Analysis of Direct Current Motors in Dynamic Regimes Using Numerical Techniques

Ioana-Gabriela Sîrbu and Lucian Mandache
University of Craiova, Faculty of Electrical Engineering, Craiova, Romania, osirbu@elth.ucv.ro

***Abstract* - The electromechanical converters and the electric drive systems where they are parts are generally nonlinear systems due to the nonlinear properties of the materials they are built from, as well as the multiple dependencies between their electrical, magnetical and mechanical parameters. However, under certain conditions (e.g. operation on the linear part of the characteristics), some simplifying hypotheses are accepted, which allow the treatment of systems as linear systems. This simplifies their analysis and solutions could be obtained with acceptable errors. So the electromechanical converters and the electric drive systems can be analyzed as linear or nonlinear systems, depending on their properties and on the equations that govern them. Thus, analytical or numerical solving techniques can be used in order to obtain the solution, i.e. the variation in time of some electric, magnetic or mechanic quantities. In this paper a separately excited direct current motor that drives a mechanical load is analyzed. The transient processes due to startup or sudden change of torque are analyzed both analytically and numerically. Two techniques – namely numerical solving using a Runge-Kutta method and respectively using an equivalent electrical scheme corresponding to the entire driving system – were used. By comparing the obtained numerical and analytical results a very good correlation between the different solving methods was proved. Thus, this paper offers alternative and viable analysis techniques, suitable for the dynamic regimes of the driving systems with direct current motors.**

Cuvinte cheie: *motoare de curent continuu, regimuri dinamice, metoda Runge-Kutta, SPICE.*

Keywords: *dc motors, dynamic regimes, Runge-Kutta method, SPICE.*

I. INTRODUCTION

Direct current motors are widely used in industry due to their advantages (high starting torque, simple regulation, reliability) [1-3]. Also they have a nonlinear behavior frequently a simplified linear model of the motor is sufficient for their design and analysis. Some authors recommend the use of nonlinear models because they do consider them suitable for the analysis of the transient regimes, to obtain more precise results [4 – 6]. Reference [1] specifies as sources of nonlinearity: the existence of friction forces, the presence of the armature reaction, the nonlinearity of some parameters (e.g. resistance and inductance of the armature), etc.

The theory of the direct current machines is well established [1-3, 7-9]. But researches are carried on in the area of mathematical description of these motors. Often, simpler models with linearized parameters and electrical equivalent diagrams with lumped parameters [10, 11] are sufficient, in a first step, to carry out steady-state studies or in preliminary analyzes of some more complex systems that include dc motors.

In the present paper we proposed to analyze separately excited direct current motors – widely used in industry. Because the operating regime of the mechanical load is often variable over time, this causes the motor to operate in dynamical regimes, with frequent starts, brakes, load variations and reversals. Thus knowing the variation in time of different quantities is important for a correct dimensioning of the system.

The model chosen for this type of motor is a linear one. The equations of the motor have been used to obtain differential equations in one unknown. Three methods were used to resolve them: the analytical method (the method that offers the exact solution), a numerical solving technique using a Runge-Kutta method and a technique based on the similarity of the equations that uses equivalent electrical circuits. For the same dc motor the results obtained by the three procedures (the variations in time of the current and of the speed) were presented in a comparative manner, observing a perfect correlation between them.

In conclusion, in the analysis of dc motors and systems containing them, any of these presented techniques can be used, the results obtained in dynamic regimes being similar.

II. ANALYTICAL AND NUMERICAL TECHNIQUES FOR THE ANALYSIS OF THE DC MOTORS

We considered the case of a separately excited direct current motor (fig. 1). The equations governing its operation in variable regime (starting, changing in the load torque at the machine spindle, etc.) are [2,3,7,8]:

$$\begin{aligned} U &= u_e + i\cdot\left(R+R_i\right)+L\frac{\mathrm{d}i}{\mathrm{d}t} \\ u_e &= K\cdot\Omega \\ M &= K\cdot i \\ M &= M_R + J\frac{\mathrm{d}\Omega}{\mathrm{d}t}, \end{aligned} \tag{1}$$

