function of the equivalent total harmonic distortions:

$THD_{eV} = \frac{V_{eH}}{V_{e1}}$ for voltage and

$THD_{eI} = \frac{I_{eH}}{I_{e1}}$ for current.

Results

$$S_{eN}^2 = \left[\frac{D_{eI}^2}{S_{e1}^2} + \frac{D_{eV}^2}{S_{e1}^2} + \frac{S_{eH}^2}{S_{e1}^2} \right] S_{e1}^2 \quad (29)$$

$$S_{eN}^2 = \left[\frac{(V_{e1} I_{eH})^2}{(V_{e1} I_{e1})^2} + \frac{(V_{eH} I_{e1})^2}{(V_{e1} I_{e1})^2} + \frac{(V_{eH} I_{eH})^2}{(V_{e1} I_{e1})^2} \right] S_{e1}^2 \quad (30)$$

and substitution gives a practical expression

$$S_{eN} = S_{e1}\sqrt{THD_{eI}^2 + THD_{eV}^2 + THD_{eI}^2 THD_{eV}^2} \quad (31)$$

which helps to evaluate separately the contributions of the three terms of S_{eN} to the harmonic pollution;

$D_{eI} = (THD_{eI})S_{e1}$,

$D_{eV} = (THD_{eV})S_{e1}$, and

$S_{eH} = (THD_{eI})(THD_{eV})S_{e1}$.

APPENDIX II

[16] **Effective apparent power (VA).**

This concept assumes a virtual balanced circuit that has exactly the same line power losses as the actual unbalanced circuit. This equivalence leads to the definition of an effective line current I_e (see [21] and [22]).

For a four-wire system, the balance of power loss is expressed in the following way:

$$r(I_a^2 + I_b^2 + I_c^2 + \rho \cdot I_n^2) = 3rI_a^2 \quad (32)$$

where:

r is the line resistance,

I_n is the neutral current (rms value),

$\rho = \frac{r_n}{r}$,

r_n is the neutral wire (or the equivalent neutral current return path) resistance.

From the previous equations, the equivalent current for a four-wire system is obtained.

$$I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2 + \rho \cdot I_n^2}{3}} \quad (33)$$

$$I_e = \sqrt{(I^+)^2 + (I^-)^2 + (1+3\rho)(I^0)^2} \quad (34)$$

In case that the value of the ratio ρ is not known, it is recommended to use $\rho = 1.0$.

For a three-wire system, $I^0 = 0$ and

$$I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2}{3}} = \sqrt{(I^+)^2 + (I^-)^2}\,. \quad (35)$$

In practical systems, ρ . is time dependent. The complicated topology of the power network as well as the unknown neutral path resistance, that is function of soil moisture and temperature, make the correct estimation of ρ nearly impossible. Since the zero-sequence resistance of three-phase lines is larger than the positive sequence resistance, it can be concluded that $\rho > 1.0$, and taking $\rho = 1.0$ will not put the customer at disadvantage when computing I_e (see [23] and DIN 40110-1997).

APPENDIX III

[15] A similar procedure is used to define the equivalent voltage V_e: the compensated hypothetical load has a unity, or close to unity, power factor. This means that only active power is supplied to the line end. The load is separated in Δ-connected loads that are supplied with the active power P_Δ (this includes also the floating neutral Y-connected loads) and the Y -connected loads with the active power P_Y (this includes all the loads connected to neutral). The Δ-connected loads are balanced and characterized by equivalent line-to-line resistances R_Δ. Similarly the Y-connected loads are represented by means of a balanced load with three line-to-neutral resistances R_Y. The equivalence of active power between the actual and the hypothetical system is

$$\frac{3V_e^2}{R_Y} + \frac{9V_e^2}{R_\Delta} = \\ = \frac{\sum_h \left(V_{ah}^2 + V_{bh}^2 + V_{ch}^2\right)}{R_Y} + \frac{\sum_h \left(V_{abh}^2 + V_{bch}^2 + V_{cah}^2\right)}{R_\Delta} \quad (36)$$

The notation $\xi = \frac{P_\Delta}{P_Y} = \frac{9V_e^2 R_Y}{3R_\Delta V_e^2} = \frac{3R_Y}{R_\Delta}$ helps rewrite the equation as follows:

$$\frac{3(1+\xi)}{R_Y}V_e^2 = \\ = \frac{1}{R_Y}\left\{\frac{\sum_h \left(V_{ah}^2 + V_{bh}^2 + V_{ch}^2\right)}{1} + \frac{\sum_h \left(V_{abh}^2 + V_{bch}^2 + V_{cah}^2\right)}{3/\xi}\right\} \quad (37)$$

From here we find the effective voltage

$$V_e = \sqrt{\frac{3\sum_h (V_{ah}^2 + V_{bh}^2 + V_{ch}^2) + \xi\sum_h (V_{abh}^2 + V_{bch}^2 + V_{cah}^2)}{9(1+\xi)}} \quad (38)$$

The separation of fundamental components from the harmonics and interharmonics using $V_e^2 = V_{e1}^2 + V_{eH}^2$ leads to the fundamental effective voltage

$$V_{e1} = \sqrt{\frac{3\left(V_{a1}^2 + V_{b1}^2 + V_{c1}^2\right) + \xi\left(V_{ab1}^2 + V_{bc1}^2 + V_{ca1}^2\right)}{9(1+\xi)}} \quad (39)$$

$$V_{e1} = \sqrt{\left(V_1^+\right)^2 + \left(V_1^-\right)^2 + \frac{\left(V_1^0\right)^2}{1+\xi}} \quad (40)$$

an expression for which the IEEE Std. 1459–2010 recommends $\xi = 1$.

The second term is the harmonic effective voltage

$$V_{eH} = \sqrt{\frac{3\sum_{h\neq 1}\left(V_{ah}^2 + V_{bh}^2 + V_{ch}^2\right) + \xi_h\left(V_{abh}^2 + V_{bch}^2 + V_{cah}^2\right)}{9\left(1+\xi_h\right)}} \quad (41)$$

$$V_{eH} = \sqrt{\sum_{h\neq 1}\left[\left(V_h^+\right)^2 + \left(V_h^-\right)^2 + \frac{\left(V_h^0\right)^2}{1+\xi_h}\right]} \quad (42)$$

$$V_{eH} = \sqrt{V_e^2 - V_{e1}^2}\,. \quad (43)$$

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