Evaluating RMS of Linearly Variable Magnitude Waveforms by Using FFT and WPT. Theory and Practice.

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***Abstract* - The paper deals with the evaluation of the Root Mean Square (RMS) indices of signals with linearly variable magnitude by using Wavelet Packet Transform (WPT) and Fast Fourier Transform (FFT). Firstly there is presented a synthesis of employed techniques and previous results of the authors with respect to synthetically generated single-harmonic signals. The signals had linearly decreasing/increasing magnitude M. M varies according to a constant slope G. The studied absolute difference between the final and initial values of M belongs to the set {2,5, 5, 7.5 and 10}% from the initial M. Results of the RMS evaluation by using both FFT and WPT in a single harmonic approach are recalled, focusing on the maximum absolute values of percent relative errors. New studies are presented now, firstly considering randomly generated synthetic multi-harmonic signals. Three cases are considered, corresponding to harmonic orders belonging to 3 distinct ranges: 3...9, 31...39 and respectively 3...40. The errors associated to the use of FFT and WPT are evaluated for them. Two real multi-harmonic signals are afterward analyzed. Small differences were noticed between the values yielded by FFT and WPT for the total RMS as compared to those computed with Riemann sums. The differences between the RMS yielded by FFT and WPT are also evaluated and discussed.**

Keywords: *Wavelet packets, Fast Fourier Transforms, power quality, numerical simulation, convergence of numerical methods.*

I. INTRODUCTION

For real-time applications, in (quasi)stationary regimes with smooth variation of parameters and an insignificant contribution of harmonics of high orders, a standard Fast Fourier Transform (FFT) analysis can provide data with an acceptable accuracy [1]. Interesting wavelet-based algorithms for the harmonic analysis in power systems were proposed in [2]-[4].

Conventional Fourier based analyzing tools have some limitations concerning frequency and time resolutions. Although Wavelet Transforms (Discrete Wavelet Transform - DWT and Wavelet Packet Transform - WPT) overcome these limitations, they suffer from the problem of spectral leakage which is related to the choice of the wavelet family and the mother wavelet used in the analysis. In order to minimize these errors, in [5] is presented to an approach to select the most suitable wavelet family and the most suitable mother wavelet to achieve accurate measurement of steady-state harmonic distortion using DWT. Because WPT is an extended version of DWT, the useful conclusions for our analysis, provided by [5], [6] are: in the case of low distortion levels the most suitable family is the 'db' (Daubechy) and the accuracy increases with increasing the wavelet order or the number of vanishing moments. Wavelet Packet Transform (WPT) provides a uniform cover of the signal and thus its frequency resolution is superior to that provided by DWT [7].

Our previous studies ([1], [6]) concerned with the accuracy of evaluating Power Quality (PQ) indices considered signals with constant magnitude over a sequence of periods. Special techniques must be used for the evaluation of PQ indices when the signals have linearly growing/decreasing magnitudes [8]. Different values are obtained for the inherent errors (accompanying any numerical method). Their study is compulsory for all applications, because they are specific to every distinct operational context. For signals obtained as sinusoidal waveforms polluted by a single harmonic, an extended study was made in [8]. In this paper, this study is continued, such as to consider sinusoidal signals with linearly variable magnitudes, polluted by more harmonics. The final goal is to estimate the accuracy provided by our original algorithms when analyzing data acquired by our Data Acquisition Systems (DAS) described in [9], which have been using to record and evaluate power quality indices for electrical waveforms acquired from power plants. The mentioned DAS provide 197 samples per period (when providing simultaneously 8 waveforms), respectively of 1576 samples per period for a single waveform.

II. ALGORITHM RELATED FEATURES

In order to improve the accuracy of analysis, spline interpolations were made, generating *NP* equally spaced intervals within each interval defined by 2 adjacent samples. *NP* is chosen in different ways, depending on the decomposition method and on the number of samples per period (*SPP*) respectively. When using FFT, *NP* was chosen depending on *SPP* as follows: for *SPP*=197, *NP*=20 and when *SPP*=1576, *NP*=3.

