Load Flow Analysis of Unbalanced Distribution Networks using Symmetrical Components Based Software

Denisa Rusinaru*, Leonardo Geo Manescu*, Marian Ciontu*, and Miron Alba†

* University of Craiova/Faculty of Electrical Engineering, Craiova, Romania, drusinaru@elth.ucv.ro

† CEZ Romania/CEZ Distribution, Craiova, Romania, Miron.Alba@cez.ro

Abstract - **This paper presents the algorithm and the main features of a Matlab (R2014a)-based software conceived by the authors for analysis of the asymmetrical operation of the three-phase distribution networks. The software's algorithm is based on an iterative procedure of Newton-Raphson type. This algorithm is independently applied on three equivalent sequence networks and includes an equivalent model for the unbalanced loads supplied from the analysed distribution grid. In order to outline the capabilities of this software tool named PFASYM and illustrate the key concepts a comprehensive case study was considered. This one summarizes the results of a three-phase load flow analysis performed for a real representative 110/20 kV distribution network supplying industrial unbalanced loads. The PFASYM's results are validated by comparison with those generated by a proprietary software package. The analysis methodology used in this illustrative case study and the results generated for different grid configurations could assist the grid operator in conducting the power flow studies of operational areas of interest. This work is part of an overall energy systems modelling and analysis project developed by the authors, in collaboration with the local power distribution operator. This one has as objective to facilitate the understanding of the power flow concepts for the present distribution networks operating in asymmetry conditions and to assist the grid operator in applying optimal mitigation measures.**

Keywords: *symmetrical components; three-phase load flow; unbalanced load; asymmetry factor.*

I. Introduction

The purpose of the three-phase analysis is to give the proper information required by the control and planning activities of the present power networks, whose operation is strongly affected by electromagnetic perturbations amplified by the extending renewable generation units. Therefore, the power networks' operators are nowadays constrained to highly consider not only safety, continuity or economic issues in their activity and in relation with customers, but also the power quality.

The large number of single phase loads connected in the distribution network, as well as the very numerous variable speed drives in industry, cause the unbalance in the line voltages at terminals in the absence of a proper line voltage regulation. Since the asymmetry of the line voltages causes poor performance of the three-phase loads and an unbalanced real and reactive power demand, the evaluation of the asymmetrical operation of the power grids, location of the asymmetry sources and evaluation and mitigation of their effects became an essential concern for the power grids' operators.

Based on these facts, a proper adjusted analysis should be performed in order to take into consideration the differences between phase values for the networks operating in asymmetrical conditions and/or supplying unbalanced loads.

So that, this task might be ensured by applying different analysis algorithms, described usually with phase components or associated to other different types of components. Tars, El-Abiad, Birt and Graffy have investigated the three-phase load flow since 1970' [1], being followed by Arrilaga and Harker [2]. There were also registered probabilistic approaches [3]. But no matter the state values representation, their non-linear dependency asks for iterative type algorithm – e.g. Gauss-Seidel, Newton-Raphson and others, whose application should take into account the different behavior and values between the phases of the unbalanced networks [4, 5, 6, 7].

The principle was applied and further developed by the authors for a three-phase load flow model, which became the core of a Matlab (R2014a)-based power system analysis and simulation tool, an in-house developed software named PFASYM [8].

II. Particularities of the Load Flow Algorithm for Three-phase Analysis of Power Grids

The principle of the power flow algorithms for analysis of unbalanced networks - PFASYM is mainly similar to those applied for one-phase approach. There are some differences required by the new level of information [5, 8].

a. The conventional PV or PQ-buses with symmetrical loads are counted further on up or down. The last indexes are assigned to the buses with unbalanced loads.

b. Power mismatches of the network buses ΔP, ΔQ are defined similarly to the symmetrical conventional case, since the nodal admittance matrix includes components describing all three phase and couplings between them, in phase terms or symmetrical components.

c. The output results contain three-phase information.

d. The voltage-controlled (generator) bus model has certain particularities describe as following.

The generator bus model includes three bus types, as in Fig.1 [1, 8]:

i. INT bus – PV type: it is an internal bus where the machine's total power ($P_A+P_B+P_C$) is injected and represents the induced internal voltage;

ii. BG bus – PQ type: it represents the terminal of the generator;

iii. P bus (optional) – PQ type: the high voltage bus at which the power transmission system is connected; it is approached as a usual PQ bus.

For those generators located in the slack buses the A-phase voltage of the BG-bus is constant and assumed as reference for phase of other buses' voltages.

If the algorithm is written in terms of symmetrical components, there are some additional particularities aiming to simplify the computing process, given as in Table I.

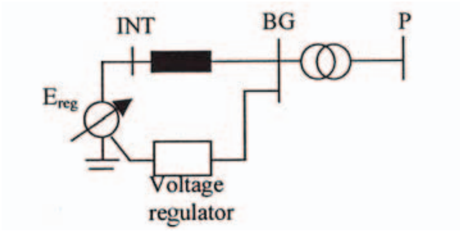

Fig. 1. Generator model for three-phase load flow analysis.

TABLE I.
CHARACTERISTICS OF THE GENERATOR BUS MODEL

Algorithm type	**Buses of generator model**	**Characteristic equations**	**State values**
Three-phase representation	***INT***	- Voltage at regulator terminals $\underline{E}_{reg}$ - Active power of generators, P_g	A-phase voltage - $\underline{E}^{A}_{INT}$
	BG	- Phase bus powers (P_a, P_b, P_c, Q_a, Q_b, Q_c)	$\underline{U}^{A}_{BG},\underline{U}^{B}_{BG},\underline{U}^{C}_{BG}$
	P	- Phase bus powers (P_a, P_b, P_c, Q_a, Q_b, Q_c)	$\underline{U}^{A}_{P},\underline{U}^{B}_{P},\underline{U}^{C}_{P}$
Symmetrical components (+, -, 0)	***INT***	- Bus voltage phasor $\underline{E}^{+}_{reg}$ - Bus active powers P^{+}_{g}	- Argument of positive sequence voltage - δ^{+}_{INT} - Positive sequence reactive power - Q^{+}
	BG	- Bus complex power $\underline{S}$ in sequence circuits	$\underline{U}^{+}_{BG},\underline{U}^{-}_{BG},\underline{U}^{0}_{BG}$

The equations describing the unbalanced operation of the power networks are:

- The power balance equations for phase circuits or equivalent sequence components circuits;
- The slack bus voltage;
- The complex power balance for all the network's buses.

