Comparison of the Conservative Power Theory (CPT) with Budeanu's Power Theory

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***Abstract* - It is shown in this paper that the Conservative Power Theory (CPT) describes power properties of electrical circuits in a way which has a strong analogy to this description in terms of Budeanu's power theory which misinterprets the power phenomena in such circuits. Also, similarly as Budeanu's power theory, the CPT does not create right fundamentals for the power factor improvement by reactive compensation. Moreover, Budeanu's power theory is no less "conservative" than the Conservative Power Theory, thus both theories can be referred to as conservative ones.**

Keywords: *power definitions, reactive current, reactive power, distortion power, Currents' Physical Components, CPC.*

I. Introduction

The first power theory of electrical systems with nonsinusoidal voltages and currents was suggested [3] by Budeanu in 1927. It was a response to Steinmetz experiment performed in 1892, which challenged the concept of the reactive power [1]. The Conservative Power Theory (CPT), presented in [15] by Tenit and his co-workers in 2003, seems to be the latest attempt aimed at developing such a theory. Comparison of these two power theories is just the subject of this paper.

The power theory of electrical systems is focused on two questions that are fundamental for the electrical engineering: ***(i) why can the apparent power S be higher than the active power P*** and ***(ii) how can this difference be reduced?*** The first question is cognitive in its nature, the second question is practical.

The period of 76 years which separates these two concepts of the power theory was filled with numerous attempts, com-piled in [22], aimed at providing answers to these two questions. In effect of these attempts the difference between the apparent and the active powers, S and P, can be now explained in terms of power related phenomena in electrical loads. Fundamentals of compensation, i.e., reduction of the difference between these two powers, in the presence of distortion and asymmetry were developed as well. Development of the Currents' Physical Components (CPC) – based power theory [9, 16] was crucial for the present state of the knowledge on the power properties of systems with nonsinusoidal and asymmetrical voltages and currents and on compensation in such systems.

The CPC – based power theory was developed in the frequency-domain, i.e., using the concept of harmonics. In this respect the development of the CPC-based theory has followed Budeanu's frequency-domain approach.

The Conservative Power Theory (CPT) has occurred after the development of the CPC – based power theory was almost completed. It seems that development of the CPT was motivated by an old postulate formulated by Fryze [4] in 1931, that the power theory should be formulated in the time-domain, i.e., without any use of the concept of harmonics and such a theory should be based on the load current decomposition into orthogonal components.

The CPT satisfies Fryze's postulates, nonetheless, it describes the power properties of electrical loads in a way, which has a strong analogy to description of these properties in terms of Budeanu's power theory. Unfortunately, as it was demonstrated in papers [11, 12], Budeanu's power theory misinterprets power phenomena in electrical circuits and it does not provide any fundamentals for their compensation. As it will be shown in this paper the same applies to the CPT.

Development of the CPT started in 2003 in paper [15], where mathematical fundamentals of the CPT for single-phase systems were presented with an extension to poly-phase networks. Later the CPT was focused mainly on three-phase systems [17, 19, 20]. It disseminates in electrical engineering and provides CPT – based interpretations of the power related phenomena in electrical systems and fundamentals for their compensation. Unfortunately, as it will be shown in this paper, the power quantities and the load current components introduced by the CPT are not associated with physical phenomena in the load. It applies first of all to the quantity called in the CPT the "reactive energy" W. The same applies to the reactive and void currents as well to the unbalanced current. These new quantities defined in the CPT can con-tribute to major misinterpretations of power phenomena and to erroneous conclusions as to methods of reactive compensators design.

The power theory of single-phase systems with nonsinusoidal voltages and currents developed by Budeanu, introduced a new definition of the reactive power Q, denoted in this paper as Q_B, and introduced a concept of the distortion power D_B to the power theory. This theory has gained almost common acceptance [10] in the electrical engineering community and was supported by some standards, such as [8], [13] or [14]. In 1987 it was challenged in [11], where it was demonstrated that the reactive power Q_B as defined by Budeanu is not associated with the energy oscillation between the load and the supply source. Moreover, it was demonstrated that the distortion power D_B is not associated with the mutual distortion of the load voltage and current. It was also demonstrated that there is no relation between the power factor improvement and reduction of the reactive power

Q_B. Consequently, Budeanu's power theory had to be abandoned for other concepts. It also disappeared from the IEEE 1459 Standard [18].

The CPT, although formulated mathematically in a substantially different way than Budeanu's power theory, shares with that theory the same incapability for explanation of the power related phenomena in electrical systems and the same incapability for providing right fundamentals for compensation. In fact, some conclusions of the CPT, formulated in the time-domain, are identical to those of Budeanu's theory, formulated in the frequency-domain. Moreover, the adjective "*conservative*", which is pivotal for the CPT to such a degree, that it is used in its name, can be applied in the same sense to Budeanu's reactive power Q_B, which does not have any physical interpretation and any practical application. In both cases *conservativeness* has nothing in common [20] with the Law of Conservation of Energy (LCE). The conservation property of the "reactive energy" W in the CPT and the reactive power Q_B in Budeanu's power theory has only mathematical, but not physical fundamentals.

