

# Wireless Power Transfer Analysis Using Scattering Parameters

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**Abstract** - Wireless power transfer systems (WPTS) can be made more efficient from the emitter-receiver and from the active power transfer efficiency point of view by using the scattering parameters  $S$ . Using electrical circuit theory and starting from the correct formulation of scattering parameters,  $S$  parameters are computed in an automated manner based on modified nodal equations corresponding to WPTS. Advanced computation software (ADS, Ansoft Extractor Q3D, Cadence, etc.) is used to compute the parameters corresponding to the magnetically coupled coils. The magnetically coupled coils can be described by the parameters of the resulting equivalent circuit, such as: impedance and admittance matrices, transfer coefficients matrix and the scattering parameters matrix. This paper presents on one hand the correct formulation of  $S$  parameters and on the other hand the practical use of these parameters to streamline the wireless energy transfer. To generate the analyzed parameters, dedicated function were developed in MATLAB and existing routines from the Microwave toolbox (MATLAB) have been used. The frequency variation of  $S$  has been studied using graphical representation and Smith diagram. The accuracy in defining the scattering parameters  $S$ , based on analog circuit theory in harmonic state, in complex domain, is confirmed by the results obtained using the program ADS, that contains routines specific for generating  $S$  parameters.

**Cuvinte cheie:** sisteme de transmisie fără fir (wireless) a energiei, parametri  $S$ , diagrame Smith.

**Keywords:** Wireless Power Transfer System (WPTS),  $S$  parameters, efficiency, Smith chart.

## I INTRODUCTION

At medium and small frequencies, in harmonic state, a linear device can be characterized by using the following parameters:  $Z$ ,  $Y$ ,  $H$ ,  $T$  (transfer coefficients and impedance matrices). At microwave frequencies these parameters cannot be easily measured because they require particular circuits with certain branches open or short circuited (otherwise the voltage or the current from the circuit cannot be determined).

The most part of the active devices (diodes, transistors, etc.) are not stable at open or short-circuits. In order to compute the input impedance of a two port network, the output's gate must be short-circuited. This condition, at high frequencies, makes the measurement almost impossible from the practical point of view. This is why the description using the scattering parameters is preferred.

$S$ -parameters can be measured by embedding the two-port network in a transmission line whose ends are con-

nected to a network analyzer. A typical network analyzer can measure  $S$ -parameters over a large frequency range, for example, the HP 8720D Vector Network Analyzer (VNA) covers the range from 50 MHz to 40 GHz. Frequency resolution is typically 1 Hz and the results can be displayed either on a Smith chart or as a conventional gain versus frequency graph [1–5].

Because there are many ways to define and introduce the scattering parameters [1-3], sometimes their understanding and physical interpretation become problematic. Usually,  $S$  parameters are defined for information transmission systems (waveguides) [2-5] and studied using the theory of electric circuits [3-8].

As for microwave theory, a similar way of thinking has been followed in circuit theory, by introducing the concept of power waves, terminology coming from their significance which is related to the dependence between the active powers absorbed by a load connected to a port and the working frequency [4, 6].

$S$  parameters can be converted to the circuit theory classical parameters (impedances  $Z$ , admittances  $Y$ ,  $H$  parameters, transfer parameters  $T$  ( $A$ ,  $B$ ,  $C$ , and  $D$ ) etc.) by using the conversion formulas, but due to the misunderstanding of their significance, some formulas are useless [5–10]. Understanding  $S$  parameters is particularly important for applications that operate at high frequencies including active and passive components in integrated circuits [7], and as well as power transfer systems "without galvanic contacts" (wireless) [10].

For different WPTS structures, the modified nodal equations generated by Circuit Symbolic Analysis Program (CSAP), [1, 11, 12], or the state equations generated using the program SYmbolic State Equation Generation, (SYSEG) [13, 14] can be used to generate automatically  $S$  parameters.

