Optimal Heating Time for Cylindrical Items Removal from the Shaft

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***Abstract* - For easy removal of cylindrical items from the shaft, they are heated on their external cylindrical surface for thermal dilatation. The best result is obtained if the temperatures gradient inside the item is highest on the separation surface. In the paper, considering a suddenly occurring constant high temperature on the external cylindrical surface of the homogeneous item, the shaft with the same thermal diffusivity and neglecting the contact thermal resistance, the transitory thermal field is analyzed and optimal time for item removal is determined. Example is given.**

Keywords: *bearing racer removal, induction heating, transient thermal field, numerical inverse Laplace transform, temperature penetration depth, characteristic length*

I. INTRODUCTION

For easy removal of cylindrical ferromagnetic items from a shaft, they are heated on their external cylindrical surface causing them to expand. For heating direct flame or induced eddy currents are used. Since at 50 Hz frequency the skin depth is about 1 mm, in both cases a suddenly occurring constant high temperature θ* on the external cylindrical surface of the homogeneous item can be considered.

On the separation from shaft surface a thin air gap δ can be considered and the temperature drop on this gap will be, according to the Fourier law for thermal flux density, as follows:

$$\Delta\theta|_r = \delta \frac{\lambda_1}{\lambda_0} \operatorname{grad}\theta|_r \qquad (1)$$

where λ_1 and λ_0 are the item and air thermal conductivities and r is the internal radius of the item. It results that this temperature drop between item and shaft is maximum when the temperature gradient $\operatorname{grad}\theta|_r$ on the internal surface of the item is maximum. For small Δθ/θ* this air gap can be neglected and temperature gradient on the internal surface of the item can be approximated with the temperature gradient in homogenous cylinder at *r* radius.

In fig. 1 the transitory thermal field is shown in the half infinite homogenous rod (wire) with *A* cross-section and *c*, γ, λ specific heat, density, thermal conductivity, for negligible heat transfer on the external surface of the rod (infinite thermal time constant of the rod), when a temperature step θ* is applied in $x = 0$. In these conditions the temperature in the point *x* will be [1]:

$$\theta(x,t) \approx \theta^{*}\operatorname{Erfc}\left(\frac{x}{2\sqrt{a\,t}}\right); \quad a = \frac{\lambda}{c\,\gamma}; \qquad (2)$$

where *a* is the thermal diffusivity of the rod material.

It can be observed that at fixed distance x_1 from the hot point, when $t = 0$ and $t = \infty$ the |grad θ| = 0, so there is an optimal time when |grad θ| is maximum at given distance from the hot point. In the paper this time is determined, neglecting the contact thermal resistance (item-shaft), i.e. the delay from the temperature step application on the external surface of the item up to the instant when the maximum temperature gradient reaches the separation surface of the item from the shaft.

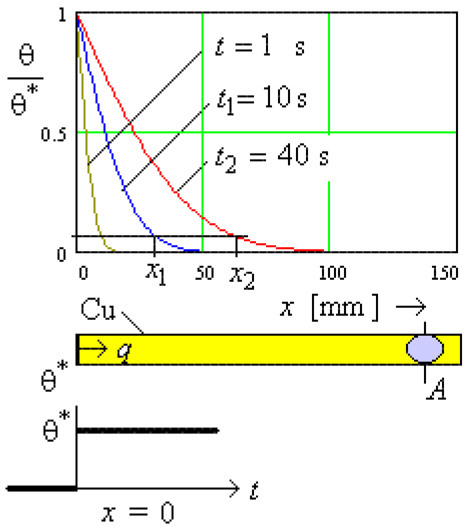

Fig. 1:Temperature distribution in a lossless semi-infinite rod when temperature step is applied at the end

II. ONE DIMENSIONAL MODEL

With given assumptions, the relative temperature θ/θ* inside the cylinder depends only on the radius *r* and the time and, close to the hot external cylindrical surface with radius r_0, is given by (2), for $x = r_0 - r$.