Conclusions on interpretations of very confusing power properties, drawn from studies of real and complex systems, where various phenomena are superimposed, might not be credible. These studies should be done on systems, where the number of different power related phenomena is reduced as much as possible. It means that to be valid and credible in poly-phase systems with a full complexity, these interpretations, definitions and conclusions have to be credible when applied to single-phase and even to purely reactive systems. A statement to be valid in the whole set of power systems has to be valid in every sub-set of such systems. Single-phase and purely reactive loads are just sub-sets of the set of three-phase loads. Therefore, to obtain credible conclusions, this paper investigates how the CPT interprets the power related phenomena in such, strongly simplified systems.

II. "Reactive Energy" W

The reactive current in the CPT is defined as

$$i_{\mathrm{rT}}(t) \overset{\mathrm{df}}{=} \frac{W}{\|\hat{u}\|^2}\hat{u}(t) \tag{1}$$

where

$$W \overset{\mathrm{df}}{=} (\hat{u}, i) \overset{\mathrm{df}}{=} \frac{1}{T}\int_0^T \hat{u}(t) i(t) dt \tag{2}$$

denotes "***a reactive energy***" as defined in the CPT. Symbol (x,y) denotes the scalar product of periodic quantities $x(t)$ and $y(t)$; symbol $\|x\|$ denotes the rms value of $x(t)$, while symbol $\hat{u}$ denotes the unbiased voltage integral:

$$\hat{u}(t) = \int_0^t u(\tau)d\tau - \frac{1}{T}\int_0^T [\int_0^t u(\tau)d\tau]dt\ . \tag{3}$$

The name of quantity W "***a reactive energy***" is written in quotation marks because the quantity W for a capacitor is negative, while energy cannot be negative. Any quantity, even with the energy dimension, that can be negative cannot be regarded as "*energy*". Index "T" in the definition (1) was used in this paper to differentiate the reactive current as defined in the CPT from the reactive currents defined in other power theories.

A new concept of the reactive current $i_{\mathrm{rT}}(t)$, introduced by the CPT as defined by (1), has the physical interpretation entirely founded on the physical interpretation of the "reactive energy" W. Thus, what the "reactive energy" is?

This term does not exist in the first papers on the CPT, meaning in [15] and [17]. Its mathematical definition was provided without any physical interpretation. Its interpretation can be found in [20], namely

> "…*the reactive energy accounts for inductive and capacitive energy stored in the load circuit.*"

To verify this interpretation of the "reactive energy", let us calculate the energy E stored in an ideal LC load, shown in Fig. 1, supplied with a sinusoidal voltage

$$u(t) = \sqrt{2}U\cos\omega_1 t.$$

The energy stored in such a reactive load is

$$E = \frac{1}{2}Li_L^2(t) + \frac{1}{2}Cu^2(t) = \frac{U^2}{\omega^2 L}\sin^2\omega t + CU^2\cos^2\omega t. \tag{4}$$

Now, let us calculate the "reactive energy" W of the same reactive load. The unbiased voltage integral is equal to

$$\hat{u}(t) = \sqrt{2}\frac{U}{\omega}\sin\omega t \tag{5}$$

thus the "reactive energy" W of such a reactive load is

$$W = (\hat{u}, i) = \frac{1}{T}\int_0^T \hat{u}(t)[i_{\mathrm{L}}(t) + i_{\mathrm{C}}(t)]dt = (\frac{1}{\omega^2 L} - C)U^2 \tag{6}$$

This is not the energy E stored, as specified by (4), in the LC load, shown in Fig. 1. Thus the interpretation of the "reactive energy" W, as presented in [20], is not right. It is even more visible at a resonance in that load, when $1/\omega L = \omega C$. At such a condition, the "reactive energy" W is zero, while the energy stored in the load is

$$E = \frac{U^2}{\omega}(\frac{1}{\omega L}\sin^2\omega t + \omega C\cos^2\omega t) = \frac{1}{\omega^2 L}U^2. \tag{7}$$

Doubts about whether the opinion expressed in [20] is right can be strengthened by results of analysis of a purely resistive circuit with a TRIAC, shown in Fig. 2.

At sinusoidal supply voltage

$$u(t) = \sqrt{2}U\sin\omega_1 t$$

the load current at the TRIAC firing angle α has the waveform as shown in Fig. 3.

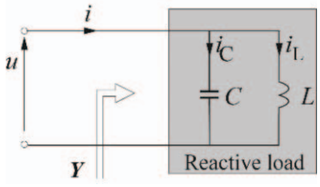

Fig. 1. Ideal reactive load.

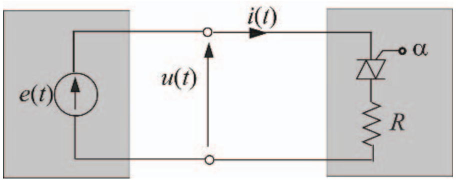

Fig. 2. Resistive load with periodic switch.