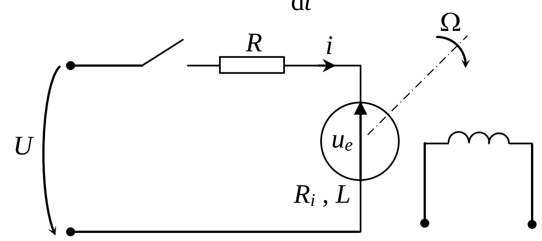


Fig. 1. Schematic circuit of a separately excited direct current machine.

where U is the constant voltage at the machine terminals, u_e is the electromotive force of the rotor winding, R_i and L are the resistance and respectively the inductance of the armature, R is the external resistance of the armature circuit. K is the constant of the machine, which depends on the number of poles pairs and respectively on the number of conductors on the current path. Ω is the angular speed of the machine, i is the current in the rotor winding. The last relation in (1) represents the motion equation. It includes the torque at the machine spindle, M , the load torque of the mechanical load, M_R , and J - the moment of inertia of the spinning bodies.

The analysis considers the motor operating at constant magnetic flux and neglects the effect of saturation. Thus its parameters can be considered constant, and the differential equation is linear. Also the moment of inertia is constant, fact that is often encountered in practice in the electric drives.

By derivating the first relation and replacing one at a time the electromotive force, the angular speed and the torque at the machine spindle from the other three relations of (1) we obtain the differential equation (2), where the only unknown is the current through the rotor winding:

$$\frac{\mathrm{d}^2 i}{\mathrm{d}t^2}+\frac{R+R_i}{L}\frac{\mathrm{d}i}{\mathrm{d}t}+\frac{K^2}{J\cdot L}\cdot i=\frac{M_R\cdot K}{J\cdot L}. \tag{2}$$

The differential equation (2) can also be written in the form [2,3]

$$\frac{\mathrm{d}^2 i}{\mathrm{d}t^2}+\frac{1}{T_e}\frac{\mathrm{d}i}{\mathrm{d}t}+\frac{1}{T_e\cdot T_m}\cdot i=\frac{i_R}{T_e\cdot T_m}, \tag{3}$$

where we used the notations:

$T_e=\dfrac{L}{R+R_i}$ for the electromagnetic time constant,

$T_m=\dfrac{J\cdot(R+R_i)}{K^2}$ for the electromechanical time constant of the system, and

$i_R=\dfrac{M_R}{K}$ is the current corresponding to the load torque of the mechanism.

Also a differential equation, but function of the angular speed, is obtained from (1) replacing in the first relation the electromotive force and then the rotor winding current and the torque of the machine:

$$\frac{\mathrm{d}^2\Omega}{\mathrm{d}t^2}+\frac{1}{T_e}\frac{\mathrm{d}\Omega}{\mathrm{d}t}+\frac{1}{T_e\cdot T_m}\cdot\Omega=\frac{\Omega_R}{T_e\cdot T_m}. \tag{4}$$

In this equation appear also the electromagnetic time constant T_e and the electromechanical time constant T_m, and

$$\Omega_R=\frac{U}{K}-\frac{R+R_i}{K^2}\cdot M_R \tag{5}$$

represents the stationary angular speed corresponding to the current i_R on a characteristic that considers the resistance R in series in the circuit of the armature [3].

The differential equations in current (3) or in angular speed (4) can be solved both analytically and numerically. The analytical solving is possible here because a linear problem was considered.

A. Analytical Solving

The analytical solution of the differential equation (3) has the form [2,3]:

$$i=A_{1i}e^{\lambda_1 t}+A_{2i}e^{\lambda_2 t}+I_0, \tag{6}$$

where the coefficients A_{1i} and A_{2i} and the current I_0 depend on the operating conditions of the system, and

$$\lambda_{1,2}=-\frac{1}{2\cdot T_e}\pm\sqrt{\frac{1}{4\cdot T_e^2}-\frac{1}{T_e\cdot T_m}} \tag{7}$$

are the solutions of the characteristic equation corresponding to the differential equation (3). These solutions are real if $T_m\geq 4\cdot T_e$ or complex otherwise.

The solution of the differential equation (4) has a similar form:

$$\Omega=A_{1\Omega}e^{\lambda_1 t}+A_{2\Omega}e^{\lambda_2 t}+\Omega_0, \tag{8}$$

where the solutions of the characteristic equation are the same as (7).