The temperature gradient at *x* distance from the hot surface is:

$$\operatorname{grad}\theta = -\frac{\theta^*}{\sqrt{\pi}\,x_t}\exp\left(\frac{-x^2}{4x_t^2}\right); \quad x_t = \sqrt{a\,t} \qquad (3)$$

Its modulus $|\operatorname{grad}\theta|/\theta^*$ reaches the maximum when the derivative is zero:

$$\frac{\partial}{\partial x_t}\frac{1}{x_t}\exp\left(\frac{-x^2}{4x_t^2}\right) = \exp\left(\frac{-x^2}{4x_t^2}\right)\frac{x^2 - 2x_t^2}{2x_t^4} = 0 \qquad (4)$$

It results the optimal diffusion time for x distance from the hot layer is:

$$x_t = \frac{x}{\sqrt{2}} \Rightarrow t_{opt} = \frac{x^2}{2a} \tag{5}$$

This result is also valid for not negligible, but constant heat transfer coefficient on the surface of the rod, i.e. finite thermal time constant τ.

Really, in this case the temperature can be written as follows [1], [2]:

$$\theta(x,t)\Big| = \frac{\theta^*}{2}\cdot\left[e^{\frac{-x}{x_\tau}}\,\mathrm{Erfc}\left(z-\sqrt{\frac{t}{\tau}}\right)+e^{\frac{x}{x_\tau}}\,\mathrm{Erfc}\left(z+\sqrt{\frac{t}{\tau}}\right)\right] \tag{6}$$

$$x_\tau = \sqrt{a\tau};\quad z = \frac{x}{2\sqrt{at}}$$

The dimensionless temperature gradient at x distance from the hot cross-section of the rod, with τ thermal time constant, results from the following equation:

$$\frac{x\,\mathrm{grad}\,\theta}{\theta^*} = \frac{X}{2}\left[e^{X}\,\mathrm{Erfc}\left(z+\frac{X}{2z}\right)-e^{-X}\,\mathrm{Erfc}\left(z-\frac{X}{2z}\right)\right] - \frac{2z}{\sqrt{\pi}}\exp\left[-\left(z^2+\frac{X^2}{4z^2}\right)\right];\quad X=\frac{x}{x_\tau}=\frac{x}{\sqrt{a\tau}} \tag{7}$$

For given x, z depends only on time, so the modulus $|x\,\mathrm{grad}\,\theta|/\theta^*$ reaches the maximum when its z derivative is equal to zero:

$$\frac{\partial}{\partial z}\left[\frac{x\,\mathrm{grad}\,\theta}{\theta^*}\right] = \frac{2}{\sqrt{\pi}}\exp\left[-\left(1+\frac{X^2}{4z^2}\right)\right]\left(2z^2-1\right)$$
$$2z^2-1=0 \Rightarrow z_{opt} = \frac{x}{2\sqrt{a t_{opt}}} = \pm\frac{1}{\sqrt{2}} \tag{8}$$

This result agrees with (5).

The maximum modulus of dimensionless temperature gradient at x distance from the hot surface (when the time constant τ is not infinite) will be:

$$\left|\frac{x\,\mathrm{grad}\,\theta}{\theta^*}\right|_{max} = \sqrt{\frac{2}{\pi}}\exp\left[-\left(\frac{1+X^2}{2}\right)\right] - \frac{X}{2}\left[e^{X}\,\mathrm{Erfc}\left(\frac{1+X}{\sqrt{2}}\right)-e^{-X}\,\mathrm{Erfc}\left(\frac{1-X}{\sqrt{2}}\right)\right] \tag{9}$$
$$X=\frac{x}{x_\tau}=\frac{x}{\sqrt{a\tau}}$$

The temperature at distance x at optimum time results from (6) and (8):

$$\theta_1(X)\Big|_{t_{opt}} = \frac{\theta^*}{2}\cdot\left[e^{-X}\,\mathrm{Erfc}\left(\frac{1-X}{\sqrt{2}}\right)+e^{X}\,\mathrm{Erfc}\left(\frac{1+X}{\sqrt{2}}\right)\right] \tag{10}$$

The dependence on X of the temperature and of the modulus of the maximum dimensionless temperature gradient at optimal time is given in the next figure.