The output data of the analysis are:

- The phase or symmetrical components of the voltages in the network's buses;
- The phase or phase or symmetrical components of the currents in the network's buses;
- The phase or phase or symmetrical components of the powers in the slack buses.

III. THE LOAD FLOW SOLUTION INDEPENDENTLY APPLIED ON THE EQUIVALENT SEQUENCE CIRCUITS OF THE ASYMMETRICAL POWER GRIDS

The significant advantages of the symmetrical components justify the frequent utilization of the load flow problem decomposition into three subsequent problems. These ones correspond to the positive, negative and zero sequence equivalent circuits, with a reduced mutual coupling between them [6].

The symmetrical components solution for the three-phase load flow has proven good performances meaning a low computing time, low computer storage requirements, stable convergence no matter the asymmetry level, as well as the validation for the mono-phase representation of the correspondent symmetrical network.

The differences between the one-phase load flow problem and the three-phase case refer to:

- the size of the bus admittance matrix;
- the number and type of variable;
- the value field of input data for the positive sequence equivalent circuit;
- the dependence of the bus voltages in each sequence circuit on all the symmetrical components of the bus voltages resulted in the previous iteration.

In addition a model of the unbalanced loads was proposed and further integrated in the three-phase load flow, allowing running independently the solution on the three sequence circuits of the studied network [9].

A. The Bus Types

The three-phase load-flow problem describes the static unbalanced operating conditions of a power system with respect to power and/or voltage constraints in the network buses. Generally, the buses are classified as in the conventional symmetrical cases, as slack/swing buses, PV buses, and PQ buses.

It should be specified that a INT-type bus will be introduced for each generator, to the back of its equivalent internal impedance, as in Fig. 1. Since the internal phase voltages are symmetrical, these buses will not be contained obviously in the negative and zero sequence equivalent circuits of the network.

The unbalanced loads will be introduced as PQ buses with non-zero power consumptions only in the positive equivalent circuit. The negative and zero sequence circuits will be connected precisely in these buses through the equivalent phase-to-phase impedance $\underline{Y}_m$, which describes the unbalance of the load accordingly to the model given in the following section.

Details of the bus classification are given in Table II.

TABLE II.
BUS DETAILS FOR THE THREE-PHASE LOAD FLOW PROBLEM

Bus type	**Number of buses**		
	Positive sequence	***Negative sequence***	***Zero sequence***
PQ or Load Bus	$N_l+N_u+N_g+N_{sl}$	$N_l+N_u+N_g+N_{sl}$	*Max* $N_l+N_u+N_g+N_{sl}$
Generator Bus (PV ≈ INT)	N_g		

Bus type	Number of buses		
	Positive sequence	***Negative sequence***	***Zero sequence***
PQ or Load Bus	$N_l+N_u+N_g+N_{sl}$		
Slack Bus (INT)	N_{sl} *(min 1)*		
TOTAL	N_+	N-	N_0

TABLE II. (CONTINUATION)

with notations:

$N_+ = N_l + N_u + 2N_g + 2N_{sl}$ – the total bus number of the positive sequence circuit of the studied network;

$N_- = N_l + N_u + N_g + N_{sl}$ – the total bus number of the negative sequence circuit;

$N_0 \leq N_l + N_u + N_g + N_{sl}$ **-** the total bus number of the zero sequence equivalent circuit and depends on the configuration type of the power transformers;

N_{sl} – the number of the slack buses;

N_g – the number of the generator INT buses;

N_l – the number of the buses with symmetrical loads;

N_u – the number of the buses with unbalanced loads.

B. The Equivalent Phase-to-phase Admittance Model of the Unbalanced Loads

The model used for the unbalanced loads in the load flow problem is given in Fig. 2. Basically this one includes two equivalent loads: a three-phase perfectly balanced load model (three phase admittances equal to $\underline{Y}_e$) and a phase-to-phase model (an admittance connected between two phases of the network, $\underline{Y}_m$), which takes over the level of the load unbalance [9, 10].

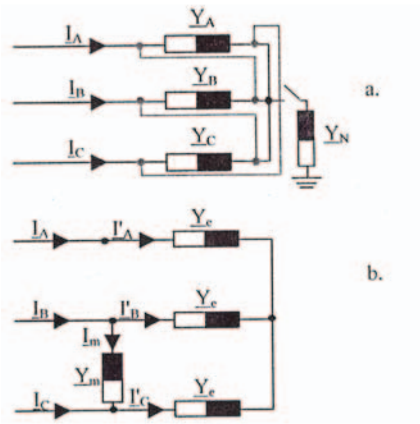

Fig. 2. The unbalanced load model: a. three-phase representation of the load; b. equivalent phase-to-phase admittance load model.

There is a correspondence between the two load representations, with matriceal terms defined by the connection type:

$$[\mathbf{I}_n] = [\mathbf{Y}_{n,unb}][\mathbf{U}_n] = [\mathbf{Y}_{n,eqv}][\mathbf{U}_n] \tag{1}$$

with:

$[\mathbf{I}_n]$, $[\mathbf{U}_n]$ – the column vector of the currents, respectively voltages at the model terminal buses;

$[\mathbf{Y}_{n,unb}]$ – the bus admittance matrix of the unbalanced load;

$[\mathbf{Y}_{n,eqv}]$ – the bus admittance matrix of the equivalent phase-to-phase admittance unbalanced load model.

