Diminishing the Computational Burden when Analyzing Electrical Signals with Long Wavelet Filters

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***Abstract* – Wavelet analysis is characterized by filters with irrational coefficients and runtimes increasing with filters order. The terminal nodes of Wavelet Packet decomposition trees exhibit good harmonics' selectivity when implemented with Daubechies wavelet mothers with filters of length 28 and respectively of length 40 (denoted in MATLAB by 'db14' and 'db20'). Therefore they can be used for power quality (PQ) analysis of electric signals. When rounded to 4 decimal places, certain coefficients from these filters become 0. In the "rounded" context, the number of iterations supporting the convolution products involved by the Wavelet analysis is restricted. Based on the number of neglected coefficients and length of wavelet filter, a formula is deduced for the percent runtime savings strictly associated to the wavelet decompositions. Statistical evaluations (considering sets of 20 consecutive programs executions) were performed considering full operational contexts, where all types of studied wavelets were involved in complex operations like evaluating PQ indices and harmonic spectra. The mean runtime savings proved to be 20.4% for 'db14' and respectively 23.2% for 'db20'. The impact of rounding over the PQ analysis's accuracy was evaluated considering three-phase waveforms acquired from a power group. The following quantities were considered: harmonic spectra and energies of the Wavelet Packet tree's terminal nodes, instantaneous values corresponding to the fundamental frequency, vectors of details and approximations yielded by the Discrete Wavelet Transform and signals reconstructed from the vectors of details and approximations. Highly acceptable small errors were obtained, recommending the rounding to 4 decimal places of the analyzed wavelet filters as a faster reliable wavelet based analysis method.**

Cuvinte cheie: *calcule specifice ingineriei energetice, calitatea puterii, transformata discretă Wavelet, transformata de tip Wavelet Packet Transform, transformata staționară Wavelet.*

Keywords: *power engineering computing, power quality, Discrete Wavelet transform, Wavelet Packet Transform, Stationary Wavelet Transform.*

I. INTRODUCTION

The classical method of power quality (PQ) analysis used in power quality monitoring systems has been the discrete Fourier transform (DFT) [1]...[4]. Nowadays new methods has been proposed in literature: Short Time Fourier Transform (STFT), Discrete Stockwell Transform (DST) [5], Discrete Wavelet Transform (DWT) ([4],[6]) Wavelet Packet Transform – WPT ([7]…[11]) , Stationary Wavelet Transform – SWT ([12], [13]) etc. Sometimes these methods are integrated with fuzzy logic, artificial neural networks (ANN) or super vector machines (SVM) [5]. The studies from [14] were focused on tuning the parameters for the DWT analysis considering filters of length 8. The parameters of interest were: the interpolation method (spline was found to provide better results) and respectively the number of decimal places used for the DWT filters' coefficients, which are irrational numbers. Three of the above mentioned methods are considered now: (a) DWT, which is faster than SWT and is able to provide time-frequency information considering frequency ranges; (b) WPT, which is able to provide (to a certain extent) the harmonic spectra when relying on wavelet mothers (WM) like Daubechies with long filters, of length 28 and 30, and (c) SWT, which counteracts the decimation phenomenon with a certain price paid to the accuracy, being the only one to yield the component of fundamental frequency with a number of instantaneous values equal to the length of the analyzed signal.

II. OPERATIONAL CONTEXT

Our study is focused on the PQ analysis of waveforms acquired from the Power Plant Turceni, Romania (Fig. 1). Two WM, both from the Daubechies family were considered for the PQ analysis. They are denoted by'db14' and 'db20' in Matlab.

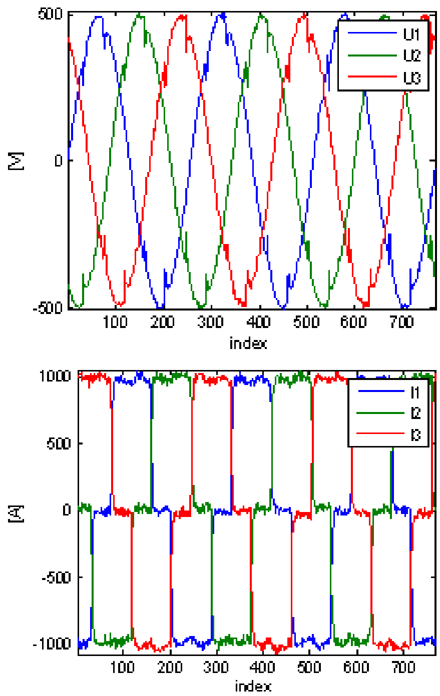

Fig. 1. Acquired waveforms (interpolated with spline). Top - currents; bottom - Voltages.