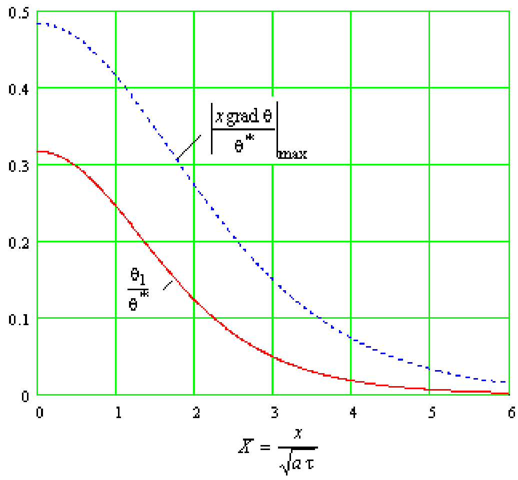

Fig. 2. Maximum temperature gradient and corresponding temperature versus the relative distance from the hot layer

For $X < 0.1$ can be considered $X = 0$ (infinite time constant) and the maximum temperature gradient in K/m inside the rod at x distance from the hot layer will be:

$$|\mathrm{grad}\,\theta|_{max} = \sqrt{\frac{2}{e\pi}}\frac{\theta^*}{x} = 0.484\frac{\theta^*}{x};\quad X<0.1 \tag{11}$$

and the temperature (2) at x distance from the hot layer will be on the t_{opt}:

$$\theta_1 = \theta(x,t_{opt}) \approx \theta^*\mathrm{Erfc}\left(\frac{1}{\sqrt{2}}\right) = 0.317\,\theta^* \tag{12}$$

The equations (5) and (6) can be used also for cylindrical items for $r > 0.85r_0$. For $r < 0.85r_0$ a two dimensional cylindrical must be used.

III. Two Dimensional Axisymmetric Model.

In this case, we will consider a homogenous cylinder at ambient temperature on which external cylindrical surface suddenly a temperature step θ^* is applied.

Due to the symmetry the temperature will be considered as function only of radius and time and the heat transfer equation becomes.

$$\frac{\partial\theta}{\partial t} = \frac{a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\theta}{\partial r}\right)-\frac{\theta}{\tau}$$
$$\tau = \frac{\iiint_V c\,\gamma\,dv}{\oint_S \alpha\,dS} = \frac{mc}{\sum_i \alpha_i S_i} \tag{13}$$

where m is the cylinder (item plus inside shaft) mass, S_i are the cooling surface areas and α_i the corresponding average heat transfer coefficients.

For the Laplace transform of the temperature with respect to the time $T(r, s)$ it results the following modified Bessel equation [1], [3]:

$$\frac{\mathrm{d}^2 T}{\mathrm{d}\xi^2}+\frac{1}{\xi}\frac{\mathrm{d}T}{\mathrm{d}\xi}-T=0$$
$$\xi=\nu r;\ \nu=\sqrt{\frac{1}{a}\left(s+\frac{1}{\tau}\right)} \tag{14}$$

We will consider the temperature of r_0 radius cylindrical surface at $t = 0$ instantly rising from 0 to θ^* and constant after that. At r equal to zero the temperature must remains finite one:

$$\theta(r_0,t)=\theta^*\cdot 1(t)\Rightarrow T(r_0,s)=\frac{\theta^*}{s}$$
$$\theta(0,t)<\infty\Rightarrow T(0,s)<\infty \tag{15}$$

The solution of this problem is expressed by modified Bessel functions of zero order:

$$T(r,s)=\frac{\theta^*}{s}\frac{\mathrm{I}_0(\nu r)}{\mathrm{I}_0(\nu r_0)};\quad r\le r_0 \tag{16}$$

In many cases, on the cooling plane surfaces the heat transfer is relatively very small and the surface heat transfer coefficient α can be considered zero (the thermal time constant τ (13) can be considered infinite).