For the symmetrical component representation, the correspondent circuit (see Fig. 3) and equations are given as following:

$$[\mathbf{I}_{sim}] = [\mathbf{Y}_{sim}][\mathbf{U}_{sim}] \tag{2}$$

with:

$[\mathbf{I}_{sim}]$, $[\mathbf{U}_{sim}]$ – the column vector of the symmetrical components of the currents, respectively voltages of the equivalent load model;

$[\mathbf{Y}_{sim}]$ – the sequence admittance matrix of the equivalent unbalanced load model.

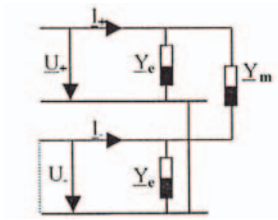

Fig. 3. Sequence circuits connection for the equivalent phase-to-phase admittance load model.

C. The Bus Admittance Matrix

A bus admittance matrix can be computed for each sequence network as following:

i. The sequence admittances of the network components are determined.

ii. The bus admittance submatrices of each network's sequence circuit are determined according to the physical links between the network components.

iii. The resulting parameters will be organized according to the nature and position of the symmetrical components into the global symmetrical components admittance matrix.

$$[\mathbf{Y}_{sim}] = \begin{bmatrix} \mathbf{Y}^{00} & \mathbf{Y}^{0+} & \mathbf{Y}^{0-} \\ \mathbf{Y}^{+0} & \mathbf{Y}^{++} & \mathbf{Y}^{+-} \\ \mathbf{Y}^{-0} & \mathbf{Y}^{-+} & \mathbf{Y}^{--} \end{bmatrix} \tag{3}$$

where $\boldsymbol{Y}^{ij}$ are the symmetrical components admittance submatrices, with i, j = 0, +, - denoting the index of the symmetrical component.

The resulting bus admittance matrix has some features that should be taken into account:

- the off diagonal submatrices (describing the mutual connection between the sequence circuits) are generally empty;

- its parameters are non-zero only for the buses with unbalanced loads (e.g. $[\mathbf{Y}^{+-}]$, $[\mathbf{Y}^{-+}]$ of the model described in the previous section), as well as those at the ends of asymmetrical transmission lines.

D. The Load Flow Algorithm Equations

A rough outline of solution of the power-flow problem supposes firstly to make an initial guess of all unknown bus voltage magnitudes and angles [8].

The initial bus voltages for the sequence circuits of the analyzed network $U^{s(0)} = \lfloor \underline{U}_1^{s(0)} \quad \underline{U}_2^{s(0)} \quad \ldots \quad \underline{U}_N^{s(0)} \rfloor$ are commonly chosen as:

$$\underline{U}_k^{+(0)} = \begin{cases} U_{sp}\angle 0^0 & k = 1,\ldots,N_g \\ 1\angle 0^0 & k = N_g+1,\ldots,N_c, N_c+1, N_+ \end{cases} \tag{4}$$

$$\underline{U}_k^{-(0)}, \underline{U}_k^{0(0)} = 0\angle 0^0 \tag{4'}$$

where s denotes the index of the symmetrical components 0, +, -;

$\underline{U}_{sp}$ – the specified initial value of the bus voltage.

The former experience has proven that such a choice is still undesirable, since the solution does not reach the convergence for some operational conditions. Instead of this, the results of load flow in the balanced network can be used as input data in the positive sequence circuit.

Further the powers' balances for the network phases and sequences are solved using the most recent iteration values of voltage angles and magnitudes, as in further equations:

$$\underline{S}_k = \underline{S}_{k,g} - \underline{S}_{k,cons} = \underline{S}_k^A + \underline{S}_k^B + \underline{S}_k^C = P_k + jQ_k \tag{5}$$

$$P_k = P_k^A + P_k^B + P_k^C ; Q_k = Q_k^A + Q_k^B + Q_k^C \tag{5'}$$

$$\underline{S}_k = 3\left(\underline{U}_k^0 \underline{I}_k^{0*} + \underline{U}_k^+ \underline{I}_k^{+*} + \underline{U}_k^- \underline{I}_k^{-*}\right) \tag{6}$$

where $\underline{S}_{k,g}$, $\underline{S}_{k,cons}$ are the complex generated/consumed power in k-th bus and the symmetrical components of voltage given by (7) for the successive iterations:

$$[\mathbf{Y}^{00}][\mathbf{U}^0] = [\mathbf{I}^0] - [\mathbf{Y}^{0+}][\mathbf{U}^+] + [\mathbf{Y}^{0-}][\mathbf{U}^0] \tag{7}$$

$$[\mathbf{Y}^{++}][\mathbf{U}^+] = [\mathbf{I}^+] - [\mathbf{Y}^{+0}][\mathbf{U}^0] + [\mathbf{Y}^{+-}][\mathbf{U}^-] \tag{8}$$

$$[\mathbf{Y}^{--}][\mathbf{U}^-] = [\mathbf{I}^-] - [\mathbf{Y}^{-0}][\mathbf{U}^0] + [\mathbf{Y}^{-+}][\mathbf{U}^+] \tag{9}$$

Taking into consideration their physical significance and weights by comparison with the other symmetrical components, the positive sequence values given by (8) will be considered as reference for developing the load flow solution.