Their filters are long, having 28 and respectively 40 components. These WM were chosen because they provide the terminal nodes of the WPT tree with a good harmonic selectivity, allowing for an estimation of the harmonic spectrum to a certain extent [7]. The implemented decomposition tree had 6 levels and the decomposed signals were interpolated by using spline [17], such as to have 256 components.

III. Mathematic Support and Benefits of the Rounding Technique

A. Mathematic Support

Wavelet analysis, in any of the forms addressed by this paper, is characterized by filters with irrational coefficients and runtimes increasing with filters order. An analysis of the filters coefficients revealed an interesting property. The low/high pass filters involved in the direct wavelet decomposition are denoted by (*L*,*H*). (*Lr*,*Hr*) are used for the reversed wavelet decomposition. Certain coefficients are very small:

- $|L_i| < 10^{-4}$ and $|Hr_i| < 10^{-4}$ for i=1…n, where n=6 for 'db14' and n=10 for 'db20';
- $|H_{l-i}| < 10^{-4}$ and $|Lr_{l-i}| < 10^{-4}$ for i=1…n, where n=6, l=28 for 'db14' and n=10, l=40 for 'db20'.

The analyzed data are waveforms with maximal values of hundreds of Amperes and thousands of Volts. Therefore one can consider that the filters coefficients can be rounded to only 4 decimal places. In this way, 6 out of 24 coefficients (at 'db14') and respectively 10 out of 40 coefficients (at 'db20') became 0. The direct consequence of this rounding technique is related to the convolution products involved by the wavelet analysis. Now they are reduced to only 22 basic multiplicative operations (out of 28 involved in the conventional 'db14') and respectively to only 30 basic multiplicative operations (out of 40 in the conventional 'db20'). Runtime savings of over 20% are obtained accordingly.

Let us consider a certain decomposition level for which the decomposed signal *S* has the length *l* and therefore *l* sums have to be performed during decomposition. *l/2* of them are needed to evaluate the vector of approximations (denoted by *a*) whilst the rest correspond to the evaluation of the vector of details (denoted by *d*). Denoting by *h* the length of *L* and *H*, the equations used are:

$$a_i = \sum_{j=1}^{h} S(i+j-1) x H(j)\ ;\ d_i = \sum_{j=1}^{h} S(i+j-1) x L(j). \quad (1)$$

According to Eq. (1), each component is computed based on a sum of *h* terms obtained from products, and therefore it involves *h* iterations in a cycle. It means that for the approached level of decomposition, *l x h* multiplications-related iterations are to be performed. When *r* products become 0 as a consequence of neglecting *r* coefficients from *H* and *L*, the sums have *h-r* terms and therefore only *l x (h-r)* iterations are involved. The "per-level" runtime is consequently diminished with a percent equal to *(l x r))/(l x h),* which actually is *r/h*. This translates to 21.4 % for 'db14r' and respectively to 25% for 'db20r'.

Different operations are involved by different types of wavelet analysis. For example DWT is using an unbalanced tree, because only the vectors of approximations are decomposed. When a tree with 6 levels is used, the number of iterations is classically computed as $\sum_{k=1}^{6} l/2^{(k-1)} \times h$. When using the rounding, the number of iterations becomes: $\sum_{k=1}^{6} l/2^{(k-1)} \times (h-r)$. The percent runtime saving is accordingly equal to $\sum_{k=1}^{6} l/2^{(k-1)} \times r / \sum_{k=1}^{6} l/2^{(k-1)} \times h$ which finally translates to *r/h*.

On the other hand, WPT uses a balanced tree where all vectors are decomposed. For it the number of iterations in the classic approach is equal to 6 *x l x h* and it is diminished to 6 *x l x (h-r)*. In this case the percent runtime saving is also *r/h*.