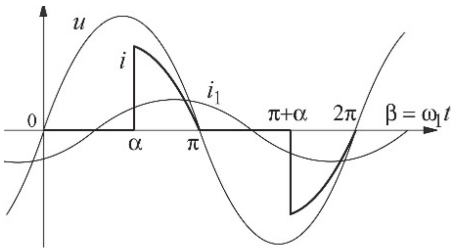

Fig. 3. Voltage, current and the current fundamental harmonic i_1 waveforms in resistive circuit with TRIAC.

The load current in such a circuit can be decomposed into harmonics

$$i(t) = \sum_{k=1}^{\infty} i_n(t) = i_1(t) + \sum_{k=2}^{\infty} i_n(t) \tag{8}$$

with the current fundamental harmonic

$$i_1(t) = \sqrt{2} I_1 \sin(\omega_1 t - \varphi_1) \tag{9}$$

i.e., shifted with respect to the voltage as shown in Fig. 3. The unbiased integral of the supply voltage is

$$\hat{u}(t) = -\sqrt{2}\,\frac{U}{\omega}\cos\omega_1 t \tag{10}$$

and consequently, the "reactive energy" W is equal to

$$\begin{aligned} W &= (\hat{u}, i) = \sum_{n=1}^{\infty} (\hat{u}_n, i_n) = (\hat{u}, i_1) = \\ &= \frac{1}{T}\int_0^T \hat{u}(t) i_1(t) dt = \frac{U I_1}{\omega_1}\sin\varphi_1. \end{aligned} \tag{11}$$

Thus, loads without any capability of energy storage could have a "reactive energy" W. This confirms the previous conclusion that the "reactive energy" W is not associated with the phenomenon of energy storage.

The "reactive energy" W was defined originally by (2) in the time-domain. In such a way the CPT follows Fryze's concept [4] of defining power quantities without any use of harmonics. This confines insight into the meaning of this quantity, however.

Thus, let us express the "reactive energy" W of a purely reactive load in the frequency-domain, assuming that the supply voltage is nonsinusoidal and composed of harmonics of the order n from a set N, namely, that it is equal to

$$u(t) = \sum_{n\in N} u_n(t) = \sqrt{2}\sum_{n\in N} U_n \cos n\omega_1 t. \tag{12}$$

The unbiased integral of such a voltage is

$$\hat{u}(t) = \sum_{n\in N} \hat{u}_n(t) = \sqrt{2}\sum_{n\in N}\frac{U_n}{n\omega_1}\sin n\omega_1 t. \tag{13}$$

A purely reactive load has the admittance for harmonic frequency of the n^{th} order harmonic equal to

$$\boldsymbol{Y}_n = G_n + jB_n = jB_n$$

i.e., with $G_n = 0$. If for the n^{th} order harmonic the load is inductive, then $B_n < 0$ and

$$i_n(t) = \sqrt{2}|B_n| U_n \sin n\omega_1 t\,.$$

If for such a harmonic the load is capacitive, i.e., $B_n > 0$, then

$$i_n(t) = -\sqrt{2}|B_n| U_n \sin n\omega_1 t\,.$$

Therefore, the current of a purely reactive load can be expressed in the form

$$i(t) = \sum_{n\in N} i_n(t) = -\sqrt{2}\sum_{n\in N} \operatorname{sgn}\{B_n\}|B_n| U_n \sin n\omega_1 t. \tag{14}$$

The "reactive energy" W of such a reactive LC load is

$$W = (\hat{u}, i) = \sum_{n\in N} (\hat{u}_n, i_n) = \sum_{n\in N} W_n = -\sum_{n\in N} \operatorname{sgn}\{B_n\}|B_n|\frac{U_n^2}{n\omega_1} \tag{15}$$

Individual terms W_n of this sum can be, depending on the sign of the load susceptance B_n, positive or negative, thus they can cancel mutually. This mutual cancelation of the harmonic "reactive energies" W_n resembles mutual cancellation of harmonic reactive powers Q_n in Budeanu's definition [3] of the reactive power Q_{B}.

$$Q_{\text{B}} = \sum_{n\in N} U_n I_n \sin\varphi_n = \sum_{n\in N} Q_n. \tag{16}$$

This mutual cancellation was of one of the major deficiencies of Budeanu's reactive power [11, 12] definition, for which it was eventually abandoned in the power theory.

Formula (15) for the "reactive energy" W has a strong analogy with definition of the reactive power Q_{B}. This is particularly visible if (16) is rearranged for reactive loads to the form.

$$Q_{\text{B}} = \sum_{n\in N} U_n I_n \sin\varphi_n = -\sum_{n\in N} \operatorname{sgn}\{B_n\}|B_n| U_n^2 \tag{17}$$

Individual terms in Budeanu's definition of the reactive power Q_{B} stand for the amplitude of the energy oscillation at the frequency of individual harmonics, since the bidirectional component of the instantaneous power $p(t)$ of the n^{th} order harmonic is equal to

$$\tilde{p}_n = U_n I_n \sin\varphi_n \sin 2n\omega_1 t = Q_n \sin 2n\omega_1 t. \tag{18}$$

The sum (16) of these amplitudes Q_n, i.e., Budeanu's reactive power Q_{B}, does not specify, as shown in [11], any physical phenomenon in the circuit, however.