In order to accomplish the WPT decomposition, two tree configurations were employed, relying on a Wavelet mother (WM) with a filter of length 40. The number of levels was 6 when *SPP*=197 and respectively 5 when *SPP*=1576. The number of calculation points (*CP)* is also variable in the WPT work frame, being computed with:

$$CP = nf \cdot 2^d \tag{1}$$

where nf =4 (it represents the length of the vectors hosted by the tree's terminal nodes) and d represents the tree's depth (the number of levels).

We also considered a numerical approximation of the total Root Mean Square (denoted by RMS_{tn}) which can be computed by using the samples, using a Riemann sum:

$$f_{RMS} = \lim_{T->\infty} \sqrt{1/T \int_0^T [f(t)]^2 dt} \tag{2}$$

We used it under the form [10]:

$$V_{RMS\,over\,T} = \sqrt{\frac{area\ under\ the\ curve\ of\ [v(t)\cdot v(t)]\ for\ a\ period\ T}{number\ of\ calculation\ points\ per\ period}} \tag{3}$$

In Eq. (3), $v(t)$ is a vector obtained: (a) only from the acquired samples from a period, and in this case CP = 197 when the smallest sampling rate is used; (b) as result of the interpolation over a period of the acquired signal (CP is equal to the number of all points, original plus those yielded by the interpolation).

III. Techniques to Evaluate Power Quality Indices and Errors Associated to Algorithms

A. Arithmetic averaged values for theoretic RMS values

For our study concerned with sine waves polluted by harmonics, both the magnitudes of the sinusoidal signal (M) and respectively of the NH polluting harmonics (H_j, j=1...NH) have linear variations all over the sequence whose length is $Nper$=10 periods. M was increased / decreased in a linear manner, along all periods. The difference between the initial and final value of M was defined in a percent relative manner (its absolute value belongs to the set of values {2.5%, 5%, 7.5%, 10%}). For a signal with a magnitude M increasing with 0.1, there is a correlation between the initial and final values of M as follows: M_{final} - $M_{initial}$=$M_{initial}$ x 1.1. From this point on we will refer this percent increase as "gain" (G).

From our point of view, a correct approach when dealing with RMS values corresponding to the whole sequence should make use of the following "theoretical reference values", defined with arithmetic averages [8]:

$$R\widetilde{M}S_{FT} = \left(\sum_{i=1}^{N_{per}\,x\,SPP} M_i / (N_{per}\,x\,SPP) \right) / \sqrt{2} \tag{4}$$

$$R\widetilde{M}S_{j,DT} = \left(\sum_{i=1}^{NPer\,x\,SPP} H_{i,j} / (Nper\,x\,SPP) \right) / \sqrt{2} \tag{5}$$

$$R\widetilde{M}S_{DT} = \sqrt{\sum_{j=1}^{NH} R\widetilde{M}S^2{}_{j,DT}} \tag{6}$$

$$R\widetilde{M}S_{TT} = \sqrt{\left(R\widetilde{M}S^2{}_{FT} + R\widetilde{M}S^2{}_{DT}\right)} \tag{7}$$

In the above formulas: M_i represents the magnitude of the pure sine wave (S) corresponding to its i-th synthetically generated sample; $H_{i,j}$ represents the magnitude of the j-th harmonic corresponding to its i-th synthetically generated sample and is raising with G. In Eqs. (4)...(7), "F" is used to denote "Fundamental", "D" is used to denote "distorting", "T" stands for "total" and "$Transf$" can be either FFT or WPT.

B. Applying FFT and WPT in a linearly variable magnitude context

When applying both FFT and WPT over the entire signals of $Nper$ periods length, unacceptable errors were obtained, mainly with respect to the value of the RMS corresponding to distortions (RMS_D). This made us apply both transforms in a "per-period" manner. That is, calculations were made for each period individually and finally the following values were computed as arithmetic averaged values over $Nper$ periods [8] with X standing for F or D:

$$RMS_{XTransf} = \sum_{per=1}^{Nper} RMS_{X\,Transf}(per) / Nper \tag{8}$$

$$RMS_{T\,Transf} = \sqrt{\left(RMS^2{}_{F\,Transf} + RMS^2{}_{D\,Transf}\right)} \tag{9}$$

The compared quantities were: (a) the "node-zero" value yielded by WPT which was compared to the RMS corresponding to the fundamental frequency RMS_F; (b) the "non-zero node" value, which was compared to the RMS_D and (c) the total RMS, denoted by RMS_T .