For SWT the decimation phenomenon is counteracted at each step by up-sampling techniques. As long as the length of the signal decomposed at each level is the same (*l*), the number of iterations dedicated only to the wavelet decomposition is *l x h* and the runtime saving corresponding to it is therefore *r/h.* The up-sampling is not speeded up by rounding.

The simple conclusion to be drawn from the above is that in all analyzed wavelet-based analysis, the runtime saving associated to the wavelet decomposition is *r/h.*

B. Runtime Statistics

Statistical evaluations (considering sets of 20 consecutive programs executions) were performed considering full operational contexts, where all types of studied wavelets were involved in complex operations like evaluating PQ indices and harmonic spectra.

To be more specific, in order to evaluate the runtimes, the tested program considered a single waveform and included: (a) signal decomposition / reconstruction of signal with DWT; (b) decomposition / reconstruction of signal with SWT; (c) decomposition / reconstruction of signal with WPT and evaluation of harmonic spectrum; (d) evaluation of all RMS with all methods.

The mean runtime savings proved to be 20.4% for 'db14' and respectively 23.2% for 'db20'. The small differences relative to the net runtime-related savings associated only to wavelet decompositions are explained by the operations not involving them (up-sampling, evaluation of nodes energies and systems solving etc.)

IV. Performing Signals Reconstruction

To see the impact of the rounding procedure over the PQ analysis accuracy, the results yielded by the analysis with unmodified filters were compared to those obtained in the 'rounded' working frame. The rounded filters will be addressed from this point on as 'db14r' and 'db20r'.

A. Reconstruction of Signals Decomposed with DWT

The acquired signals were decomposed with DWT considering the 6 level tree with 'db14r' and 'db20r'. The vectors of approximation and details were afterward used to reconstruct the signals with filters also rounded to 4 decimal places. Figs. 2 and 3 depict the waveforms representing the difference "(signal decomposed with DWT) – (reconstructed signal)", when using 'db14r', respectively

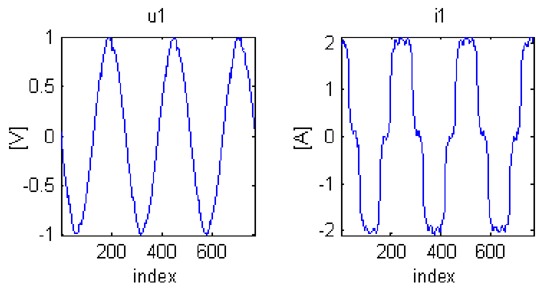

Fig. 2. Waveforms representing reconstruction errors when using DWT and ‘db14r’.

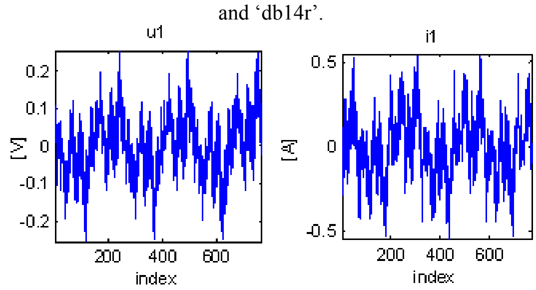

Fig. 3. Waveforms representing reconstruction errors when using DWT and ‘db20r’.

‘db20r’ for the pair of acquired waveforms u_1 and i_1. Similar results were obtained for the rest of waveforms.

Denoting by S the decomposed signal, by $rec14_{DWT}$ the error introduced by ‘db14r’ and respectively by $rec20_{DWT}$ the error introduced by ‘db20r’, both in the framework of a DWT decomposition, we could estimate the magnitude of errors by using two ratios, as follows:

$$e_{rec14\,,DWT} = \max(abs(rec14_{DWT}))/\max(abs(s))\cdot 100\ ; \quad (2)$$

$$e_{rec20\,,DWT} = \max(abs(rec20_{DWT}))/\max(abs(s))\cdot 100 \quad (3)$$

The errors introduced in the evaluation of reconstructed signals by ‘db14r’ and ‘db20r’ when using DWT are small: $e_{rec14,DWT}$ is ≈ 2% and $e_{rec20,DWT}$ is ≈ 0.05%.