The solution of the previous equation system associated to the load flow problem is reached by applying an iterative procedure to (8), (9) and (7), with respect to this order. For the positive sequence circuit, the iterated bus powers are given by (10):

$$\underline{S}_k^+ = \underline{U}_k^+ \left(\sum_{n=1}^{N+} \underline{Y}_{kn}^{++} \underline{U}_n^+ \right)^* + \underline{U}_k^+ \left(\sum_{n=1}^{N0} \underline{Y}_{kn}^{+0} \underline{U}_n^0 + \sum_{n=1}^{N-} \underline{Y}_{kn}^{+-} \underline{U}_n^- + \right)^* = \tag{10}$$
$$= \frac{1}{3}\left(\underline{S}_{k,g} - \underline{S}_{k,cons}\right) - \underline{U}_k^0 \underline{I}_k^{0*} - \underline{U}_k^{-*} \underline{I}_k^{-*}$$

E. The Load Flow Algorithm Steps

The load flow study involves the following steps:

1. The state values of the positive sequence circuit are iteratively calculated according to a Newton-Raphson algorithm [2, 8], described by (11):

$$\begin{bmatrix} \mathbf{J}_1(i) & \mathbf{J}_2(i) \\ \mathbf{J}_3(i) & \mathbf{J}_4(i) \end{bmatrix} \begin{bmatrix} \Delta\delta(i) \\ \Delta\mathbf{U}(i) \end{bmatrix} = \begin{bmatrix} \Delta\mathbf{P}(i) \\ \Delta\mathbf{Q}(i) \end{bmatrix} \tag{11}$$

with:

$\mathbf{J}_1$, $\mathbf{J}_2$, $\mathbf{J}_3$, $\mathbf{J}_4$ – the Jacobian term at the *i-th* iteration;

$\Delta\mathbf{U}$, $\Delta\delta$ – the magnitude, respectively phase of the bus voltages at the *i-th* iteration;

$\Delta\mathbf{P}$, $\Delta\mathbf{Q}$ – the bus power mismatches at the *i-th* iteration.

2. The power mismatches having the initial values set according to (10) are given in Table III, while the Jacobian components are summarized in Table IV.

TABLE III.
BUS POWER MISMATCHES

Mismatch type	**Equation**	**Number of equations**
ΔP_k	$\frac{1}{3}\left(P_{k,g} - P_{k,cons}\right) - U_k^+ \sum_{n=1}^{N+}\left[U_n^+\left(G_{kn}^{++}\cos\delta_{kn}^+ + B_{kn}^{++}\sin\delta_{kn}^+\right)\right] - U_k^+\left[\left(\sum J_{\mathrm{Re},k}\right)\cos\delta_k^+ - \left(\sum J_{\mathrm{Im},k}\right)\sin\delta_k^+\right]$ (12)	$N_+ - N_{sl}$
ΔQ_k	$-\left(Q_{k,g}\right) - U_k^+ \sum_{n=1}^{N+}\left[U_n^+\left(G_{kn}^{++}\sin\delta_{kn}^+ - B_{kn}^{++}\cos\delta_{kn}^+\right)\right] - U_k^+\left[\left(\sum J_{\mathrm{Re},k}\right)\sin\delta_k^+ + \left(\sum J_{\mathrm{Im},k}\right)\cos\delta_k^+\right]$ (13)	$N_+ - N_g - N_{sl}$

with:

$$J_{\mathrm{Re},k} \overset{def}{=} J_{\mathrm{Re},k}^{+0} + J_{\mathrm{Re},k}^{+-}, \quad \sum J_{\mathrm{Im},k} \overset{def}{=} J_{\mathrm{Im},k}^{+0} + J_{\mathrm{Im},k}^{+-} \tag{14}$$

$$J_{\mathrm{Re},k}^{+0} \overset{def}{=} \mathrm{Re}\left(\sum_{n=1}^{N0} \underline{Y}_{kn}^{+0} \underline{U}_n^0\right)^* = f_1\left(\mathbf{U}^0, \delta^0\right) \tag{15}$$

$$J_{\mathrm{Im},k}^{+0} \overset{def}{=} \mathrm{Im}\left(\sum_{n=1}^{N0} \underline{Y}_{kn}^{+0} \underline{U}_n^0\right)^* = f_2\left(\mathbf{U}^0, \delta^0\right) \tag{15'}$$

$$J_{\mathrm{Re},k}^{+-} \overset{def}{=} \mathrm{Re}\left(\sum_{n=1}^{N-} \underline{Y}_{kn}^{+-} \underline{U}_n^-\right)^* = f_3\left(\mathbf{U}^-, \delta^-\right) \tag{16}$$

$$J_{\mathrm{Im},k}^{+-} \overset{def}{=} \mathrm{Im}\left(\sum_{n=1}^{N-} \underline{Y}_{kn}^{+-} \underline{U}_n^-\right)^* = f_4\left(\mathbf{U}^-, \delta^-\right) \tag{16'}$$

$$G_{k,n}^{++} = \mathrm{Re}\left(\underline{Y}_{k,n}^{++}\right) \quad B_{k,n}^{++} = \mathrm{Im}\left(\underline{Y}_{k,n}^{++}\right) \quad \delta_{k,n}^{++} = \delta_k^+ - \delta_n^+ \tag{17}$$

TABLE IV.
THE JACOBIAN OF THE POSITIVE SEQUENCE CIRCUIT

Bus ID	**Jacobian Component**
n≠k	$\mathbf{J}1_{kn} = \frac{\partial P_k}{\partial \delta_n^+}; \mathbf{J}2_{kn} = U_n^+ \frac{\partial P_k}{\partial U_n^+}; \mathbf{J}3_{kn} = \frac{\partial Q_k}{\partial \delta_n^+}; \mathbf{J}4_{kn} = U_n^+ \frac{\partial Q_k}{\partial U_n^+}$ (18)
n=k	$\mathbf{J}1_{kk} = \frac{\partial P_k}{\partial \delta_k^+}; \mathbf{J}2_{kk} = U_k^+ \frac{\partial P_k}{\partial U_k^+}; \mathbf{J}3_{kk} = \frac{\partial Q_k}{\partial \delta_k^+}; \mathbf{J}4_{kk} = U_k^+ \frac{\partial Q_k}{\partial U_k^+}$ (19)

3. The Jacobian system (11) is iteratively solved and the voltage values in the buses of the positive sequence circuit are updated, resulting the components $\mathbf{U}^{+(i+1)}$ and $\delta^{+(i+1)}$ of the voltages' vector at the *i+1-th* iteration.