The relations (6) – (8) are valid for any transient process: starting, braking, reversal or sudden torque variation. For example, in this paper the analysis was made only for two of these cases, namely: starting under load and sudden torque variation.

1) Starting. In this case the initial conditions of the system with respect to the current and the angular speed are:

$$t=0:\quad i=0,\quad \frac{\mathrm{d}i}{\mathrm{d}t}=U/L;\qquad \Omega=0,\quad \frac{\mathrm{d}\Omega}{\mathrm{d}t}=0. \tag{9}$$

It was assumed that the system has a mechanism that prevents the reversal in rotation. The starting is considered to be completed when the stationary regime has been reached. Also, $I_0=i_R$ and $\Omega_0=\Omega_R$.

Considering these conditions, from the relation (6) and its derivative we obtain a system which, by solving, offers the values of the constants A_{1i} and A_{2i} :

$$A_{1i}=\frac{\lambda_2 I_0+U/L}{\lambda_1-\lambda_2},\ A_{2i}=-\frac{\lambda_1 I_0+U/L}{\lambda_1-\lambda_2}. \tag{10}$$

Similarly, from (8) the constants are obtained:

$$A_{1\Omega}=\frac{\lambda_2\Omega_0}{\lambda_1-\lambda_2},\ A_{2\Omega}=-\frac{\lambda_1\Omega_0}{\lambda_1-\lambda_2}. \tag{11}$$

The constants (10) and (11) respectively replaced in (6) and (8) give the analytical expressions of the current and of the speed during the starting.

2) Sudden torque variation. It is encountered at some working machines that cause sudden variations of the torque from a constant value M_1 to another value M_2. Therefore, according to (1), the current will vary between the constant values

$$I_1 = M_1 / K\,,\ \ I_2 = M_2 / K\,, \tag{12}$$

and the angular speed between the values

$$\Omega_1 = \frac{U-(R+R_i)I_1}{K}\,,\ \Omega_2 = \frac{U-(R+R_i)I_2}{K}\,. \tag{13}$$

The initial conditions in this regime are:

$$t=0:\ \ i = I_1, \frac{\mathrm{d}i}{\mathrm{d}t} = \delta = \frac{1}{L}\left(U - K\Omega_1 - (R+R_i)I_1\right); \\ \Omega = \Omega_1, \frac{\mathrm{d}\Omega}{\mathrm{d}t} = 0. \tag{14}$$

Also, $I_0 = I_2$ and $\Omega_0 = \Omega_2$.

The relation (6) together with its derivative form a system from which the constants corresponding to the new operating regime are obtained:

$$A_{1i} = \frac{\delta+\lambda_2(I_2-I_1)}{\lambda_1-\lambda_2}\,,\ A_{2i} = -\frac{\delta+\lambda_1(I_2-I_1)}{\lambda_1-\lambda_2}\,. \tag{15}$$

Similarly, (8) together with the new conditions give the constants:

$$A_{1\Omega} = \frac{\lambda_2(\Omega_2-\Omega_1)}{\lambda_1-\lambda_2}\,,\ A_{2\Omega} = -\frac{\lambda_1(\Omega_2-\Omega_1)}{\lambda_1-\lambda_2}\,. \tag{16}$$

The constants (15) and (16) respectively with the notations (12) and (13), replaced in (6) and (8), give the analytical expressions of the current and of the speed when the torque is changed suddenly.

We considered for example a separately excited direct current motor having the following parameters: rated power P =11 kW, rated voltage U =230V, efficiency η=91.5 %, n =500 rot/min, 2 pairs of poles, armature parameters $R_i = 1.4\Omega$ and $L = 0.209\mathrm{H}$ and, for the system, $R = 0.5\Omega$, $M_r = 35\,\mathrm{N\cdot m}$ and $J = 30\,\mathrm{kg{\cdot}m^2}$.

By calculations, using the above formulas, resulted: $K = 4.0193$, $T_e = 0.11\,\mathrm{s}$, $T_m = 3.85\,\mathrm{s}$, $M_n = 210\ \mathrm{N{\cdot}m}$ (rated torque), $\lambda_1 = -0.2676$, $\lambda_2 = -8.8233$.