Since there is not closed form of the original of (16), using the best located nodes [4] it can be enough exact calculated, with the equation:

$$\theta(r,t,\tau)\approx\theta^*\sum_{i=1}^{n}A_i\frac{\mathrm{I}_0\left(\frac{r}{\sqrt{a t}}\sqrt{p_i+\frac{t}{\tau}}\right)}{\mathrm{I}_0\left(\frac{r_0}{\sqrt{a t}}\sqrt{p_i+\frac{t}{\tau}}\right)} \tag{17}$$

The coefficients A_i and the nodes p_i are calculated in [4] with 20 significant digits and approximately given in the next table.

TABLE 1
p AND *A* VALUES [4]

p =

	0
0	0
1	5.225+15.73i
2	5.225-15.73i
3	8.776+11.922i
4	8.776-11.922i
5	10.934+8.41i
6	10.934-8.41i
7	12.226+5.013i
8	12.226-5.013i
9	12.838+1.666i
10	12.838-1.666i

A =

	0
0	0
1	-10.349+4.111i
2	-10.349-4.111i
3	186.327-253.322i
4	186.327+253.322i
5	$-858.652+2.322i\cdot10^3$
6	$-858.652-2.322i\cdot10^3$
7	$1.552\cdot10^3-8.44i\cdot10^3$
8	$1.552\cdot10^3+8.44i\cdot10^3$
9	$-868.461+1.546i\cdot10^4$
10	$-868.461-1.546i\cdot10^4$

We can write simpler this equation using a basic time unit t_b:

$$\frac{\theta(\rho,t^*,\tau^*)}{\theta^*}=\sum_{i=1}^{n}A_i\frac{\mathrm{I}_0\left(\rho\sqrt{\frac{p_i}{t^*}+\frac{1}{\tau^*}}\right)}{\mathrm{I}_0\left(\sqrt{\frac{p_i}{t^*}+\frac{1}{\tau^*}}\right)}$$
$$t^*=\frac{at}{r_0^2}\quad \tau^*=\frac{a\tau}{r_0^2}\quad \rho=\frac{r}{r_0}\quad t_\mathrm{b}=\frac{r_0^2}{a} \tag{18}$$

For infinite time constant the values given by last equation agree with the exact solution given in [3].

$$\theta(\rho,t^*)=\theta^*\left(1-2\sum_{i=1}^{\infty}e^{-\beta_i^2 t^*}\frac{J_0(\rho\beta_i)}{\beta_i J_1(\beta_i)}\right)$$
$$J_0(\beta_i)=0,\quad i=1,2,... \tag{19}$$

where J_0, J_1 are the Bessel functions and β_i the roots of J_0. For several values of t^* the temperature distributions when $\tau^*=\infty$ are shown in fig. 3.

The temperature gradient results from (18) [5]

$$\mathrm{grad}\,\theta=\frac{\partial\theta}{\partial r}=$$
$$\frac{\theta^*}{\sqrt{at}}\sum_{i=1}^{n}A_i\sqrt{p_i+\frac{t}{\tau}}\frac{\mathrm{I}_1\left(\frac{r}{\sqrt{at}}\sqrt{p_i+\frac{t}{\tau}}\right)}{\mathrm{I}_0\left(\frac{r_0}{\sqrt{at}}\sqrt{p_i+\frac{t}{\tau}}\right)} \tag{20}$$

We will write this equation as dimensionless quantity, using the basic time t_b:

$$\frac{r_0\,\mathrm{grad}\theta}{\theta^*}=\sum_{i=1}^{n}A_i\sqrt{\frac{p_i}{t^*}+\frac{1}{\tau^*}}\frac{\mathrm{I}_1\left(\rho\sqrt{\frac{p_i}{t^*}+\frac{1}{\tau^*}}\right)}{\mathrm{I}_0\left(\sqrt{\frac{p_i}{t^*}+\frac{1}{\tau^*}}\right)} \tag{21}$$
$$=G(\rho,t^*,\tau^*)$$

The above quantity is a function of three variables and for given ρ and τ^* can be found an optimal value t^*_{opt} of t^*, using the Minerr function from Mathcad.