The definitions for these indices computed by using WPT are given in [11] and respectively those computed by using FFT are given in [12]. The counterpart definitions in the case when WPT is used consider the following rule: RMS_F is calculated by using the energy of the leftmost node from the bottom level of the binary tree, whilst RMS_D is using the energies of the rest of the nodes from the same level [6].

We considered percent relative errors:

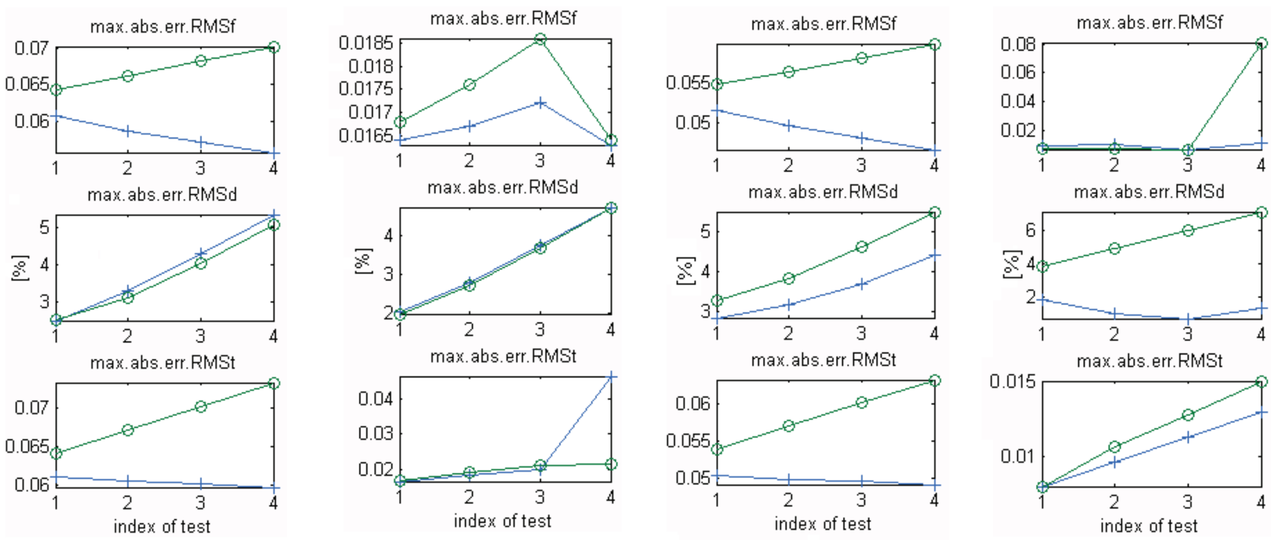
$$err = (val_t - val_c) / val_c \cdot 100 \tag{10}$$

where val_t represents the theoretic values (yielded by Eqs. (4)...(7)) and val_c represents the computed values (the counterpart of val_t , computed with the Eqs. (8), (9)).

IV. PQ Evaluation for Single Harmonic Signals with Linearly Variable Magnitude

Simulations were performed with FFT and WPT for the slope defining the variation of M (G) following the rule: G=($index\ of\ test$) * 2.5% , both for increasing and respectively decreasing M, in a single harmonic context. Fig. 1 [8], [13] provides graphical representations of the maximum absolute value of the percent relative error (MAVPRE) considering maximum 39 harmonics reaching at most 0.1 from the fundamental's magnitude. The symbol '+' was used for the ascending slope whilst 'o' corresponds to the descending one.

The highest absolute errors are associated to RMS_D. They appear at small harmonic orders with small magnitudes (Fig. 2). The following symbols were used: „M↑" denotes "M raises"; „M↓" denotes "M falls". Table I depicts the mean values of the MAVPRE (averaged across all values of G. This table reveals that the level of all MAVPREs is low, denoting that both methods provide appropriate results for practical applications. The values represented with italicized fonts are used to denote "better accuracy" as compared to the other decomposition technique, for the same value of SPP.