Considering the starting of the dc motor by direct connection to the supply voltage U until the permanent operating mode is reached, followed by a sudden variation of the torque from M_1 (the torque corresponding to the first permanent regime) to $M_2 = 0.5 \cdot M_1$, the variations in time of the current and of the angular speed resulted as shown in fig. 2. Because in the analyzed case $T_m > 4 \cdot T_e$, the solutions $\lambda_{1,2}$ given by (7) resulted real, so that there are no oscillations of the current or the speed during the transient regime.

B. Solving Using an Equivalent Electrical Scheme

This procedure is based on the observation that there is similarity between the differential equations of the current (3) and the speed (4) of the dc motor and the differential

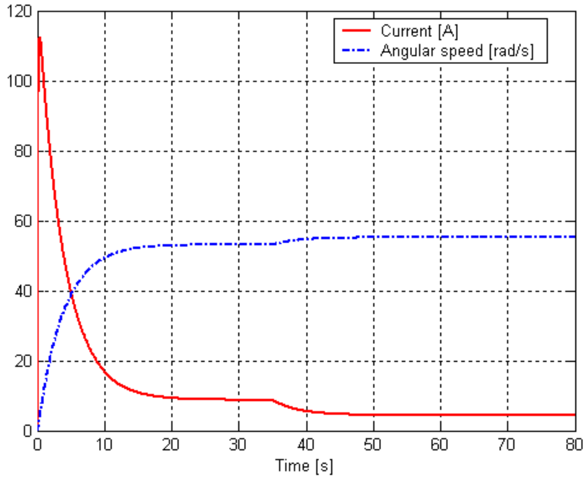


Fig. 2. Time variations of the current and of the angular speed during starting and torque variation (analytical method).

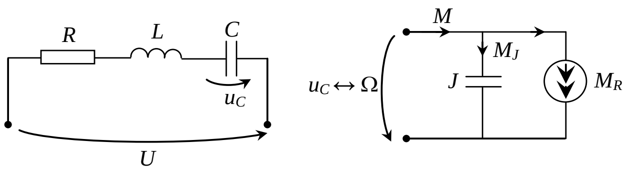
equation corresponding to the *RLC* series circuit in time-varying regime [12] (fig. 3 (a)), having as unknown the voltage at the capacitor terminals, u_C:

$$\frac{\mathrm{d}^2 u_C}{\mathrm{d}t^2} + \frac{R}{L}\frac{\mathrm{d}u_C}{\mathrm{d}t} + \frac{1}{L\cdot C}\cdot u_C = \frac{U}{L\cdot C}\,. \tag{17}$$

Table I shows the correspondence between the quantities that appear in the equations (3), (4) and (17) respectively.

Based on this analogy, the determination of the variation in time of the current (or of the angular speed) is reduced to the determination of the time variation of the voltage u_C that occurs as a result of the application of the voltage U to the *RLC* series circuit terminals.

The initial conditions related to the value of the current or of the speed and respectively to their derivatives are introduced as initial conditions for the capacitor and respectively for the coil in the equivalent circuit.



(a) (b)

Fig. 3. RLC series circuit (a) and the complementary equivalent electrical diagram for the motion equation (b).

TABLE I.
CORRESPONDENCE BETWEEN THE COEFFICIENTS OF THE DIFFERENTIAL EQUATIONS

Differential Equation	Equivalent Electric Circuit
$T_e = L / (R+R_i)$	$T_e = L / R$
$T_m = J \cdot (R+R_i) / K^2$	$T_m = R \cdot C$
i_R (or Ω_R)	U
i (or Ω)	u_C

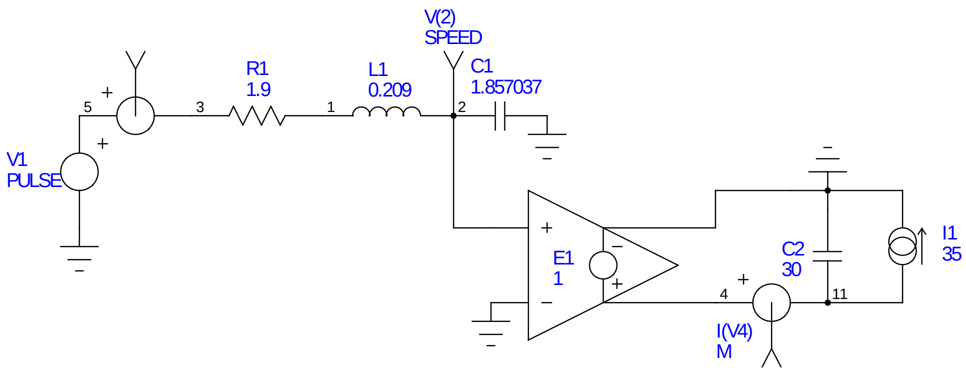


Fig. 4. RLC series circuit and motion equation implemented in SPICE.