Thus the "reactive energy" W, when expressed in the frequency-domain, look a lot like the reactive power suggested at the beginning of the power theory development. In particular, it occurs to be almost identical with the reactive power Q_{I} defined in 1925 [2] by Illovici.

Namely, according to Illovici, the reactive power should be defined as the quantity measured by a wattmeter with the resistor in the voltage branch replaced by an inductor L.

Such a device, assuming that it is ideal and lossless, measures the quantity

$$Q_{\mathrm{I}} = \sum_{n\in N} \frac{1}{n} U_n I_n \sin\varphi_n \tag{19}$$

According to Illovici, just this is one of the quantities that should be regarded as the reactive power at nonsinusoidal supply voltage.

Assuming that the voltage branch is lossless, then at terminals of a purely reactive LC load such an instrument measures the quantity

$$Q_{\mathrm{I}} = \sum_{n\in N} \frac{1}{n} U_n I_n \sin\varphi_n = -\sum_{n\in N} \mathrm{sgn}\{B_n\}|B_n|\frac{U_n^2}{n} = \omega_1 W \tag{20}$$

Thus, Illovici's reactive power Q_{I} and the "reactive energy" W differ mutually only by the dimensional coefficient ω_1. Consequently, there is no physical phenomenon in the load that could be characterized by the quantity W, called in the CPT "a reactive energy".

III. Conservability of "Reactive Energy" W

The "reactive energy" W satisfies the Conservative Property. It means that in any circuit confined by a sphere with zero energy transfer and composed of K branches, as shown in Fig. 4,

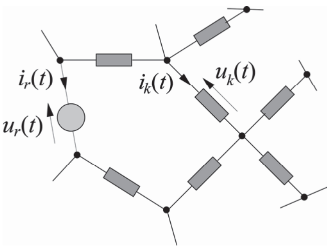

Fig. 4. Circuit with K branches.

the sum of "reactive energies" of individual branches W_k is equal to zero, i.e.,

$$\frac{1}{T}\int_0^T \sum_{k=1}^{K} \hat{u}_k(t) i_k(t) dt = \sum_{k=1}^{K} W_k = 0 \tag{21}$$

This is a very important property. It enables balancing the "reactive energies" and verification of its calculation. Also, if a quantity satisfies the conservative property, this might indicate that this quantity has a physical nature. Such argument was sometimes used in discussions on the physical nature of Budeanu's reactive power. It also satisfies the conservative property, i.e.,

$$\sum_{k=1}^{K}\sum_{n\in N} U_{kn} I_{kn} \sin\varphi_{kn} dt = \sum_{k=1}^{K} Q_{\mathrm{B}k} = 0 \tag{22}$$

The conservative property can be an outcome of one of two more fundamental principles. One of them is the Law of Conservation of Energy (LCE). The second principle is the Tellegen Theorem [5]. According to the LCE, if in any circuit confined by a sphere with zero energy transfer and composed of K branches, and if energy E_k is transferred to the k-branch, then

$$\sum_{k=1}^{K} E_k = \mathrm{Const.} \tag{23}$$

Since the instantaneous power of the k-branch is

$$\frac{dE_k}{dt} = p_k(t) \tag{24}$$

thus the conservative property of the instantaneous power

$$\sum_{k=1}^{K} \frac{dE_k}{dt} = \sum_{k=1}^{K} p_k(t) = 0 \tag{25}$$

is a direct conclusion from the Law of Conservation of Energy.

As emphasized in [15], the conservative property of the "reactive energy" W, with the importance of this property reflected in the name of the Conservative Power Theory, is a conclusion from the Tellegen Theorem.

This Theorem, concluded by Tellegen from Kirchoff Laws in [5], seems to be not commonly known because it was developed not long ago. Since it is crucial for this discussion on the conservative property of the "reactive energy" W, its meaning is explained below.

According to this Theorem, if we have two circuits of the identical topologies, as shown in Fig. 5,

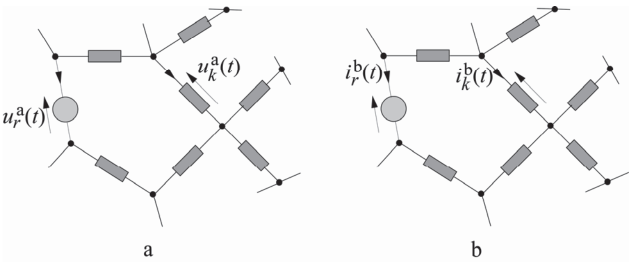

Fig. 5. Two circuits with identical topology.