(a) (b) (c) (d)

Fig. 1. Maximum absolute values of the percent relative error : (a) , (b) : when using FFT, *SPP*=197, respectively *SPP*=1576. (c), (d) : when using WPT, *SPP*=197, respectively *SPP*=1576.

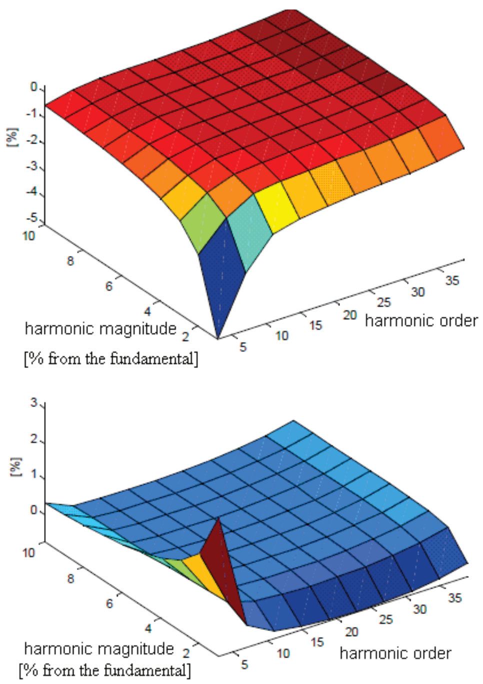


Fig. 2. Minimum (top) and maximum (bottom) of relative percent errors when evaluating RMS_D with FFT. *SPP*=197, *G*=10%, *M* falls.

TABLE I.
MEAN VALUES OF THE MAXIMUM ABSOLUTE VALUES OF THE PERCENT RELATIVE ERRORS

PQ index / mean value of MAVPRE		SPP = 197		SPP = 1576	
		FFT	WPT	FFT	WPT
RMS_F	M↑	0.0581	*0.0489*	0.0167	*0.01*
	M↓	0.0672	*0.0573*	*0.0174*	0.02
RMS_D	M↑	3.8568	*3.5436*	3.3108	*1.288*
	M↓	*3.6738*	4.3108	*3.2618*	5.482
RMS_T	M↑	0.0604	*0.0497*	0.0253	*0.0104*
	M↓	0.0687	*0.0586*	0.0197	*0.0116*

No systematic trend could be deduced from the quantitative point of view. None of the methods can be declared superior to the other one from the MAVPRE point of view. With 2 exceptions, both yielded by FFT (RMS_D computed when *SPP*=197 and RMS_T computed when *SPP*=1576), MAVPREs are higher for decreasing *M* as compared with the cases when *M* is raising.

The use of more samples has advantages for both decomposition techniques when computing RMS_F and RMS_T (it reduces the associated MAVPREs by a factor of at least 3). Yet, due to the sensibility of WPT analysis relative to phase differences, higher maximum absolute values for the percent relative errors were generated by WPT when more samples were used during the evaluation of RMS_D, corresponding to certain particular phase differences. For the most critical case (*G*=10%), actually the mean value of the absolute values for the percent relative errors associated to RMS_D was around 0, with a "peak" of 2% for the lowest harmonic orders with very small magnitude.

V. STUDY OF ERRORS IN A MULTI-HARMONIC CONTEXT

Considering the results from Section IV, our attention was afterward focused on 3 test signals spanning over 10 periods, obtained synthetically by superposing over a perfect sinusoid (with the magnitude *M* =10), sets of 4 harmonics with randomly generated magnitudes (correlated to *M*) and respectively with randomly generated phase differences, as depicted by Table II.

The ranges of harmonic orders were chosen such as to cover all areas of interest: the 1-st signal is polluted only by low odd harmonic orders (3, 5, 7 and 9), the 2-nd signal is polluted only by high odd harmonic orders (33, 35, 37 and 39), whilst the 3rd signal is polluted with harmonic orders from all the range of interest from the European standard point of view (3, 13, 27 and 39). Similar to our previous studies from Section IV, *M* and the harmonics' magnitudes were increased/decreased in a linear manner, along all 10 periods, considering a gain *G* calculated with: *G*=(*index of test*) * 2.5% , both for increasing and respectively decreasing magnitudes.