B. Reconstruction of Signals Decomposed with SWT

Similar to the procedure applied when reconstructing signals decomposed with DWT relying on ‘db14r’ and ‘db20r’, signals were decomposed and reconstructed with SWT. The waveforms representing the reconstruction errors when using ‘db14r’ and SWT are almost identical to those from Fig. 2. Fig. 4 represents the reconstruction errors when using ‘db20r’ and SWT, for the pair of acquired waveforms u_3 and i_3. Similar results were obtained for the other 2 pairs of acquired waveforms.

Similar to the decomposition with DWT, we defined the following indices:

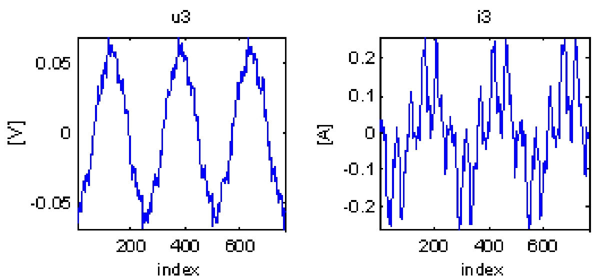

Fig. 4. Waveforms representing reconstruction errors when using SWT and ‘db20r’.

$$e_{rec14\,,SWT} = \max(abs(rec14_{SWT}))/\max(abs(S))\cdot 100\,; \quad (4)$$

$$e_{rec20\,,SWT} = \max(abs(rec20_{SWT}))/\max(abs(S))\cdot 100\,, \quad (5)$$

where $rec14_{SWT}$ represents the error introduced by ‘db14r’ and respectively $rec20_{SWT}$ represents the error introduced by ‘db20r’, both associated to the SWT decomposition.

The errors introduced in the evaluation of reconstructed signals by ‘db14r’ and ‘db20r’ when using SWT are small: $e_{rec14,SWT}$ is around 2% and $e_{rec20,SWT}$ is around 0.012% at voltages and respectively 0.026% at currents.

V. Rounding Effects over the WPT Analysis

The WPT analysis performs an evaluation of the harmonic spectrum by using the energy of nodes from the final level of the decomposition tree. We performed the WPT decomposition in 2 scenarios: in the 1-st one ‘db14’ and ‘db20’ were used whilst in the other one ‘db14r’ and ‘db20r’ were used. The differences between the nodes energies were evaluated in a percent relative manner:

$$e_{quant\,,F} = (quant_F - quant_{F_r})/quant_F \cdot 100\,, \quad (6)$$

where by *quant* we denoted either the nodes energies, or the harmonic weights , by F we denoted one of the Daubechies filters (‘db14’ or ‘db20’) and respectively by F_r we denoted either ‘db14r’, or ‘db20r’.

Figs. 5 and 6 depict the percent relative errors in the

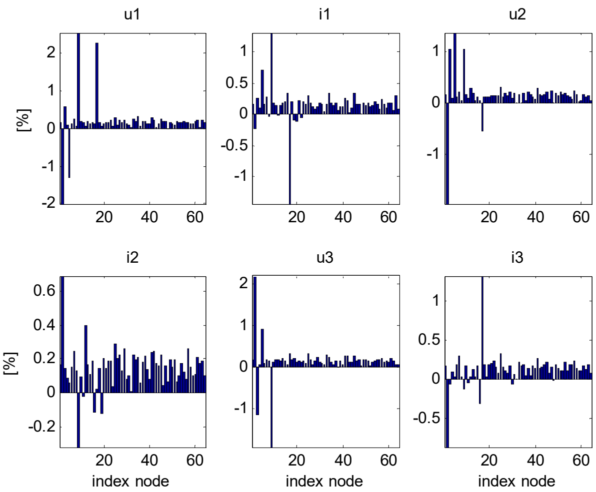

Fig. 5. Percent relative errors for nodes energies, considering ‘db14r’ relative to ‘db14’.