Advanced computation software (Cadence [15], ADS [16], Ansoft Extractor Q3D [17], Feko [2, 18] etc.) is used to generate the parameters corresponding to magnetically coupled coils. If we consider the two magnetically coupled coils as a passive linear 2-port network in harmonic regime, it can be described by a number of equivalent circuit's parameters, such as: the transfer matrix  $T$ , the impedance matrix  $Z$ , the admittance matrix  $Y$  and the matrix corresponding to the parameters  $S$ . This paper presents the relationships that allow the transition from one matrix to another. There are also presented several procedures for calculating these parameters [12-14].

Using the theory of electric circuits, we present: a correct formulation of  $S$  parameters, the automatic generation of  $S$  matrix and of the scattering parameters and then,

based on this matrix, the computation of  $\mathbf{Z}$ ,  $\mathbf{Y}$  and  $\mathbf{T}$  matrices. Next, considering the two magnetically coupled coils (resonators), used in WPTS [19–21, 22, 23], as a linear passive 2–port network, there are determined: the input and output complex impedances,  $Z_{in}$  and  $Z_{out}$ , the reflection coefficients from the generator  $\Gamma_G$ , from the load  $\Gamma_L$ , from the input  $\Gamma_{in}$  and from the output  $\Gamma_{out}$  and the relationships between these quantities and S parameters are being deduced.

These parameters were used to make more efficient the propagation and transmission processes of information and electromagnetic wireless energy transfer (from transmitting the signal (transmitter to receiver) point of view of and from active power transfer at the input of WPTS at the loads connected to its output point of view). The maximum active power transmitted by a WPTS to its load is computed using the maximum active power transfer theorem. The performance coefficients such as: transducer power gain  $G_t$ , available power gain  $G_a$  and the power gain  $G_p$ , also called the operating gain, and has been also computed.

Based on the developed procedures, were implemented in MATLAB programming routines to compute the above-mentioned parameters and the results obtained by simulation were compared with those existing in literature and with the experimental ones. The frequency dependence of  $\underline{S}$  parameters is graphically presented on Smith chart.

## II SCATTERING PARAMETERS

To compute the efficiency of signal transmission in microwave networks and in WPTS, the scattering parameters S are used. The literature presents efficient methods and techniques for measuring  $\underline{S}$  parameters [1, 24], so these parameters can be used to make the transmission and propagation processes of wireless electromagnetic energy transfer more efficient [19–23].

To formulate and define, in a correct manner,  $\underline{S}$  parameters, let's consider the circuit from Figure 1 [1].

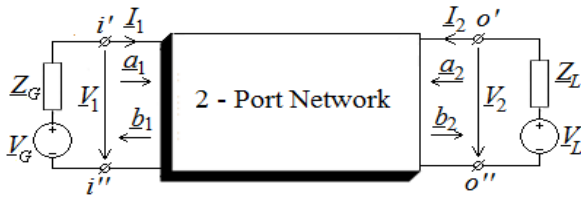


Fig. 1. The circuit used to define S parameters.

Firstly, it takes place the substitution of variables pair  $(V_1, I_1)$  and  $(V_2, I_2)$  with the pair  $(a_1, b_1)$  and, respectively  $(a_2, b_2)$ , according to the relationships:

$$\begin{aligned} V_1 &= \sqrt{Z_0}(a_1 + b_1), \quad I_1 = \frac{1}{\sqrt{Z_0}}(a_1 - b_1) \\ V_2 &= \sqrt{Z_0}(a_2 + b_2), \quad I_2 = \frac{1}{\sqrt{Z_0}}(a_2 - b_2), \end{aligned} \quad (1)$$

where  $Z_0$  is a real, positive variable, named–*reference (characteristic) impedance*. In practice, the reference impedance is chosen to be  $Z_0 = 50$  ohm, [2–8]. By simi-