If $\tau^* > 100$ for temperature gradient evaluation can be used the equation which results from (19):

$$G_\infty(\rho,t^*)=2\sum_{i=1}^{\infty}e^{-\beta_i^2 t^*}\frac{J_1(\rho\beta_i)}{J_1(\beta_i)}$$
$$\frac{\partial G_\infty(\rho,t^*)}{\partial t^*}=-2\sum_{i=1}^{\infty}e^{-\beta_i^2 t^*}\beta_i^2\frac{J_1(\rho\beta_i)}{J_1(\beta_i)} \tag{22}$$

The dimensionless temperature gradient G_∞ (for $\tau^*=\infty$) versus relative diffusion time is shown in fig. 4 for several values of ρ.

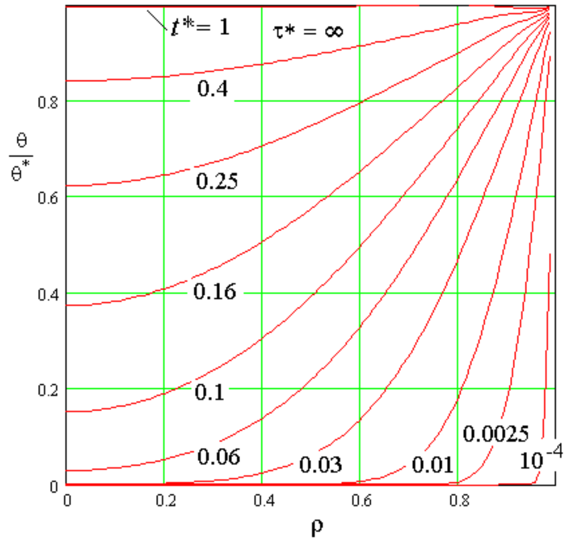

Fig. 3 Temperature distribution in homogenous cylinder with $\tau^*=\infty$ at several times t^* after temperature step application on external cylindrical surface

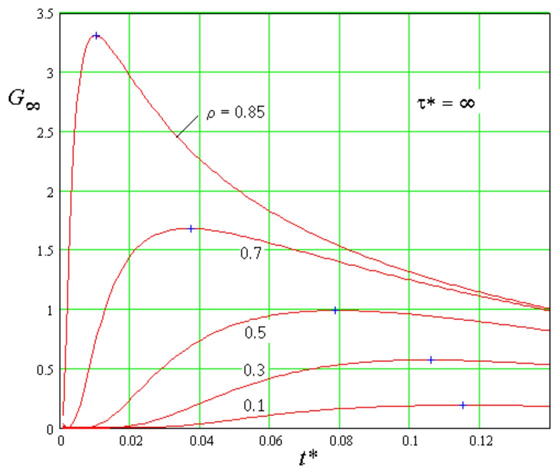

Fig. 4 Dimensionless temperature gradient G_∞ versus relative diffusion time, $\rho = r/r_0$ parameter, + . G_∞ at optimal time (23)

Calculations made for $\tau^* > 0.02$ showed that the optimal time t^*_{opt} is independent on τ^* and depends only on ρ. Example of calculation and the obtained values of t^*_{opt} for two different values of τ^* are given in Annex 1.

The obtained optimal values of the dimensionless diffusion time can be approximated as follows:

$$t^*_{opt}(\rho) \approx \left(\frac{1-\rho}{2.95 - 3.18\rho + 1.68\rho^2} \right)^2; \ \rho = \frac{r}{r_0} \qquad (23)$$

For $r > 0.85r_0$ the one-dimensional model (5) can be applied and a following simpler equation for optimal time relative values can be used:

$$t^*_{opt}(\rho) \approx t^*_{1opt} = \frac{(1-\rho)^2}{2}; \quad \rho = \frac{r}{r_0} > 0.85 \qquad (24)$$

The optimal diffusion time relative values, approximated with (23) and (24) are given in fig. 5.