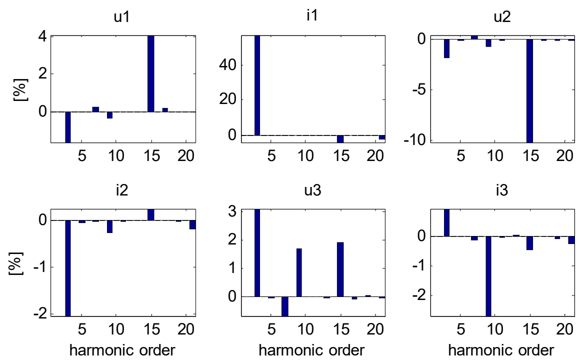

Fig. 6. Percent relative errors for harmonic spectra, considering ‘db14r’ relative to ‘db14’.

case of nodes' energies, respectively harmonic spectrum when a filter of length 28 is used whilst Figs. 7 and 8 represent their counterparts when considering the filter of length 40. Fig. 9 depicts the harmonic spectrum estimated with 'db20', which is similar to the one estimated with 'db14'.

The analysis of the percent relative errors (PRE) introduced by 'db14r' and by 'db20r' respectively when evaluating the nodes energies reveals that they do not exceed an absolute value of 2%, being either positive or negative in an apparently random manner. When addressing the PRE relative to the harmonic spectra, similar maximal deviations of 2% in absolute value were noticed for the significant harmonic weights. As revealed by the harmonic spectra, the weights of the harmonic orders 3, 9, 15 and 21 are very small and therefore the sometimes high differences recorded in their case are translated to absolute values of maximum 0.8% from the fundamental harmonic magnitude. Considering the oscillating nature of the vectors involved in the WPT decomposition they are highly acceptable from a practical point of view.

VI. Extracting the Components at Fundamental Frequency

SWT is unique among the other types of wavelet analysis considering the ability of extracting the instantaneous values of the component oscillating at the fundamental frequency. We will address the waveform composed by these components as "*IC*".

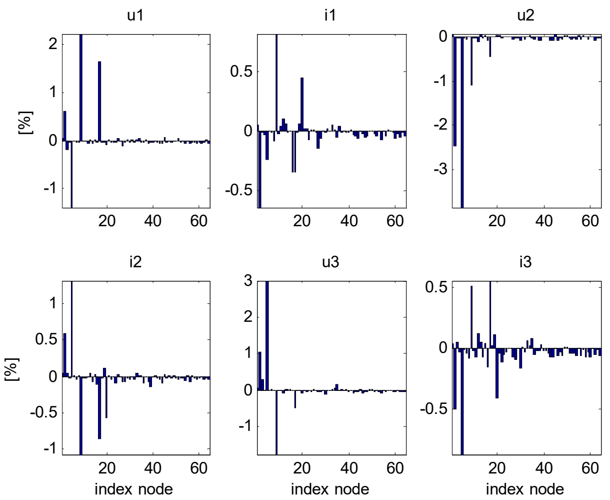

Fig. 7. Percent relative errors for nodes energies, considering 'db20r' relative to 'db20'.

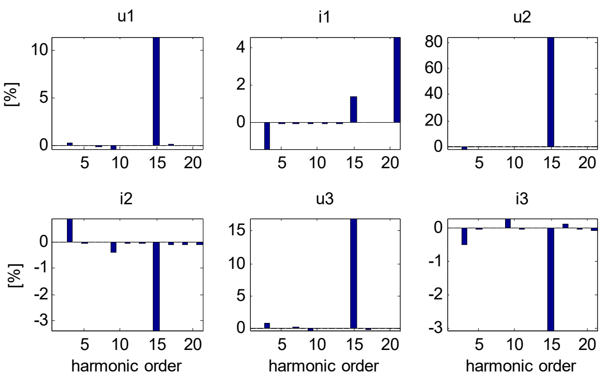

Fig. 8. Diagrams representing percent relative errors for harmonic spectra, considering 'db20r' relative to 'db20'.

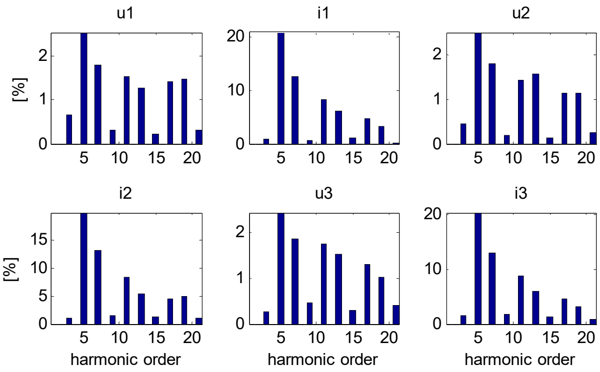

Fig. 9. Harmonic spectra generated with 'db20'.