larity with the wave equation, the solution  $a_1$  ( $a_2$ ) represents the *direct wave* from gate  $i' - i''$  ( $o' - o''$ ), and  $b_1$  ( $b_2$ ) is the *inverse wave* at the same gate. For linear circuits, the variables associated to each gate, can be considered as a superposition of direct waves and of the inverse waves [6, 22–25]. The modules of the new variables have the magnitude  $\sqrt{AV}$ , showing that the square of these modules have the dimension of the electric power. Usually, the reference impedance is equal to the load impedance module. By solving the equations (1), function of the new variables  $(a_1, b_1)$  and  $(a_2, b_2)$ , one obtains:

$$\begin{aligned} a_1 &= \frac{1}{2} \left( \frac{1}{\sqrt{Z_0}} V_1 + \sqrt{Z_0} I_1 \right) = \frac{1}{2\sqrt{Z_0}} (V_1 + Z_0 I_1) \\ b_1 &= \frac{1}{2} \left( \frac{1}{\sqrt{Z_0}} V_1 - \sqrt{Z_0} I_1 \right) = \frac{1}{2\sqrt{Z_0}} (V_1 - Z_0 I_1) \end{aligned} \quad (2)$$

respectively,

$$\begin{aligned} a_2 &= \frac{1}{2} \left( \frac{1}{\sqrt{Z_0}} V_2 + \sqrt{Z_0} I_2 \right) = \frac{1}{2\sqrt{Z_0}} (V_2 + Z_0 I_2) \\ b_2 &= \frac{1}{2} \left( \frac{1}{\sqrt{Z_0}} V_2 - \sqrt{Z_0} I_2 \right) = \frac{1}{2\sqrt{Z_0}} (V_2 - Z_0 I_2) \end{aligned} \quad (3)$$

The scattering parameters  $\underline{S}$  of a 2–port network (Fig. 1) satisfy the following relations between the direct and inverse signals:

$$\begin{aligned} b_1 &= \underline{S}_{11} a_1 + \underline{S}_{12} a_2 \\ b_2 &= \underline{S}_{21} a_1 + \underline{S}_{22} a_2 \end{aligned} \quad (4)$$

A linear circuit can be analyzed as a multi–port network. Each gate is sequentially excited using small signals and the answer is transformed into  $\underline{S}$  parameters. Fig. 1 offers the representation of a wave as a 2–port network. For each of the two gates, the following signals are being defined [1–10]:  $a_1$  – input signal at gate 1;  $b_1$  – output signal at gate 1;  $a_2$  – input signal at gate 2;  $b_2$  – output signal at gate 2.

The  $\underline{S}$  parameters are correct defined in the paper [1].  $\underline{S}$  parameters for the 2–port network (Fig.1) have the following significance:  $\underline{S}_{11}$  – the reflection coefficient from gate 1;  $\underline{S}_{22}$  – the reflection coefficient from gate 2;  $\underline{S}_{21}$  – the transmission coefficient from gate 1 to gate 2;  $\underline{S}_{12}$  – the transmission coefficient from gate 2 to gate 1. The reflection factor  $\underline{S}_{11}$  and the transmission factor  $\underline{S}_{21}$  can be measured using VNA [13–15, 24].

$\underline{S}$  parameters generation for linear analog circuits and for nonlinear analog, piecewise linear around a working point, in precise polarization and temperature conditions for electronic devices, can be done by small signal simulations [3, 26]. To generate  $\underline{S}$  parameters for nonlinear circuits it is used the simulator – Large-Signal  $\underline{S}$ -Parameter Simulation (SPP) [5], that uses the harmonic balance analysis method. This is a simulation of big signal for which the solutions include also the effects due to the nonlinearities of the electronic components.  $\underline{S}$  parameters both for small and big signals are defined as ratios between the direct and inverse waves [1, 5].