4. The negative and zero sequence bus currents are consequently updated, as in (20):

$$\begin{bmatrix} \underline{I}_k^0 \\ \underline{I}_k^- \end{bmatrix}^{(i+1)} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \end{bmatrix}\left[\underline{I}_k^A \quad \underline{I}_k^B \quad \underline{I}_k^C\right]^{(i+1)} \tag{20}$$

with:

$$\begin{bmatrix} \underline{I}_k^A \\ \underline{I}_k^B \\ \underline{I}_k^C \end{bmatrix}^{(i+1)} = \begin{bmatrix} \underline{S}_{k,cons}^A / \underline{U}_k^{A(i+1)} \\ \underline{S}_{k,cons}^B / \underline{U}_k^{B(i+1)} \\ \underline{S}_{k,cons}^C / \underline{U}_k^{C(i+1)} \end{bmatrix}^*$$ - the column vector of the bus phase currents for the *i+1-th* iteration;

$$\begin{bmatrix} \underline{U}_k^A \\ \underline{U}_k^B \\ \underline{U}_k^C \end{bmatrix}^{(i+1)} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \underline{U}_k^{0(i)} \\ \underline{U}_k^{+(i+1)} \\ \underline{U}_k^{-(i)} \end{bmatrix}^*$$ - the column vector of the bus phase voltages for the *i+1-th* iteration.

5. The reactive powers of the voltage-controlled buses in the positive sequence circuit $\left(Q_k^{+(i+1)}\right)$ are iteratively calculated, as well as the associated injected powers: $Q_{k,g}^{(i+1)} = Q_k^{+(i+1)} + Q_{k,cons}$. If the resulting values exceed their limits Q_{MAX} or Q_{MIN} for any iteration, then the PV-type bus is changed as a PQ-type one, with the value of $Q_{k,g}$ set to its limit.

6. The negative and zero sequence bus voltages are updated at the *i+1-th* iteration:

$$\left[\mathbf{U}^0\right]^{(i+1)} = \left[\mathbf{Y}^{00}\right]^{-1}\left[\left[\mathbf{I}^0\right]^{(i+1)} - \left(\left[\mathbf{Y}^{0+}\right]\left[\mathbf{U}^+\right]^{(i+1)} + \left[\mathbf{Y}^{0-}\right]\left[\mathbf{U}^-\right]^{(i)}\right)\right] \quad (21)$$

$$\left[\mathbf{U}^0\right]^{(i+1)} = \left[\mathbf{Y}^{00}\right]^{-1}\left[\left[\mathbf{I}^0\right]^{(i+1)} - \left(\left[\mathbf{Y}^{0+}\right]\left[\mathbf{U}^+\right]^{(i+1)} + \left[\mathbf{Y}^{-+}\right]\left[\mathbf{U}^+\right]^{(i+1)}\right)\right] \quad (22)$$

7. The problem converges to a valid solution in the positive sequence circuit at once the mismatch condition is fulfilled (ΔP^+, $\Delta Q^+ \leq \varepsilon$) or the rate of convergence is exceeded.

8. Once the load flow solution was obtained, the active and reactive powers of the slack buses are determined, as well as the phase bus currents and the current and power flows along the network's branches.

The solution of the three-phase load flow problem can be written as in Table V.

TABLE V.
THREE-PHASE LOAD FLOW SOLUTION DISPLAY

Bus ID	Voltage		Current		Power			
	$\underline{U}^{+,-,0}$	$\underline{U}^{A,B,C}$	$\underline{I}^{+,-,0}$	$\underline{I}^{A,B,C}$	$P^{+,-,0}$	$Q^{+,-,0}$	$\underline{S}^{+,-,0}$	$\underline{S}^{A,B,C}$

The three-phase load flow method is based on a double iterative algorithm: one is related to the powers of the positive sequence circuit buses; the second one is related to the state values and currents in the buses of the negative and zero sequence circuits.

IV. SUBROUTINES OF THE THREE-PHASE LOAD FLOW PROGRAM

The three-phase load flow algorithm makes the core of the MATLAB-based software analysis tool PFASYM.

Though PFASYM does not have any GUI, it has some other certain advantages such as an information query system designed to access load flow parameters and other electrical system information, integrated unbalanced loads' models and a robust and efficient algorithm to solve three-phase load flow. The simulation results can be obtained as static reports (text output), as well as plots (graphical output).

The reason behind developing this computational tool with MATLAB programming environment is the easiness of matrix-oriented programming, attractive graphical capabilities and the integration with MATLAB Simulink.

PFASYM contains 12 subroutines [8]. These are designed to compute the network parameters for the sequence circuits of the network, as well as the bus and branch state values describing the asymmetrical operation: voltages, currents and powers on phases and sequence circuits, as well as the voltage asymmetry factors.

The PFASYM subroutines are described in Table VI.

TABLE VI.
COMPONENTS OF PFASYM LOAD FLOW PROGRAM

Subroutine	Description
PARIN.mat	Reads and validates the input network branch parameters
TRANSFORM.m	Computes the sequence parameters and the primitive matrix for the network's power transformers
LINE.m	Computes the sequence parameters and the primitive matrix for the network's symmetrical (**Linsim**) and asymmetrical lines (**Linnes**)
SHUNTS.m	Computes the sequence admittance matrix for the network's reactive shunt components
CONDEZ.m	Computes the admittance matrix of the coupled sequence networks in buses with unbalanced loads
YNODSECV.mat	Computes and stores the bus sequence submatrices **YPP**, **YNN**, **YZZ**, **YPN**, **YNP**, **YPZ**, **YZP**, **YNZ**, **YZN** with sparsity technique
INVYNZ.m	Computes the inverse bus admittance matrices for the negative and zero sequence circuits **ZNN**, **ZZZ**
MSTIN.mat	Reads and validates input bus data
CSPOZ.m	Computes the Jacobian for the positive sequence circuit, as well as the correspondent iteration values
MSTNZ.m	Computes the present iteration negative and zero components of bus voltages and currents
FINAL.m	Validates the solution convergence and computes the phase bus and branch values, as well as asymmetry factors
EDIT.mat	Writes the output data

V. SIMULATION RESULTS

In order to outline the capabilities of PFASYM software an asymmetrical electric utility system is considered in order to validate a power flow case study suitable for simulating and evaluating alternative scenarios for the system's phases loading [11].