then the sum of voltage-currents products over all K branches with voltages taken from the circuit in Fig. 5(a) and the currents taken from the circuit in Fig. 5(b) is equal to zero, i.e.,

$$\sum_{k=1}^{K} u_k^{\mathrm{a}}(t)\, i_k^{\mathrm{b}}(t) \equiv 0 \tag{26}$$

The voltage-current products in (26) do no not stand for any physical quantity, however, because voltages are taken from one circuit while the currents are taken from the other one. Nonetheless, such non-physical products have the conservative property. This property is also valid for any integral operations performed on voltages and currents in these two circuits. Therefore, assuming that

$$u_k^{\mathrm{a}}(t) \equiv \hat{u}_k(t), \quad i_k^{\mathrm{b}}(t) \equiv i_k(t) \tag{27}$$

from the Tellegen Theorem (26) we obtain

$$\sum_{k=1}^{K} u_k^{\mathrm{a}}(t) i_k^{\mathrm{b}}(t) = \sum_{k=1}^{K} \hat{u}_k(t) i_k(t) \equiv 0, \tag{28}$$

and hence

$$\frac{1}{T}\int_0^T \sum_{k=1}^{K} \hat{u}_k(t) i_k(t) dt = \sum_{k=1}^{K} < \hat{u}_k, i_k > = \sum_{k=1}^{K} W_k \equiv 0. \tag{29}$$

It means that the conservative property of the reactive energy W does not strengthen arguments for its physical nature. This has a strong analogy with the conservative property of the reactive power as defined by Budeanu.

Budeanu's reactive power Q_{B} can be expressed as demonstrated in [7]

$$Q_{\mathrm{B}} = \sum_{n\in N} U_n I_n \sin\varphi_n = \frac{1}{T}\int_0^T u(t) \mathrm{H}\{i(t)\} dt \tag{30}$$

where

$$\mathrm{H}\{i(t)\} = \mathrm{PV}\frac{1}{\pi}\int_{-\infty}^{\infty} \frac{i(\tau)}{\tau - t} dt \tag{31}$$

is the Hilbert Transform of the load current $i(t)$. Symbol PV denotes the principal value of the integral.

Assuming that in circuits in Fig. 5(a) and (b)

$$u_k^{\mathrm{a}}(t) \equiv u_k(t), \quad i_k^{\mathrm{b}}(t) \equiv \mathrm{H}\{i_k(t)\} \tag{32}$$

then from Tellegen Theorem

$$\sum_{k=1}^{K} u_k^{\mathrm{a}}(t) i_k^{\mathrm{b}}(t) = \sum_{k=1}^{K} u_k(t) \mathrm{H}\{i_k(t)\} \equiv 0. \tag{33}$$

Hence

$$\frac{1}{T}\int_0^T \sum_{k=1}^{K} u_k(t) \mathrm{H}\{i_k(t)\} dt = \sum_{k=1}^{K} \frac{1}{T}\int_0^T u_k(t) \mathrm{H}\{i_k(t)\} dt = \\ = \sum_{k=1}^{K} Q_{\mathrm{B}k} = 0. \tag{34}$$

The conservative property of Budeanu's reactive power Q_{B} is not a consequence of the LCE, i.e., a physical principle, but only the Tellegen Theorem, which is a sort of mathematical, but not a physical property of electrical systems. Consequently, the CPT is no more conservative than Budeanu's power theory. From the fact, that the "reactive energy" W has a conservative property, we should not draw the conclusion that it is a physical quantity. The same was with Budeanu's reactive power Q_{B}.

IV. The Reactive Current $i_{\mathrm{rT}}(t)$

The previous section has demonstrated that the physical interpretation of reactive current $i_{\mathrm{rT}}(t)$ in the CPT cannot be founded on the "reactive energy", since it does not have such interpretation. Thus, what the reactive current $i_{\mathrm{rT}}(t)$ is?

Definition (1) of this current shows that it can be regarded as a current of an ideal inductor, since

$$i_{\mathrm{rT}}(t) = \frac{W}{\|\hat{u}\|^2}\hat{u}(t) = \frac{1}{L_e}\hat{u}(t) \tag{35}$$

where

$$L_e = \frac{\|\hat{u}\|^2}{W}. \tag{36}$$

It means that with respect to the "reactive energy" W at the supply voltage $u(t)$, the purely reactive load is equivalent to an inductor of inductance L_{e}. Such an inductor draws the current $i_{\mathrm{rT}}(t)$ from the supply source.

Since the physical meaning of the "reactive energy" W in the CPT is not clear, not clear is also the physical meaning of the reactive current $i_{\mathrm{rT}}(t)$. Its meaning can be clarified using the Currents' Physical Components (CPC) power theory [16]. Namely, at the supply voltage

$$u(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \boldsymbol{U}_n e^{jn\omega_1 t} \tag{37}$$

the reactive current defined in the CPT is

$$i_{\mathrm{rT}}(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} \frac{1}{jn\omega_1 L_e}\boldsymbol{U}_n e^{jn\omega_1 t}. \tag{38}$$

This is not the reactive current as defined by Shepherd and Zakikhani [6], namely the current

$$i_{\mathrm{r}}(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} jB_n \boldsymbol{U}_n e^{jn\omega_1 t} \tag{39}$$

meaning, the current which occurs in the supply lines due to a phase-shift between the voltage and current harmonics. The current $i_{\mathrm{rT}}(t)$ is only a part of that reactive current $i_{\mathrm{r}}(t)$.