TABLE II.
CHARACTERISTIC FEATURES OF THE SYNTHETIC WAVEFORMS USED FOR TESTS - MULTI-HARMONIC CASES

Synthetic waveform index	Harmonic orders				Harmonic magnitudes [% from the magnitude of the pure sine wave]				Phase differences of harmonics [rad.]			
	1-st	2-nd	3-rd	4-th	1-st.	2-nd.	3-rd	4-th	1-st	2-nd	3-rd	4-th
1	3	5	7	9	13	3.5	2.8	0.7	-2.2	-1.56	2.14	-1.54
2	33	35	37	39	7.7	0.63	0.09	0.8	0.24	3.12	-2.65	-0.36
3	3	13	27	39	4.9	0.49	0.35	0.02	0.19	1.75	2.73	-2.32

Figs. 3 and 4 depict the percent relative errors associated to the use of FFT, respectively WPT. The left column corresponds to positive values of *G* and the right column is dedicated to the negative ones. The symbol '+' depicts the results for *SPP*=197 whilst 'o' is used for *SPP*=1576.

Table III depicts the mean values of the absolute percent relative errors, averaged across all values of *G*. Similar to the single-harmonic case, for the multi-harmonic pollution the greatest errors are associated the evaluation of RMS_D. Another similarity is related to the "descending over ascending slope" comparison. In all cases, higher errors were recorded for the descending slope, irrespective to the method used for analysis.

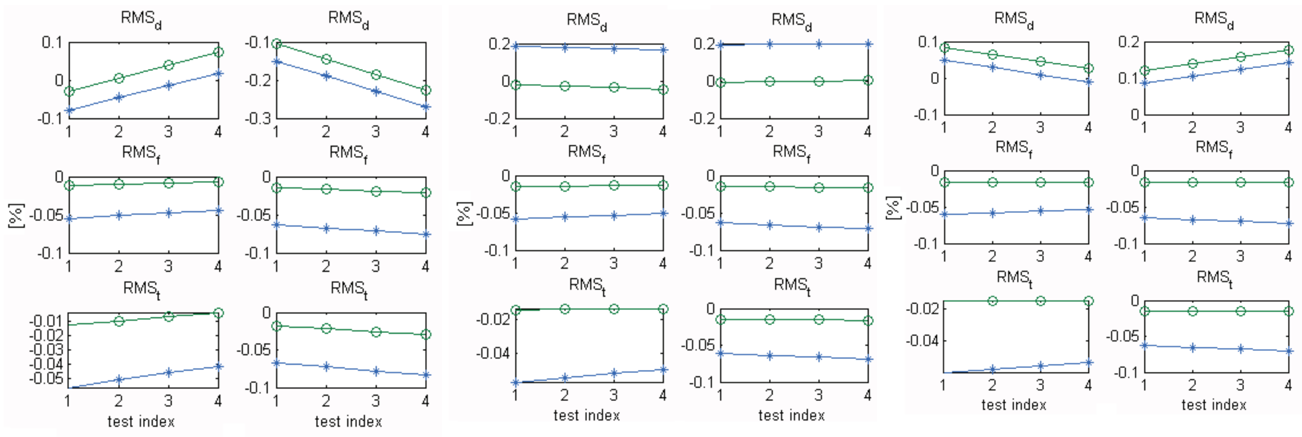
Usually WPT provides slightly lower errors (see the italicized fonts). In only 8 out of 72 cases (see the bolded fonts), the FFT yielded slightly smaller errors. All of them are associated to RMS_d.

The magnitudes of errors in all cases are very low, highly acceptable for industrial applications. Moreover, they did not exceed the maximal values evaluated during the single-harmonic study.