The selection of a local 110 kV distribution network given in Fig. 4 has taken into consideration two criteria: (1) the technical complexity of the network configuration, which allowed all major concepts pertaining to power flow studies to be addressed; (2) the proximity of the network position to our research center, which facilitated site visits and access to data.

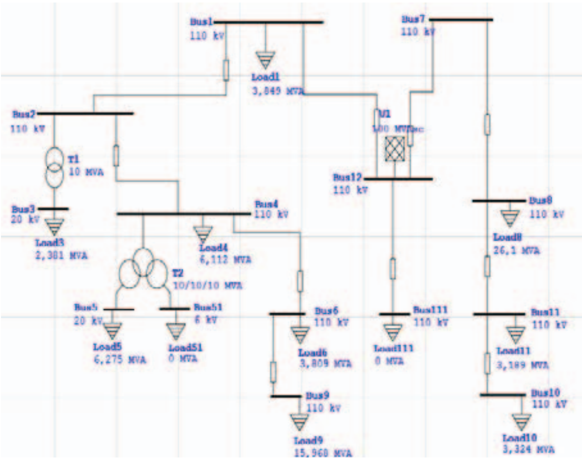

Fig. 4. The test network configuration.

The network includes 12 buses and 9 lines, with branch data given in Table VII.

TABLE VII.
BRANCH DATA OF THE TEST NETWORK

Branch		R (Ω)	X (Ω)	Y (mS)
From	*To*			
#1	#2	0.7336	1.5298	0.0711
#1	#12	0.8263	1.7231	0.0820
#2	#4	0.9885	2.0612	0.0960
#4	#6	0.9885	2.0612	0.0960
#6	#9	0.1421	0.2963	0.0129
#7	#8	0.1421	0.2963	0.0129
#7	#12	3.1044	6.4736	0.3014
#8	#11	0.8263	1.7231	0.0800
#11	#10	4.2241	8.0860	0.4103

TABLE VIII.
BUS DATA OF THE TEST NETWORK

Bus ID	P (MW)	Q (MVAr)
#1	3.464	1.679
#2	0.000	0.000
#3	2.176	0.966
#4	5.887	1.642
#5	6.001	1.833
#6	3.615	1.201
#7	0.000	0.000
#8	22.498	12.231
#9	14.400	6.900
#10	3.226	0.803
#11	3.087	0.799

Three buses are connecting points for railway traction substations (#1, #10, #11) for which an asymmetry higher than 2.5% was registered. The bus input data are measurements of the local distribution operator, given in Table VIII. For this purpose, the following six study cases are considered:

Case I. The network described by the previous information in accordance with the real field data is studied. This case configuration includes 3 buses (#1, #10, #11) with unbalanced loads and is considered as reference (named here initial configuration) for the following ones. By running PFASYM for this case, the simulation results are given in Table IX.

Case II. The load in bus #10 is replaced by a balanced one in the initial configuration, with results given in Table X.

Case III. The load in bus #1 is replaced by a balanced one in the initial configuration, with results given in Table XI.

Case IV. The load in bus #1 is added to one in #2 in the initial configuration, with results given in Table XII.

Case V. The load value in bus #1 is added to one in #6 in the initial configuration, with results given in Table XIII.

Case VI. The asymmetry of the load in bus #1 is doubled in the initial configuration, with results given in Table XIV.

TABLE IX.
BUS RESULTS FOR THE INITIAL CONFIGURATION (CASE I)

Bus ID	PA	QA	PB	QB	PC	OC
	(MW)	*(MAVr)*	*(MW)*	*(MAVr)*	*(MW)*	*(MAVr)*
#1	1.223	1.823	2.215	0.157	0.000	0.000
#2	0.000	0.000	0.000	0.000	0.000	0.000
#3	0.724	0.326	0.704	0.304	0.710	0.320
#4	2.543	0.509	1.906	0.550	1.945	0.550
#5	1.968	0.580	1.867	0.578	2.010	0.628
#6	1.128	0.405	1.178	0.400	1.223	0.365
#7	0.000	0.000	0.000	0.000	0.000	0.000
#8	7.635	4.736	7.347	4.612	7.730	4.007
#9	4.662	2.234	4.679	2.242	4.681	2.243
#10	1.950	0.447	0.000	0.000	1.166	1.258
#11	2.068	1.895	0.556	0.582	0.626	0.523
	Pgen (MW) 250.783			Qgen (MVAr) 211.541		

TABLE X.
BUS RESULTS FOR CASE II

Bus ID	PA	QA	PB	QB	PC	OC
	(MW)	*(MAVr)*	*(MW)*	*(MAVr)*	*(MW)*	*(MAVr)*
#1	1.223	1.823	2.2150	0.1570	0.000	0.0000
#2	0.000	0.0000	0.0000	0.0000	0.000	0.0000
#3	0.724	0.3260	0.7040	0.3040	0.710	0.3200
#4	2.543	0.5090	1.9060	0.5500	1.945	0.5500
#5	1.968	0.5800	1.8670	0.5780	2.010	0.6280
#6	1.128	0.4050	1.1780	0.4000	1.223	0.3650

Bus ID	PA	QA	PB	QB	PC	OC
	(MW)	*(MAVr)*	*(MW)*	*(MAVr)*	*(MW)*	*(MAVr)*
#7	0.000	0.0000	0.0000	0.0000	0.000	0.0000
#8	7.635	4.7360	7.3470	4.6120	7.730	4.0070
#9	4.662	2.2340	4.6790	2.2420	4.681	2.2430
#10	1.950	0.4470	1.9770	0.4540	1.981	0.4550
#11	2.068	1.8950	0.5560	0.5810	0.626	0.5250
	Pgen (MW) 253.552			Qgen (MVAr) 209.355		

TABLE X. (Continuation)

In the Case II, the system phase loading determines a slight change repartition of the powers in the vicinity bus #11. It also minimally discharges the injection sources of reactive power.