Therefore the *IC*-s evaluated with 'db14', 'db14r', 'db20' and 'db20r' and the differences between the results yielded by the same class of WM were compared.

Fig. 10 depicts the *IC*-s evaluated with 'db14' and 'db14r', as well as the differences among them for the pair u_1 and i_1. Similar results were obtained for the other 2 pairs of acquired waveforms. Fig. 11 has the same role as Fig. 10, but for 'db20' and 'db20r'. The specifications "low" and "high" from these figures are associated to the use of 4, respectively 12 decimal places.

The differences between the components of the different evaluations of *IC* are highly acceptable and somehow proportional to *IC* as evaluated with 12 decimal places.

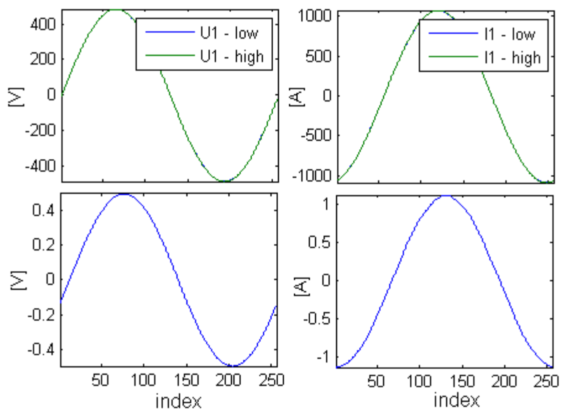

Fig. 10. Top: *IC*-s (overlapped) evaluated with 'db14' and 'db14r'
Down: differences among them.

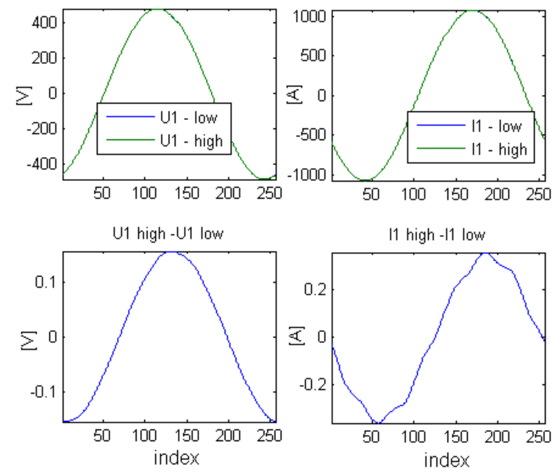

Fig. 11. Top: *IC*-s (overlapped) evaluated with 'db20' and 'db20r'.
Down: differences among them.

For ‘db14’ the ratios defined as those from Section IV are ≈ 0.1% , whilst for ‘db20’ they are smaller (≤0.03%).

VII. ROUNDING EFFECTS OVER THE VECTORS OF DETAILS YIELDED BY DWT

DWT is mainly used for the detection of faults, providing “time-frequency” information by means of the vectors of details (*VD*). Therefore a comparison between *VD*–s evaluated with (‘db14’,’db14r’) and respectively with (‘db20’,’db20r’) had to be done, in order to see the effects of the rounding technique.

Fig. 12 depicts the *VD* evaluated with ‘db14’ (for i_3 and u_3) and Fig. 13 depicts the differences between *VD* evaluated with ‘db14’ and *VD* evaluated with ‘db14r’ for the same pair of acquired waveforms. Similar information are provided by Figs. 14 and 15, but now for ‘db20’ and ‘db20r’ respectively.