According to the CPT the reactive current $i_{\mathrm{rT}}(t)$ can be compensated entirely by a capacitor connected as shown in Fig. 6.

The "reactive energy" of the capacitor is

$$W_C = (\hat{u}, i_C) = -\sum_{n\in N} \frac{n\omega_1 C}{n\omega_1} U_n^2 = -C\sum_{n\in N} U_n^2 = -C\|u\|^2. \tag{40}$$

Thus a shunt capacitor of capacitance

$$C = \frac{W}{\|u\|^2} \tag{41}$$

compensates the "reactive energy" W entirely. It changes the CPT reactive current $i_{\mathrm{rT}}(t)$ to

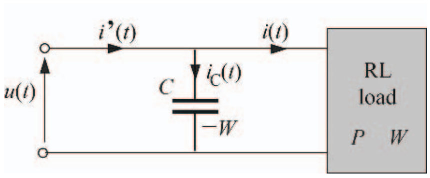

Fig. 6. RL load with a capacitor which compensates the "reactive energy" W.

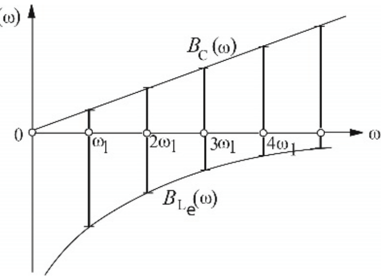

Fig. 7. Change of inductor and capacitor susceptance with harmonic order.

$$i_{rT}^{'}(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} j(n\omega_1 C - \frac{1}{n\omega_1 L_e})\boldsymbol{U}_n e^{jn\omega_1 t}\,. \quad (42)$$

The susceptance of the capacitor C changes with the harmonic order in a different way than the susceptance of the equivalent inductance L_e, as shown in Fig. 7, however.

Thus, reduction of the reactive current $i_{rT}(t)$ does not result from (42), but from reduction of the "reactive energy" W to zero and an increase of the equivalent inductance L_e to infinity. The true reactive current $i_r(t)$, as defined by (39), is not compensated, however. The CPT ignores the fact that the compensating capacitor can affect also the void current.

V. THE VOID CURRENT $i_V(t)$

The load current according to the CPT is composed of the active, reactive and the void currents

$$i(t) = i_a(t) + i_{rT}(t) + i_V(t) \quad (43)$$

where the void current is defined as

$$i_V(t) = i(t) - [i_a(t) + i_{rT}(t)]\,. \quad (44)$$

The void current $i_V(t)$, as defined by (29), is not expressed in terms of voltage and the load parameters, which are specified in the frequency-domain, however, but in the time-domain. The physical meaning of this current is not clear. This meaning can be clarified in the frequency-domain, with the CPC–based power theory.

Since the active current $i_a(t)$ is equal to

$$i_a(t) \stackrel{\mathrm{df}}{=} G_e u(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} G_e \boldsymbol{U}_n e^{jn\omega_1 t}, \quad G_e = \frac{P}{\|u\|^2} \quad (45)$$

while the reactive current $i_{rT}(t)$ is given by (38), thus the void current can be expressed as

$$i_V = i - i_a - i_{rT} =$$
$$= \sqrt{2}\,\mathrm{Re}\sum_{n\in N}[(G_n + jB_n) - G_e - \frac{1}{jn\omega_1 L_e}]\,\boldsymbol{U}_n e^{jn\omega_1 t}. \quad (46)$$

This formula shows that the void current is in fact a compound quantity. It contains in-phase component

$$i_s(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N}(G_n - G_e)\boldsymbol{U}_n e^{jn\omega_1 t} \quad (47)$$

revealed [9] in CPC and called ***scattered current***. It contains also a quadrature component, i.e., composed of current harmonics shifted by π/2 with respect to the voltage harmonics

$$i_{vr}(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} j(B_n + \frac{1}{n\omega_1 L_e})\boldsymbol{U}_n e^{jn\omega_1 t} \quad (48)$$

Thus,

$$i_v(t) = i_s(t) + i_{vr}(t)\,. \quad (49)$$

The quadrature component of the void current has the rms value

$$\|i_{vr}\| = \sqrt{\sum_{n\in N}(B_n + \frac{1}{n\omega_1 L_e})^2 U_n^2}\,. \quad (50)$$

When a capacitor is connected as shown in Fig. 6 to compensate the "reactive energy" W, then the supply current does not contain the reactive current $i_{rT}(t)$. The quadrature component of the void current changes to

$$i_{vr}^{'}(t) = \sqrt{2}\,\mathrm{Re}\sum_{n\in N} j(B_n + n\omega_1 C)\boldsymbol{U}_n e^{jn\omega_1 t}\,. \quad (51)$$

Its rms value changes to

$$\|i_{vr}^{'}\| = \sqrt{\sum_{n\in N}(B_n + n\omega_1 C)^2 U_n^2} \quad (52)$$

Thus capacitive compensation of the reactive current $i_{rT}(t)$ changes the void current rms value. Moreover, this change increases with the harmonic order n. Thus, compensation of the reactive current $i_{rT}(t)$ cannot be separated from its effect on the void current $i_{vr}(t)$ rms value increase. This is illustrated numerically on an example of effects of compensation of the "reactive energy" W of RL load shown in Fig. 8. To have these effects clearly visible, it was assumed that the fifth order harmonic of the supply voltage has the rms value U_5 equal to the fundamental harmonic rms value U_1. It is, of course, unrealistically strong distortion, but we could expect that conclusions of the CPT are valid irrespective of the level of the supply voltage distortion.