Thus, the initial value of the current (speed) corresponds to the initial condition of the capacitor, while its derivative, multiplied by the value of the equivalent capacitor's capacitance, corresponds to the initial condition of the equivalent circuit coil.

If u_C corresponds to the speed Ω, then the electrical diagram of fig. 3 (a) can be completed with an equivalent electrical scheme for the motion equation of (1), which gives as output quantity the motor torque M, too (fig. 3 (b)). The moment of inertia J corresponds to the capacitance of a capacitor passed by a current corresponding to the dynamic torque and fed with a voltage proportional to the angular speed. The load torque M_R is introduced as an independent current source so that the sum of the load torque and the dynamic torque give the motor torque *M*.

These simple electrical diagrams can be simply entered into an electric circuit simulation program (as SPICE), that provides graphical variations in time of the selected quantities [13]. The SPICE version of the diagrams of fig. 3 is shown in fig. 4. The parameters of the same machine as in the analytical method were used. Figs. 5 and 6 show the results obtained with SPICE at the motor starting, i.e. the variation in time of armature current *i*, of angular speed Ω and of torque M.

It is observed that, in all cases, the variations similar to those in fig. 2, both in terms of quality and value, are obtained.

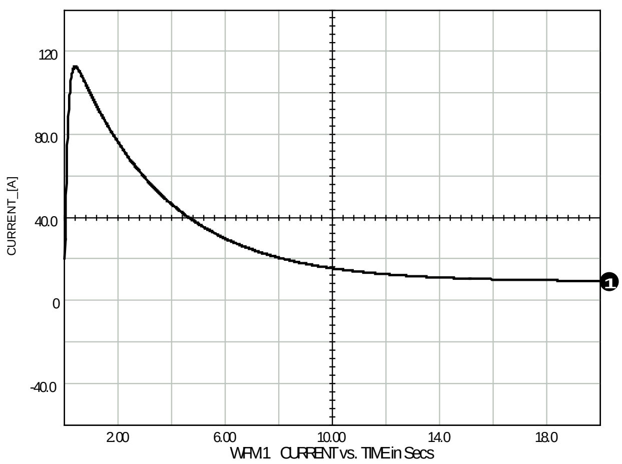


Fig. 5. The time variation of the armature current during the starting, obtained in SPICE.

C. Solving Using Runge-Kutta Method

It was possible to observe the similarity between the differential equations of the armature current (3) and of the angular speed (4). For this reason the presentation of this method will only be related to the armature current *i*.

The differential equation (3) is transformed into a system of differential equations [14] of the form:

$$\begin{cases} \dfrac{\mathrm{d}i}{\mathrm{d}t} = y \\ \dfrac{\mathrm{d}y}{\mathrm{d}t} + \dfrac{1}{T_e} y + \dfrac{1}{T_e \cdot T_m} \cdot i = \dfrac{i_R}{T_e \cdot T_m}, \end{cases} \tag{18}$$

where it was noted with y the derivative of first order of the armature current.

The derivatives of the two quantities will be expressed then as follows:

$$\begin{cases} i' = y \\ y' = -\dfrac{1}{T_e} y - \dfrac{1}{T_e \cdot T_m} \cdot i + \dfrac{i_R}{T_e \cdot T_m} \end{cases} \tag{19}$$

The system can be resolved if the initial conditions are known. In the case of starting, they are:

$$t = 0: \; i = 0, \; y = U / L, \tag{20a}$$

and in the case of torque variation:

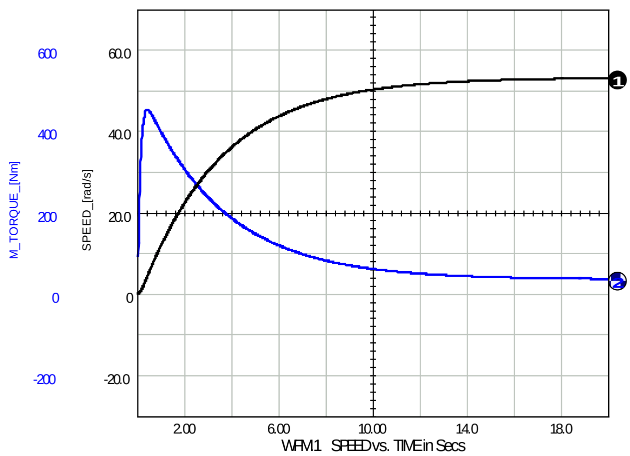


Fig. 6. The time variations of the angular speed (1) and of the motor torque (2) during the starting, obtained in SPICE.

$$t = 0 : \; i = I_1 , \; y = \delta . \qquad (20b)$$

Because of its accuracy, it is often preferred to solve the first order differential equations and systems by using a Fourth Order Runge-Kutta technique [14, 15].

The procedure is iterative, the values being calculated with approximation (numerically) at each moment t^{j+1} as function of the values of the previous time t^j,

$$t^{j+1} = t^j + h , \qquad (21)$$

where h is the time step (of rather small value).

Using this technique, at each iteration, the current i and its derivative y will be calculated with the relations [14], [15]:

$$\begin{aligned} i^{j+1} &= i^j + \frac{1}{6}k_1^1 + \frac{1}{3}k_2^1 + \frac{1}{3}k_3^1 + \frac{1}{6}k_4^1 \\ y^{j+1} &= y^j + \frac{1}{6}k_1^2 + \frac{1}{3}k_2^2 + \frac{1}{3}k_3^2 + \frac{1}{6}k_4^2 . \end{aligned} \qquad (22)$$

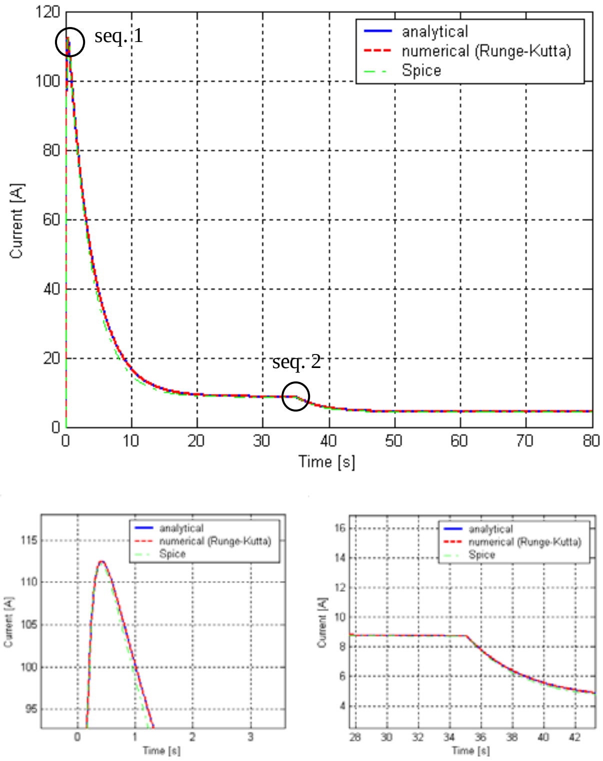
In (22), for the separately excited dc motor and taking into account the system (19), the terms k have the following expressions:

$$\begin{aligned} k_1^1 &= h \cdot y^j \\ k_1^2 &= h \cdot \left(-\frac{1}{T_e \cdot T_m} \cdot i^j - \frac{1}{T_e} y^j + \frac{i_R}{T_e \cdot T_m} \right) \\ k_2^1 &= h \cdot \left(y^j + k_1^2 / 2 \right) \\ k_2^2 &= h \cdot \left(-\frac{1}{T_e \cdot T_m} \cdot \left(i^j + k_1^1 / 2 \right) - \frac{1}{T_e} \left(y^j + k_1^2 / 2 \right) + \frac{i_R}{T_e \cdot T_m} \right) \\ k_3^1 &= h \cdot \left(y^j + k_2^2 / 2 \right) \\ k_3^2 &= h \cdot \left(-\frac{1}{T_e \cdot T_m} \cdot \left(i^j + k_2^1 / 2 \right) - \frac{1}{T_e} \left(y^j + k_2^2 / 2 \right) + \frac{i_R}{T_e \cdot T_m} \right) \\ k_4^1 &= h \cdot \left(y^j + k_3^2 \right) \\ k_4^2 &= h \cdot \left(-\frac{1}{T_e \cdot T_m} \cdot \left(i^j + k_3^1 \right) - \frac{1}{T_e} \left(y^j + k_3^2 \right) + \frac{i_R}{T_e \cdot T_m} \right) . \end{aligned} \qquad (23)$$