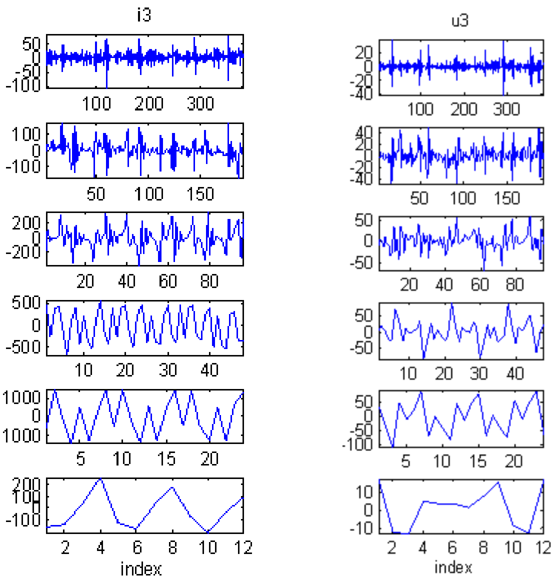

Fig. 12. Vectors of details generated with ‘db14’. Levels increasing from top to down.

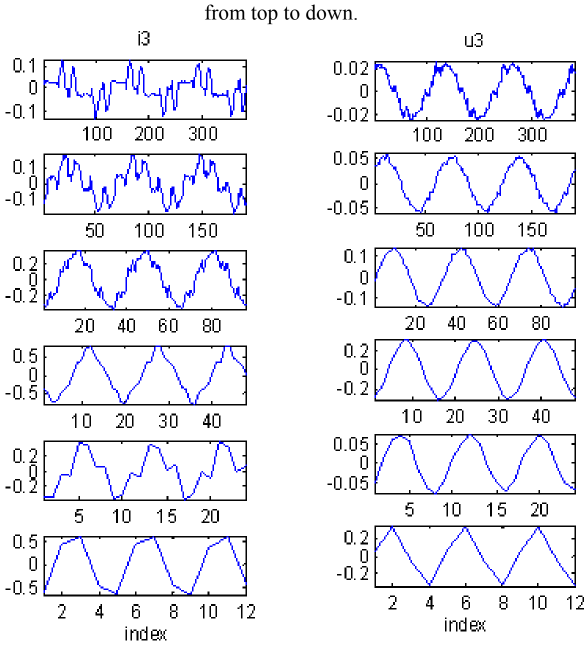

Fig. 13. Differences between the vectors of details generated with ‘db14’ and ‘db14r’. Levels increasing from top to down.

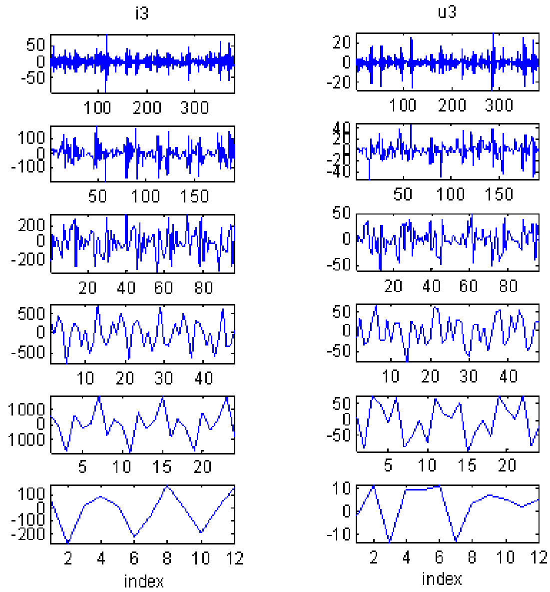

Fig. 14. Vectors of details generated with ‘db20’. Levels increasing from top to down.

Table I gathers the ratios used to evaluate the influence of the rounding procedures. They are evaluated with:

$$ratios\_for\_diff_{l,wm} = \frac{\max\left(abs\left(VD_{l,wm} - VD_{l,wmr}\right)\right)}{\max\left(abs\left(VD_{l,vm}\right)\right)}, \quad (7)$$

where *l* represents the level, *wm* represents the WM represented with 12 decimal places and *wmr* represents the WM represented with 4 decimal places.

An analysis of the Table I reveals that highly acceptable small differences were introduced by the rounding procedure in the *VD*. They are characterized by ratios evaluated with Eq.6 belonging to the range [0.02 … 1]% for currents and respectively [0.125...4] % for voltages. With a single exception (*VD* for the voltages in the 4-th level), the smallest differences were recorded when using ‘db20r’.