At the supply voltage harmonics complex rms (crms) values

$$\boldsymbol{U}_1 = \boldsymbol{U}_5 = 100e^{j0^{\mathrm{o}}}\ \mathrm{V}, \quad \|u\| = 100\sqrt{2}\ \mathrm{V}$$

the crms values of the load current harmonics are

$$\boldsymbol{I}_1 = 70.7e^{-j45^{\mathrm{o}}}\ \mathrm{A}, \quad \boldsymbol{I}_5 = 19.9e^{-j79^{\mathrm{o}}}\ \mathrm{A}, \quad \|i\| = 73.4\ \mathrm{A}$$

so that, assuming that the supply voltage frequency is normalized to $\omega_1 = 1$ rad/s, the "reactive energy", is

$$W = \mathrm{Re}\sum_{n=1,5}\frac{\boldsymbol{U}_n}{jn\omega_1}(\boldsymbol{Y}_n\boldsymbol{U}_n)^* = 0.538\times 10^4\ \mathrm{J}.$$

Capacitance of a shunt capacitor for the "reactive energy" W of the load compensation is equal to

$$C = \frac{W}{\|u\|^2} = 0.269\ \mathrm{F}\,.$$

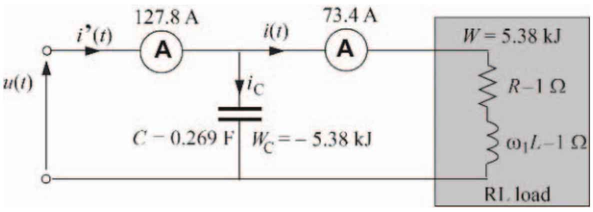

Fig. 8. Results of compensation of the "reactive energy" W of RL load.

The capacitor compensates the "reactive energy" W of the load, but it changes the crms values of the supply current harmonics to

$$\boldsymbol{I}'_1 = 55.1e^{-j24.8^\circ}\ \text{A},\ \boldsymbol{I}'_5 = 115.3e^{j88.0^\circ}\ \text{A},\ \|i'\| = 127.8\ \text{A}$$

The results of compensation of this "energy" are shown in Fig. 8.

The "reactive energy" W of the compensated load is zero, but the compensator increases the void current rms value. Consequently, instead of improving the power factor, it was worsened.

VI. Distortion Power

According to the CPT, the load current of a purely reactive single-phase LC load is composed only of the reactive $i_{\text{rT}}(t)$ current and the void $i_{\text{v}}(t)$ current.

$$i(t) = i_{\text{rT}}(t) + i_{\text{v}}(t) \tag{53}$$

The supply current of a purely reactive load contains neither the active current, as defined in Fryze's power theory [4], nor the scattered current, as defined in the CPC–based power theory [9, 16].

The reactive and void currents are mutually orthogonal, so that their rms values satisfy the relationship

$$\|i\|^2 = \|i_{\text{rT}}\|^2 + \|i_{\text{v}}\|^2 . \tag{54}$$

Multiplying this formula by the square of the supply voltage rms value $\|u\|$, the power equation of reactive loads is obtained. It has the form

$$S^2 = Q_{\text{T}}^2 + D_{\text{T}}^2 . \tag{55}$$

According to [20], the quantity

$$D_{\text{T}} = \|i_{\text{v}}\| \times \|u\| \tag{56}$$

is a distortion power of the load. In some papers on the CPT, such as [17], this quantity is called a void power.

The concept of a distortion power occurred for the first time in Budeanu's power theory. It was defined as

$$D_{\text{B}} \stackrel{\text{df}}{=} \sqrt{S^2 - P^2 - Q_{\text{B}}^2}\ . \tag{57}$$

Indices T and B were used in (55 – 57) to distinguish distortion powers in Budeanu's and in the CPT power theories. Despite having the same name, these are two different quantities.

Distortion power D_{B} is interpreted as a measure of the effect of the voltage and current mutual distortion on the apparent power S of the load. This interpretation was challenged in [11, 12], where it was demonstrated that such interpretation was not right. There is no relation between distortion power D_{B} and the voltage and current mutual distortion.