TABLE XI.
BUS RESULTS FOR CASE III

Bus ID	PA	QA	PB	QB	PC	OC
	(MW)	*(MAVr)*	*(MW)*	*(MAVr)*	*(MW)*	*(MAVr)*
#1	1.223	1.823	1.224	1.823	1.224	1.822
#2	0.000	0.000	0.000	0.000	0.000	0.000
#3	0.724	0.326	0.704	0.304	0.710	0.320
#4	2.423	0.518	2.006	0.545	2.045	0.545
#5	1.968	0.580	1.864	0.578	2.006	0.627
#6	1.128	0.405	1.178	0.400	1.220	0.465
#7	0.000	0.000	0.000	0.000	0.000	0.000
#8	7.635	4.736	7.347	4.612	7.730	4.007
#9	4.662	2.234	4.673	2.239	4.672	2.238
#10	1.950	0.447	0.000	0.000	1.166	1.258
#11	2.068	1.895	0.556	0.582	0.626	0.523
	Pgen (MW) 250.985			Qgen (MVAr) 215.347		

TABLE XII.
BUS RESULTS FOR CASE IV

Bus ID	PA	QA	PB	QB	PC	OC
	(MW)	*(MAVr)*	*(MW)*	*(MAVr)*	*(MW)*	*(MAVr)*
#1	1.222	1.819	2.214	0.157	0.000	0.000
#2	1.213	1.805	2.200	0.156	0.000	0.000
#3	0.722	0.325	0.703	0.303	0.710	0.320
#4	1.912	0.507	1.903	0.549	1.907	0.551
#5	1.965	0.578	1.864	0.577	2.010	0.628
#6	1.127	0.405	1.170	0.399	1.223	0.365
#7	0.000	0.000	0.000	0.000	0.000	0.000
#8	7.635	4.736	7.256	4.555	7.683	3.983
#9	4.655	2.230	4.672	2.239	4.682	2.243
#10	1.950	0.447	0.000	0.000	1.955	1.254
#11	2.057	1.934	0.556	0.592	0.626	0.534
	Pgen (MW) 259.932			Qgen (MVAr) 215.9640		

In the Case III, a balanced repartition of the phase loads has a minimal influence over the vicinity load buses #4, #6 and leads to a higher network injection, which seems to be influenced by the low overall load on the B-phase.

TABLE XIII.
BUS RESULTS FOR CASE V

Bus ID	PA	QA	PB	QB	PC	OC
	(MW)	*(MAVr)*	*(MW)*	*(MAVr)*	*(MW)*	*(MAVr)*
#1	1.220	1.823	2.215	0.157	0.000	0.000
#2	0.000	0.000	0.000	0.000	0.000	0.000
#3	0.722	0.325	0.703	0.303	0.710	0.320
#4	1.906	0.506	1.902	0.549	1.907	0.551
#5	1.959	0.570	1.862	0.577	2.010	0.628
#6	2.831	2.170	3.335	0.245	1.223	0.365
#7	0.000	0.000	0.000	0.000	0.000	0.000
#8	7.635	4.730	7.256	4.555	7.683	3.983
#9	4.628	2.210	4.663	2.235	4.682	2.243
#10	1.950	0.440	0.000	0.001	1.166	1.258
#11	2.068	1.895	0.568	0.595	0.626	0.523
	Pgen (MW) 260.381			Qgen (MVAr) 216.005		

For the Cases IV, V, the aggravation of the overall asymmetry following the increase of the loads unbalance leads to a higher network injection up to 3% compared to the initial case.

TABLE XIV.
BUS RESULTS FOR CASE VI

Bus ID	PA	QA	PB	QB	PC	OC
	(MW)	*(MAVr)*	*(MW)*	*(MAVr)*	*(MW)*	*(MAVr)*
#1	2.444	3.638	4.427	0.314	0.000	0.000
#2	0.000	0.000	0.000	0.000	0.000	0.000
#3	0.723	0.325	0.704	0.304	0.710	0.320
#4	1.910	0.508	1.905	0.550	1.945	0.550
#5	1.969	0.579	1.865	0.578	2.010	0.628
#6	1.129	0.405	1.177	0.400	1.223	0.365
#7	0.000	0.000	0.000	0.000	0.000	0.000
#8	7.635	4.736	7.347	4.612	7.730	4.007
#9	4.663	2.235	4.679	2.240	4.681	2.243
#10	1.950	0.447	0.000	0.000	1.166	1.258
#11	2.068	1.895	0.556	0.582	0.626	0.534
	Pgen (MW) 254.206			Qgen (MVAr) 213.241		

In the Case VI, the active power injection in the network has a lower increase by comparison with the two previous cases (≈ 1.3% compared to the initial case).

Based on the results given in Table IX…XIV, the voltage asymmetry factors are given in Fig. 5, Fig. 6 and Fig. 7.

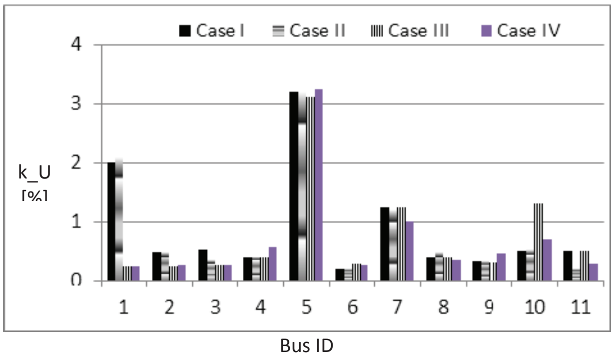

Fig. 5. Voltage asymmetry factors for Cases I…IV.