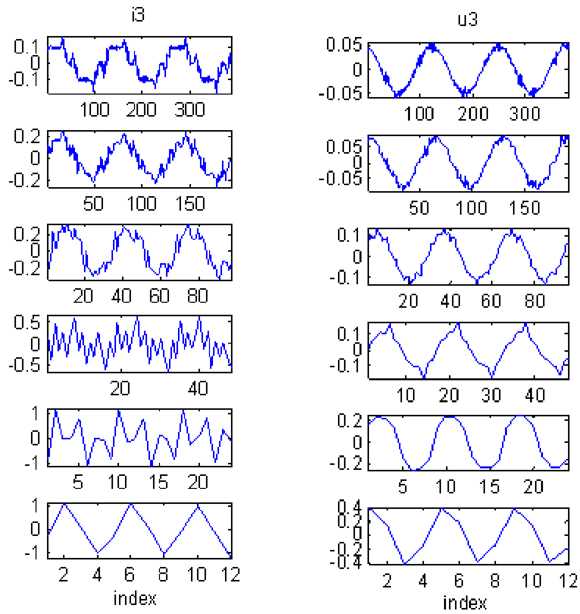

Fig. 15. Differences between the vectors of details generated with ‘db20’ and ‘db20r’. Levels increasing from top to down.

TABLE I.
RATIOS USED TO EVALUATE THE INFLUENCE OF THE ROUNDING PROCEDURES OVER THE INSTANTANEOUS VALUES OF DETAILS

Level	Wavelet mother	For currents [%]	For voltages [%]
1	*'db14'*	0.2	0.25
	'db20'	0.2	0.1
2	*'db14'*	0.2	0.125
	'db20'	0.1	0.1
3	*'db14'*	0.1	0.4
	'db20'	0.1	0.2
4	*'db14'*	0.1	0.2
	'db20'	0.1	0.4
5	*'db14'*	0.1	0.4
	'db20'	0.02	0.1
6	*'db14'*	1	4
	'db20'	0.5	2

VIII. CONCLUSIONS

The wavelet based analysis represents a useful tool, but the use of long filters involves a significant computational burden. The paper is concerned with the Daubechies WM with filters of length 28 and 40. Runtime savings of over 20% are obtained when performing rounding of filters' coefficients to the 4-th decimal place in all operations involving wavelet transforms. Statistics obtained with programs involving wavelet-based complex evaluations of PQ indices revealed runtime savings of 20.4% for 'db14' and respectively of 23.2% for 'db20'. The impact of rounding over the PQ analysis's accuracy was evaluated considering three-phase currents (significantly distorted) and voltages (with small to moderate harmonic content) acquired from the primary winding of the excitation transformer in a power group. The errors introduced by the rounding procedure were evaluated.

The PRE introduced in the evaluation of reconstructed signals by 'db14r' and 'db20r' when using DWT were found to be small, with maximum absolute values of around 2% for 'db14' and respectively of around 0.05% for 'db20', relative to the maximum absolute values of the decomposed signals. In the same context, it was demonstrated that SWT introduces even smaller differences (around 2% for 'db14r' and 0.012% at voltages and respectively 0.026% at currents for 'db20r').

The PRE introduced by 'db14r' and by 'db20r' respectively when evaluating the nodes' energies do not exceed an absolute value of 2%, being either positive or negative in an apparently random manner. As for the harmonic spectra, similar maximal deviations of 2% in absolute value were noticed for the significant harmonic weights.

The differences between different evaluations of *IC*-s are highly acceptable and somehow proportional to *IC* as evaluated with 12 decimal places. For 'db14' the ratios defined as those from Section IV are around 0.1%, whilst for 'db20' they did not exceed 0.03%.

Highly acceptable small differences were introduced by the rounding procedure in the vectors of details (*VD*). They are characterized by ratios evaluated with Eq.7 belonging to range [0.02 … 1] % for currents and respectively [0.125…4%] for voltages. In most of the cases the smallest differences were recorded when using 'db20r'.

In all types of wavelet analysis, the smallest errors were introduced in the case of 'db20r'.

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Contribution of authors:

First author – 45%

First coauthor – 30%

Second coauthor – 25%

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