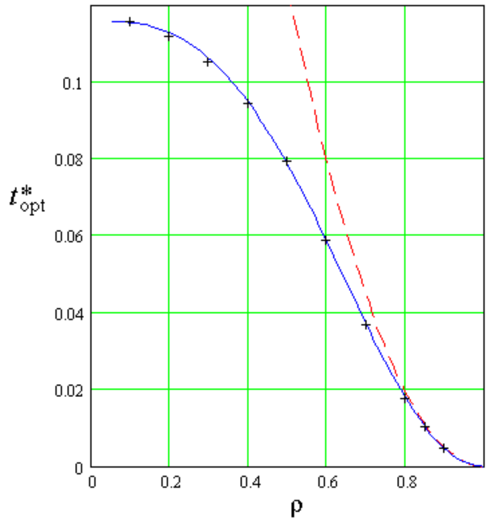

Fig. 5 Optimal heat diffusion time versus $\rho = r/r_0$ (dash line – the one dimensional model, + Table Annex 1 values)

The optimal heat diffusion time in seconds will be:

$$t_{opt} = \frac{r_0^2}{a} t^*_{opt} \quad [s] \qquad (25)$$

where t^*_{opt} is given in fig, 5 or by (23) and for $r > 0.85r_0$

$$t_{opt} \cong \frac{(r_0 - r)^2}{2a}; \quad r > 0.85\, r_0 \qquad (26)$$

The temperature and the temperature gradient on separation surface at this time will be (18), (21) and (23):

$$\theta_1(\rho,\tau^*) = \theta(\rho, t^*_{opt}(\rho), \tau^*)$$
$$\left.\frac{r_0 \operatorname{grad}\theta}{\theta^*}\right|_{max} = G(\rho, t^*_{opt}(\rho), \tau^*) \qquad (27)$$

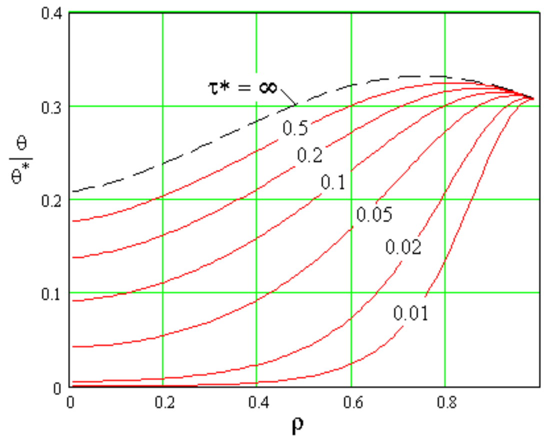

Fig. 6 Average temperature in the separation layer θ/θ^* at optimal time versus r/r_0, τ^* parameter

For one-dimensional model these functions are (12) and (11):

$$\theta_1 = 0.317\,\theta^*$$
$$G(\rho) = \sqrt{\frac{2}{e\pi}\frac{1}{1-\rho}}; \quad \rho = \frac{r}{r_0} \tag{28}$$

The average temperature in the separation layer and its dimensionless gradient are given in Fig, 6 and 7 for several values of thermal time constant

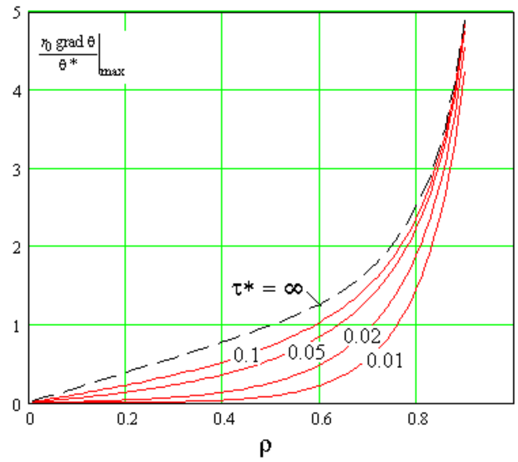

Fig. 7 Maximal dimensionless temperature gradient on separation surface versus. r/r_0, τ^* parameter