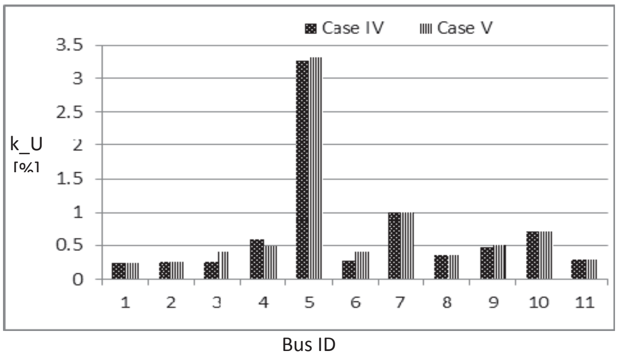

Fig. 6. Voltage asymmetry factors for Cases IV vs. Case V.

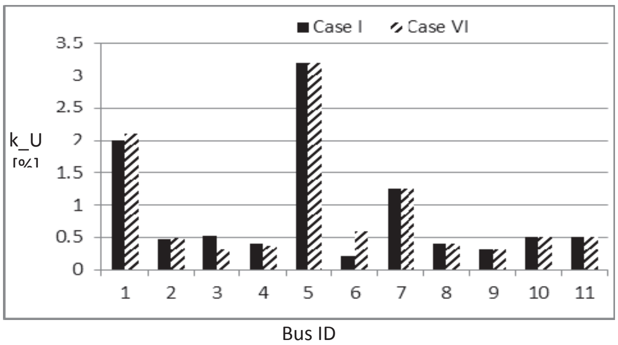

Fig. 7. Voltage asymmetry factors for Case I vs. Case VI.

The validation of the PFASYM results was made by running the load flow program of ETAP 14.1.0 [12].

For the studied network, the PFASYM load flow solution exhibited a maximum deviation of 3.184% (in bus # 1) from the results generated by ETAP program.

VI. Conclusions

This paper presents a novel load flow method adjusted to the particularities of the real power distribution network operating in unbalanced conditions. The method's algorithm was developed using a Newton-Raphson solution in the equivalent positive sequence circuit. The negative and zero sequence components are independently determined as a function of the positive sequence components. For the unbalanced loads supplied by the network a particular model is used. This one includes an equivalent phase-to-phase load connected between of the negative and positive sequence circuits. This approach leads to a lower convergence rate. Here are also summarized the results of a load flow analysis performed with PFASYM program, the in-house Matlab-based software for three-phase power system analysis. The program solves a singular power quality aspect, but it has the great advantage of a low cost/ no cost for the owner of a Matlab license, by comparison with the commercial software packages. The analysis of data generated by PFASYM outlined that the load symmetrization in a bus with unbalanced load leads to decreasing of the voltage asymmetry in the buses of its vicinity, while the voltage asymmetry tends to increase in those buses nearby those with unbalanced load. The longer is the distance between the highly unbalanced loads and the system injection bus, the higher is the voltage asymmetry. In some buses of the studied network the voltage asymmetries exceeds the standards limits [13, 14]. The voltage asymmetry is high in the buses with unbalanced loads, but mostly in the bus placed centrally in the network configuration related to the three railway traction substations. The PFASYM results were validated by comparison with those of an ETAP load flow on the same network configuration. Though the convergence of solution was reached more rapidly (up to 5 iterations) in the case of the proprietary tool, the PFASYM's results are similar, with an average error within 3.2% related to the ETAP ones.

Received on July 19, 2016
Editorial Approval on November 15, 2016

References

[1] K.A Birt, J.J. Graffy, J.D. McDonald, A.H.El-Abiad, "Three phase load flow program", IEEE Trans. On Power Apparatus and Systems, vol.PAS-95, no.1, Jan/Febr. 1976, p. 59-65.

[2] J. Arrillaga, C.P. Arnold, and B.J. Harker, Computer Modelling of Electrical Power Systems, Chicester Wiley, 2nd edition, 2001.

[3] R.N. Allan, A.M. Leite da Silva, and R.C. Burchett, "Evaluation methods and accuracy in probabilistic load flow solutions", IEEE Trans. On Power Apparatus and Systems, vol. PAS-100, no.5,1981, p.2539-2546.

[4] A Garces, "A linear three phase load flow for power distribution networks", IEEE Trans. on Power Systems, vol.31, issue 1, Jan. 2016, p. 827-828.

[5] T.H.Chen, and Y.L. Chang X.P. Zhang, "A novel and fast three-phase load flow for unbalanced radial distribution systems", ", IEEE Trans. on Power Systems, vol.17, issue 4, Nov. 2002, p. 1238-1244.

[6] J. Arrillaga, and C.P. Arnold, Three-phase load flow, in Computer Analysis of Power Systems, John Wiley & Sons, Ltd., West Sussex, England. doi: 10.1002/9781118878309.ch3, 1990.

[7] E. Demirok, S.B. Soren, D. Sera, and R. Teodorescu, "Three-Phase Unbalanced Load Flow Tool for Distribution Networks", Proceedings of the 2nd International Workshop on Integration of Solar Power Systems. Energynautics GmbH, 2012.

[8] D. Rusinaru, The permanent unbalanced operation of the power networks (*in Romanian*), Craiova, Universitaria Publ., 2005.

[9] D. Rusinaru, L.G. Manescu, and R.C. Dinu, "Compensator configurations for load currents' symmetrization", International Conference on Applied Sciences ICAS2015, Book Series: IOP Conference Series-Materials Science and Engineering, Vol: 106, 2016, 012030.

[10] D. Rusinaru D, L G Manescu, and M. Merfu, "The Load Unbalance Influence on the Power Factor Value in Three-Phase Distribution Networks", Recent Advances in Electrical Engineering Series 22 314-319, 2013.

[11] D. Rusinaru, L.G. Manescu, M. Merfu, and P. Postolache, "Power quality general levels in distribution networks", 16th International Conference on Harmonics and Quality of Power (ICHQP), 2014 IEEE, Romania, 2014, p: 58 – 62.

[12] ETAP 14.1. User Guide. ETAP Operation Technology, Inc., 2015.

[13] Romanian Performance Standard for Electricity Distribution Service, ANRE, 2015.

[14] EN 50160, Voltage characteristics of electricity supplied by public distribution systems, 1999.