Let us check whether distortion power D_{T} defined in the CPT is related to the load voltage and current mutual distortion. This is done below with a numerical analysis of a purely reactive load shown in Fig. 9

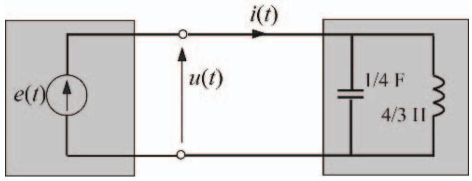

Fig. 9. Circuit with reactive load.

supplied with the voltage:

$$u(t) = \sqrt{2}(100 \sin \omega_1 t + 30 \sin 3\omega_1 t)\ \text{V},\ \ \omega_1 = 1\ \text{rad/s}.$$

The admittances of such a load for the voltage harmonics are $\boldsymbol{Y}_1 = -j1/2$ S and $\boldsymbol{Y}_3 = j1/2$ S. The "reactive energy" W of such load is equal to

$$W = -\sum_{n\in\{1,\,3\}} \text{sgn}\{B_n\}|B_n|\frac{U_n^2}{n\omega_1} = 4.85\ \text{kJ}.$$

Since

$$\|\hat{u}\| = \sqrt{\sum_{n\in\{1,\,3\}} \left(\frac{U_n}{n\omega_1}\right)^2} = \sqrt{\left(\frac{U_1}{\omega_1}\right)^2 + \left(\frac{U_3}{3\omega_1}\right)^2} = 100.50\ \text{V s}$$

the rms value of the reactive current $i_{\text{rT}}(t)$ is

$$\|i_{\text{rT}}\| = \left\|\frac{W}{\|\hat{u}\|^2}\hat{u}(t)\right\| = \frac{|W|}{\|\hat{u}\|} = 48.26\ \text{A}\ .$$

The load current rms value is

$$\|i\| = \sqrt{\sum_{n\in\{1,3\}} (Y_n U_n)^2} = \sqrt{(0.5\times 100)^2 + (0.5\times 30)^2} = 52.20\ \text{A}\ .$$

Since the active current does not exist in this circuit, the rms value of the void current is equal to

$$\|i_{\text{v}}\| = \sqrt{\|i\|^2 - \|i_{\text{rT}}\|^2} = \sqrt{52.2^2 - 48.26^2} = 19.90\ \text{A}$$

so that the distortion power

$$D_{\text{T}} = \|i_{\text{v}}\|\,\|u\| = 19.90 \times 104.40 = 2.08\ \text{kVA}.$$

The load current is equal to

$$i(t) = \sqrt{2}[50 \sin(\omega_1 t - \frac{\pi}{2}) + 15 \sin(3\omega_1 t + \frac{\pi}{2})] =$$

$$= \sqrt{2}[50 \sin \omega_1 (t - \frac{T}{4}) + 15 \sin 3\omega_1 (t - \frac{T}{4})]\ \text{A} =$$

$$= \frac{1}{2} u(t - \frac{T}{4}).$$

The load current is only shifted versus the voltage by $T/4$, as shown in Fig. 10. In spite of non-zero distortion power D_{T}, the voltage and current are not mutually distorted.

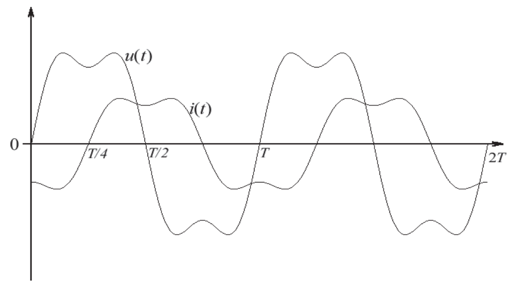

Fig. 10. Waveforms of the voltage and current.

It demonstrates that there is no relation between distortion power D_T and distortion of the load current with respect to the supply voltage.

This conclusion has a strong analogy to the conclusion on the distortion power D_B in Budeanu's power theory. Both in the CPT and in Budeanu's power theory, the name "distortion power" of D_B and D_T quantities suggests a relationship between these powers and the voltage and current mutual distortion. There is not such a relationship between these powers and the voltage and current distortion, however. The concept of these powers in both cases contributes to misinterpretation of power related phenomena in systems with nonsinusoidal voltage.

VII. CONCLUSIONS

It was demonstrated in this paper that the Conservative Power Theory occurs to be a sort of return to its initial phase, to Budeanu concept. Although, unlike Budeanu's power theory, it is formulated in the time-domain and generalized to unbalanced three-phase loads, it has all deficiencies of Budeanu's power theory. The CPT follows Fryze's approach to power theory, meaning it is based on the current orthogonal decomposition, but repeats some of its deficiencies. Namely, just as Fryze's concept did not explain the physical meaning of the reactive current, $i_{rF}(t)$, the CPT also does not provide physical interpretation of the reactive current $i_{rT}(t)$, because the "reactive energy" W is not a physical quantity. Consequently, the void current $i_v(t)$ also does not have any physical meaning. It is associated in the CPT with distortion power D_T, but similarly as it was with Budeanu's distortion power D_B, there is no relationship between distortion power D_T and the voltage and current mutual distortion. It means that the Conservative Power Theory misinterprets power related phenomena in electrical circuits. Moreover, the Conservative Power Theory is no more "conservative" than Budeanu's power theory.

Like Budeanu's power theory the CPT does not provide right fundamentals for reactive compensation, because compensation of the reactive current $i_{rT}(t)$ as defined in the CPT can change the rms value of the void current and consequently, its compensation, as shown in this paper, can increase the supply current rms value, thus degrade the power factor.

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