The relations (22) together with (23) allow obtaining in a numerical manner the variations in time of the current i and even of the speed Ω, depending on the considered initial values (20). They can be implemented in a computational program such as MATLAB, which, in addition to the iterative numerical computation, allows graphical display of the results. The numerical results obtained by this method, for the same dc motor chosen as an example, are shown in the next section.

III. COMPARATIVE RESULTS

The results obtained by using the three analysis procedures for dynamic regimes of the separately excited dc motors were compared graphically. As a case study the dc motor with the parameters presented in the analytical method was chosen.



(a) Sequence 1 (b) Sequence 2

Fig. 7. Comparative representation of the time variation of the current.

Fig. 7 shows the time variations of the armature current during the starting (the first 35 seconds), respectively at a torque variation from $M_1 = M_{steady}$ to $0.5 \cdot M_1$ (the next 45 seconds).

It is found that the three approaches of the problem offer identical results, the three curves being perfectly overlapped.

At a closer look, by enlarging different zones of the curves, we noticed that in the most parts the curves coincide. However, during the starting (sequence 1) and at the beginning of the torque variation (sequence 2), the three procedures offer slightly different results. This difference is not important, so it can be neglected.

The same thing is observed in fig. 8.

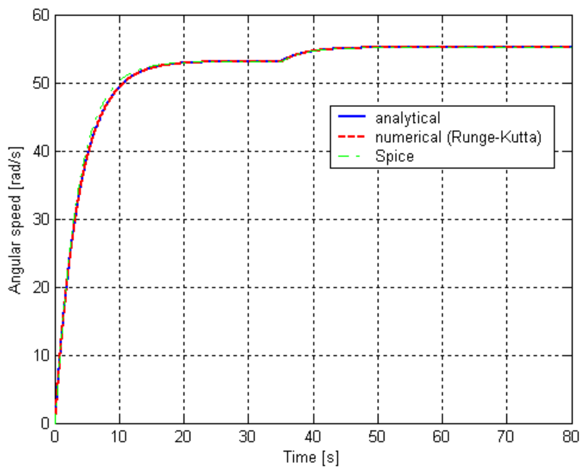


Fig. 8. Comparative representation of the time variation of the angular speed.

TABLE II.
VALUES OF THE MAXIMUM ABSOLUTE ERRORS

Current *i* [A]		Speed Ω [rad/s]	
Spice	Runge-Kutta	Spice	Runge-Kutta
3.8097	0.0105	1.7752	0.0049

Here the variation of the angular speed as function of time, obtained by the three calculation techniques, was represented comparatively. The three variation curves overlap almost perfectly over their entire length.

To make a clearer distinction between the obtained results, a calculation of the errors was made. Table II gives the maximum absolute errors obtained in the numerical results compared to the analytical results (considered to be the reference).

It can be noticed that the errors obtained through the equivalent electric scheme (SPICE) are slightly higher than those obtained by the Runge-Kutta method. This may be due to the choice of a precision-performing Runge-Kutta method, while in SPICE the implicit calculation options were considered without choosing a better method or a lower computational error. However, the obtained errors (max. 3.8097 A for the current and max. 1.7752 rad/s for the angular speed, achieved during the starting process) are small enough to be accepted.

IV. CONCLUSION

The paper presents alternative techniques for solving the separately excited direct current motor equations in dynamic regimes. For this, a linearized model was chosen for it, in which all parameters are considered to be constant. It has been described how to obtain the relations of the variations in time of the armature current and of the speed in dynamic regime, for two of the cases met in practice, namely: motor starting and sudden torque variation.

Three procedures of analysis were considered: the analytical method, the technique based on the equivalent electric diagram and the technique based on a Runge-Kutta method.

The obtained results and the comparative study have shown that any procedure can be used successfully to obtain the desired results, with a good accuracy.

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First author – 60%
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