IV. Thermal Time Constant

Since the lateral cylindrical surface of the item is maintained at constant temperature, the heat losses will be considered only on the two plane sides. The thermal time constant of such homogenous cylindrical item with the heat transfer coefficients α_1 and α_2 on the two plane sides and with out heat transfer on lateral surface will be:

$$\tau = \frac{m\,c}{\sum_i \alpha_i\,S_i} = \frac{c\,\gamma\,h}{\alpha_1 + \alpha_2}$$
$$\tau^* = \frac{\tau}{t_b} = \frac{\lambda}{\alpha_1 + \alpha_2}\frac{h}{r_0^2} \tag{29}$$

where c, λ and γ are the specific heat, the thermal conductivity and the density of the item material and h the axial length of the item.

The cooling effect of the shift with the constant radius r and the length l_1 can be considered by replacing the heat transfer coefficient α_1 with an equivalent one α_{1e}.

If on the rod surface the heat transfer coefficient is α, taking into account the steady-state temperature variation along the rod (shaft), the equivalent heat transfer coefficient on the item surface occupied by the shaft cross-section results from the equation:

$$\pi\,r^2\alpha_{eq} = 2\,\pi\,r\,\alpha\int_0^{l_1}\frac{\cosh(\nu_1\,(l_1 - x))}{\cosh(\nu_1\,l_1)}\,\mathrm{d}\,x$$
$$\nu_1 = \sqrt{\frac{c\,\gamma}{\lambda\,\tau_1}} = \sqrt{\frac{2\,\alpha}{\lambda\,r}} \tag{30}$$

It results

$$\alpha_{eq} = 2\,\alpha\,\frac{\tanh(\nu_1\,l_1)}{\nu_1\,r} = \sqrt{\frac{2\,\alpha\,\lambda}{r}}\,\tanh(\nu_1\,l_1) \tag{31}$$

The equivalent α_{1e} coefficient will be:

$$\alpha_{1e} = \alpha + \rho^2(\alpha_{eq} - \alpha) \tag{32}$$

V. Conclusions

1. When the temperature step is applied to the lateral external surface of cylindrical item, a delay is necessary for temperature diffusion up to separation layer, up the instant when the temperature gradient at internal surface of the item reaches its maximum (Fig. 4).
2. This optimal heating time, which is independent on item thermal constant, for homogenous cylinder is shown in Fig. 5 and can be evaluated by eq. (23) in relative units and (25) in seconds. For $r > 0.85r_0$ the one dimensional model is more exact and the simpler eq. (24), (26) are recommended.
3. The maximum temperature gradient and corresponding average temperature in the separation layer are shown in Fig. 7 and 6 for cylindrical model and Fig. 2 and eq. (9) (10) for one-dimensional model.

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ANNEX 1

$$to(\rho) := \left(\frac{1-\rho}{2.95 - 3.18\cdot\rho + 1.68\cdot\rho^2}\right)^2$$

$$\rho 1 := (0.1\ \ 0.2\ \ 0.3\ \ 0.4\ \ 0.5\ \ 0.6\ \ 0.7\ \ 0.8\ \ 0.85\ \ 0.9)^T$$

$$y := to(\rho 1)$$

Given

$$\frac{1}{\overrightarrow{G(\rho 1, y, 0.1)}} = 0$$

$$t_{opt}^{\langle 0\rangle} := \text{Minerr}(y)$$

Given

$$\frac{1}{\overrightarrow{G(\rho 1, y, 100)}} = 0$$

$$t_{opt}^{\langle 1\rangle} := \text{Minerr}(y)$$

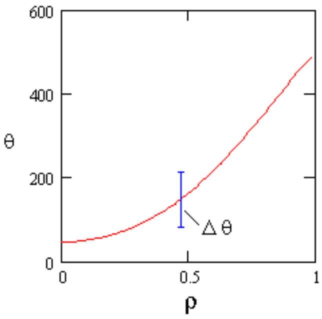
$\rho 1 =$

	0
0	0.1
1	0.2
2	0.3
3	0.4
4	0.5
5	0.6
6	0.7
7	0.8
8	0.85
9	0.9

$t_{opt} =$

	0	1
0	115.707	115.722
1	111.831	111.829
2	105.004	104.996
3	94.492	94.483
4	79.231	79.234
5	58.904	58.926
6	36.833	36.83
7	17.739	17.746
8	10.323	10.332
9	4.729	4.735