VI. STUDY ON REAL DATA

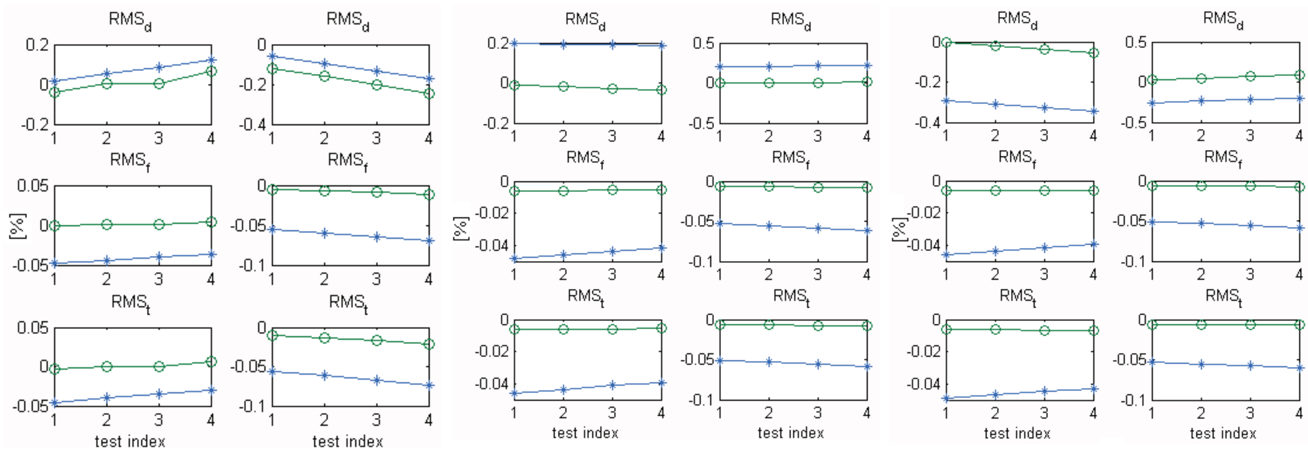
The next step was to perform analysis over real data, acquired from a test stand. Data correspond to a driving system using a chopper and a DC motor. The 1st set of real data corresponds to unfiltered currents (Fig. 5). The 2nd set of real data (Fig. 6) contains currents with reduced harmonic content, but with a "zig-zag" variation of magnitudes (alternately fallings/risings of signal's magnitude). The sampling frequency was 19200 Hz.

For a global picture, data were also analyzed considering the reversed order, the final goal being to address both the "ascending" specific and respectively "descending



(a) (b) (c)

Fig. 3. Percent relative errors yielded by the FFT analysis. (a) – low harmonic orders; (b) high harmonic orders; (c) mixed harmonic orders.



(a) (b) (c)

Fig. 4. Percent relative errors yielded by the WPT analysis. (a) – low harmonic orders; (b) high harmonic orders; (c) mixed harmonic orders.

TABLE III.
MEAN VALUES OF PERCENT RELATIVE ERRORS (ABSOLUTE VALUES)

PQ index / mean value of MAVPRE			SPP = 197		SPP = 1576	
			FFT	WPT	FFT	WPT
Low harmonic orders	RMS_F	M↑	0.0495	*0.0425*	0.0090	*0.0013*
		M↓	0.0696	*0.0627*	0.0180	*0.0079*
	RMS_D	M↑	**0.0399**	0.0684	0.0369	*0.0295*
		M↓	0.2106	*0.1151*	**0.1663**	0.1834
	RMS_T	M↑	0.0487	*0.0380*	0.0077	*0.0022*
		M↓	0.0754	*0.0648*	0.0240	*0.0151*
High harmonic orders	RMS_F	M↑	0.0549	*0.0448*	0.0141	*0.0057*
		M↓	0.0676	*0.0575*	0.0157	*0.0073*
	RMS_D	M↑	**0.1764**	0.1911	0.0330	*0.0233*
		M↓	**0.1964**	0.2099	0.0043	*0.0055*
	RMS_T	M↑	0.0526	*0.0425*	0.0142	*0.0059*
		M↓	0.0650	*0.0549*	0.0156	*0.0072*
Mixed harmonic orders	RMS_F	M↑	0.0570	*0.0429*	0.0159	*0.0057*
		M↓	0.0687	*0.0547*	0.0165	*0.0055*
	RMS_D	M↑	**0.0241**	0.3206	0.0553	*0.0233*
		M↓	**0.1149**	0.2294	0.1487	*0.0073*
	RMS_T	M↑	0.0562	*0.0457*	0.0152	*0.0059*
		M↓	0.0668	*0.0565*	0.0148	*0.0072*

specific features.

An interesting aspect is related to the invariance of results yielded by FFT with respect to the sense of variation (identical values were obtained at magnitude's raising and falling respectively). Small differences were instead revealed in particular cases by the WPT analysis.