10^{-3}



Temperature distribution and temperature drop at optimal time

ANNEX 2

Example

$$r := \begin{pmatrix} 114 \\ 54 \end{pmatrix}\cdot mm \quad a := 12\cdot\frac{mm^2}{sec} \quad \alpha := 15\cdot\frac{W}{m^2\cdot K} \quad \lambda := \begin{pmatrix} 0.032 \\ 52 \end{pmatrix}\cdot\frac{W}{m\cdot K}$$

$$l1 := 0.8\cdot m \quad h := 110\cdot mm \quad \delta := 0.02\cdot mm \quad \alpha d := 11.5\cdot\frac{10^{-6}}{K}$$

$$\theta' := 500\cdot K$$

Equivalent heat transfer coefficient

$$v1 := \sqrt{\frac{2\cdot\alpha}{\lambda_1\cdot r_1}} \quad v1 = 3.269\,m^{-1} \quad \alpha eq := 2\cdot\alpha\cdot\frac{\tanh(v1\cdot l1)}{v1\cdot r_1} \quad \alpha eq = 168.157\frac{W}{m^2\cdot K}$$

$$\rho := \frac{r_1}{r_0} \quad \rho = 0.474 \quad \alpha 1e := \alpha + \rho^2\cdot(\alpha eq - \alpha) \quad \alpha 1e = 49.365\frac{W}{m^2\cdot K}$$

Thermal time comstant

$$\tau' := \frac{\lambda_1}{\alpha 1e + \alpha}\cdot\frac{h}{(r_0)^2} \quad \tau' = 6.838 \quad tb := \frac{(r_0)^2}{a} \quad tb = 1.083\times 10^3\,s$$

$$\tau := \tau'\cdot tb \quad \tau = 2.057\,hr$$

Optimal heating delay

$$t'opt := \left(\frac{1-\rho}{2.95 - 3.18\cdot\rho + 1.68\cdot\rho^2}\right)^2 \quad t'opt = 0.084 \quad topt := t'opt\cdot tb$$

$$topt = 90.505\,s$$

Maximum temperature gradient

$$G(\rho, t, \tau) := \sum_{i=1}^{nn}\left(A_i\cdot\sqrt{\frac{p_i}{t}+\frac{1}{\tau}}\cdot\frac{I1\left(\rho\cdot\sqrt{\frac{p_i}{t}+\frac{1}{\tau}}\right)}{I0\left(\sqrt{\frac{p_i}{t}+\frac{1}{\tau}}\right)}\right)$$

$$G(\rho, t'opt, \tau') = 0.924 \quad grad\theta := 0.924\cdot\frac{\theta'}{r_0} \quad grad\theta = 4.053\frac{K}{mm}$$

Avrage temperature in the separation layer

$$\theta(\rho, t', \tau') := \theta'\cdot\left[\sum_{i=1}^{nn}\left(A_i\cdot\frac{I0\left(\rho\cdot\sqrt{\frac{p_i}{t'}+\frac{1}{\tau'}}\right)}{I0\left(\sqrt{\frac{p_i}{t'}+\frac{1}{\tau'}}\right)}\right)\right] \quad \theta(\rho, t'opt, \tau') = 148.587\,K$$

$$\theta(0, t'opt, \tau') = 46.453\,K$$

Temperature drop and dilaltation

$$\Delta\theta := \delta\cdot\frac{\lambda_1}{\lambda_0}\cdot grad\theta \quad \Delta\theta = 131.711\,K \quad \Delta r_1 := \alpha d\cdot\Delta\theta\cdot r_1 \quad \Delta r_1 = 0.082\,mm$$