Tables IV and V gather the results of the joint analysis. The differences between the results yielded by different methods (FFT, WPT and Riemann sum) are evaluated in a percent relative manner. For example the difference "FFT vs WPT" is given by:

$$\left(value_{FFT} - value_{WPT}\right) / \, value_{FFT} \times 100 \qquad (11)$$

For the 1-st set of real data, all methods provide almost identical values for all types of RMS. Because the method relying on Riemann sums provides the most accurate value for RMS_T when considering the theoretical approach, one can deduce that the phenomenon of "spectral leakages" is faced by both FFT and WPT methods. Fortunately the errors associated to it are very small.

For the 2-nd set of real data, WPT is less affected by the spectral leakage phenomena. On the other hand, an over-evaluation of RMS_T can be noticed when FFT is used, but for both methods the errors are lower than those

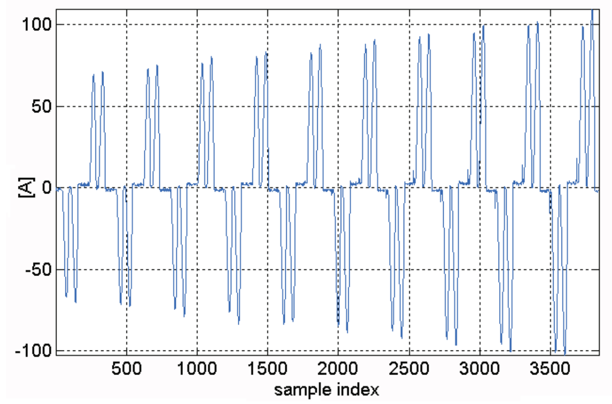


Fig. 5. Phase current from the 1-st set of real data.

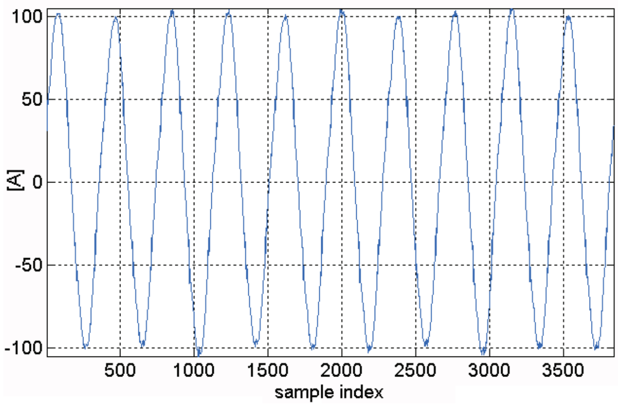


Fig. 6. Phase current from the 2-nd set of real data.

associated to the 1-st set of data.

The opposite signs of errors associated to RMS_T are in correlation with the significant percent relative differences noticed between the values of RMS_D yielded by FFT and WPT, mainly for the current flowing through the 1-st phase. The explanation for them might rely on 2 reasons:
- both FFT and WPT can yield positive, respectively negative errors when evaluating RMS_D which are higher for low harmonic orders with low harmonic magnitudes. They do not occur at the same phase difference and therefore the effects can add, providing differences for counterpart values;
- FFT is more affected by the non-symmetry between the 1-st and 2-nd half-period of the same period and this kind of non-symmetries are frequently noticed in the waveforms from the 2-nd set of data.

Yet, considering the small absolute values of the distorting residues these differences should not be a concern with respect to the accuracy provided by both methods.

VII. CONCLUSIONS

When dealing with harmonically polluted signals in the context of variable magnitudes, test signals must be generated synthetically such as to simulate as accurate as possible the real signals for which the PQ analysis will be performed by using algorithms with clear specifications relative to the internal data structures and sampling ratios. By varying in a systematic manner the harmonic orders and magnitudes, the phase differences and the value of the slope associated to the magnitude variation it is possible to estimate the level of maximum absolute values of the percent relative errors (MAVPRE) associated to the evaluation of the most important RMSs.

In a single harmonic work frame, the representations of MAVPRE revealed interesting aspects:
- none of the methods can be declared superior to the other one from the MAVPRE point of view;
- the level of all MAVPREs is low;
- usually MAVPREs are higher for decreasing M ;
- unlike the case when M raises, when M falls the behavior of MAVPREs is more predictable;
- the 3D representations revealed that the most significant errors appear when evaluating RMS_D;
- a better sampling ratio is always beneficial from the point of view of evaluating with better accuracy RMS_F and RMS_T. For certain combinations of phase differences it can instead result into higher MAVPREs associated to RMS_D, mainly when WPT is used.

TABLE IV.
VALUES YIELDED BY FFT AND WPT ANALYSIS AND PERCENT RELATIVE DIFFERENCES FOR THE FIRST SET OF REAL DATA

Current / PQ index	RMS_D					RMS_F					RMS_T						
	FFT [A]	DWT↑ [A]	FFT vs DWT↑ [%]	DWT↓ [A]	FFT vs DWT↓ [%]	FFT [A]	DWT↑ [A]	FFT vs DWT↑ [%]	DWT↓ [A]	FFT vs DWT↓ [%]	Riemann [A]	FFT [A]	FFT vs R [%]	DWT↑ [A]	DWT↑ vs R [%]	DWT↓ [A]	DWT↓ vs R [%]
I1	26.97	26.95	0.07	26.96	0.04	32.86	32.81	0.15	32.82	0.12	42.89	42.51	0.89	42.46	1.00	42.47	0.98
I2	27.69	27.67	0.07	27.67	0.07	34	33.97	0.09	33.97	0.09	44.26	43.86	0.90	43.81	1.02	43.81	1.02
I3	27.00	26.99	0.04	26.99	0.04	32.96	32.96	0.00	32.96	0.00	43.04	42.61	1.00	42.60	1.02	42.60	1.02

TABLE V.
VALUES YIELDED BY FFT AND WPT ANALYSIS AND PERCENT RELATIVE DIFFERENCES FOR THE SECOND SET OF REAL DATA

Current / PQ index	RMS_D					RMS_F					RMS_T						
	FFT [A]	DWT↑ [A]	FFT vs DWT↑ [%]	DWT↓ [A]	FFT vs DWT↓ [%]	FFT [A]	DWT↑ [A]	FFT vs DWT↑ [%]	DWT↓ [A]	FFT vs DWT↓ [%]	Riemann [A]	FFT [A]	FFT vs R [%]	DWT↑ [A]	DWT↑ vs R [%]	DWT↓ [A]	DWT↓ vs R [%]
I1	4.55	3.86	15.16	3.74	17.80	69.44	69.4	0.06	69.4	0.06	69.52	69.58	-0.09	69.51	0.01	69.51	0.01
I2	4.29	4.17	2.80	3.8	11.42	70.99	70.95	0.06	70.98	0.01	71.09	71.12	-0.04	71.08	0.01	71.08	0.01
I3	4.89	4.39	10.22	4.73	3.27	69.27	69.27	0.00	69.25	0.03	69.42	69.44	-0.03	69.41	0.01	69.41	0.01

The tests on randomly generated multi-harmonic pollutions revealed that:

- the greatest errors are associated to RMS_D;
- another similarity to the single harmonic cases is related to the "descending over ascending slope" comparison. In all cases, higher absolute errors were recorded for the descending slope, irrespective to the method ;
- usually WPT provides slightly lower errors. All exceptions are associated to RMS_d;
- the magnitudes of errors in all cases are very low, highly acceptable for industrial applications. Moreover, they did not exceed the maximal values evaluated during the single-harmonic study.

For the 1-st set of real data, all methods provide almost identical values for all types of RMS. The phenomenon of "spectral leakages" is faced by both FFT and WPT methods. Fortunately the errors associated to it are very small.

For the 2-nd set of real data, WPT is less affected by the spectral leakage phenomena. On the other hand, an over-evaluation of RMS_T can be noticed when FFT is used, but for both methods the errors are lower than those associated to the 1-st set of data. The opposite signs of errors associated to RMS_T are in correlation with the significant percent relative differences noticed between the values of RMS_D yielded by FFT and respectively WPT, mainly for the current flowing through the 1-st phase.

A final conclusion is relative to the good practice of applying any of the analyzed transform in a per-period manner and performing arithmetic averages.

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