Very simply, we can prove that:

1. The complex impedance matrix  $\underline{\mathbf{Z}} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$  can be expressed function of  $\underline{\mathbf{S}}$  parameters matrix,  $\underline{\mathbf{S}} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$ , using the relation:

$$\underline{\mathbf{Z}} = (\mathbf{I}_2 - \underline{\mathbf{S}})^{-1} \cdot (\mathbf{I}_2 + \underline{\mathbf{S}}) \mathbf{Z}_0 \quad (5)$$

where  $\mathbf{I}_2$  is the order 2 unit matrix;

2. The matrix  $\underline{\mathbf{S}}$  is expressed function of matrix  $\underline{\mathbf{Z}}$  using the relation:

$$\underline{\mathbf{S}} = (\underline{\mathbf{Z}} - Z_0 \mathbf{I}_2) \cdot (\underline{\mathbf{Z}} + Z_0 \mathbf{I}_2)^{-1} \quad (6)$$

3. The transfer matrix  $\underline{\mathbf{T}} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$  has the following expression, function of matrix  $\underline{\mathbf{Z}}$  elements:

$$\underline{\mathbf{T}} = \frac{1}{Z_{21}} \begin{bmatrix} Z_{11} & Z_{11} \cdot Z_{22} - Z_{12} \cdot Z_{21} \\ 1 & Z_{22} \end{bmatrix} \quad (7)$$

4. The expression of matrix  $\underline{\mathbf{Z}}$  function of  $A$ ,  $B$ ,  $C$  and  $D$  parameters is:

$$\underline{\mathbf{Z}} = \frac{1}{C} \begin{bmatrix} A & A \cdot D - B \cdot C \\ 1 & D \end{bmatrix} \quad (8)$$

### III. COMPUTING THE EFFICIENCY OF WPTS USING $\underline{\mathbf{S}}$ PARAMETERS

To define the input and output complex impedances and the reflection coefficients  $\Gamma_{in}$  (input),  $\Gamma_{out}$  (output),  $\Gamma_G$  (generator) and  $\Gamma_L$  (load), the system consisting of the two magnetically coupled coils, used in WPTS, is equivalent to the one represented in Fig. 2 (passive linear 2-port network in harmonic state) [1–9].

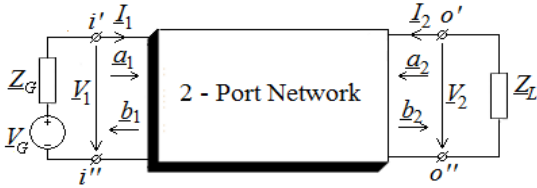


Fig. 2. Passive linear 2-port network connected to generator and to the load.

Input impedance ( $i' - i''$ )  $\underline{\mathbf{Z}}_{in}$ :

$$\underline{\mathbf{Z}}_{in} = \left. \frac{V_1}{I_1} \right|_{V_G=0, V_2=-Z_L I_2} \quad (9)$$

Taking into consideration the 2-port network impedances equations:

$$\begin{cases} V_1 = Z_{11} I_1 + Z_{12} I_2 \\ V_2 = Z_{21} I_1 + Z_{22} I_2, V_2 = -Z_L I_2 \end{cases} \quad (10)$$

results:

$$\underline{\mathbf{Z}}_{in} = Z_{11} - \frac{Z_{12} Z_{21}}{Z_L + Z_{22}} \quad (11)$$

Output impedance ( $o' - o''$ )  $\underline{\mathbf{Z}}_{out}$ :

$$\underline{\mathbf{Z}}_{out} = \left. \frac{V_2}{I_2} \right|_{V_G=0, V_1=-Z_G I_1} \quad (12)$$

Taking into consideration the 2-port network impedances equations:

$$\underline{\mathbf{Z}}_{out} = Z_{22} - \frac{Z_{12} Z_{21}}{Z_G + Z_{11}} \quad (13)$$

The reflection coefficient from the input  $\Gamma_{in}$ :

$$\Gamma_{in} = \left. \frac{b_1}{a_1} \right|_{V_G=0, V_1=Z_{in} I_1, V_2=-Z_L I_2} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} \Rightarrow \quad (14)$$

$$\Rightarrow Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}}$$

The reflection coefficient from the generator  $\Gamma_G$  has:

$$\Gamma_G = \left. \frac{a_1}{b_1} \right|_{V_G=0, V_G=-Z_G I_1, V_2=-Z_L I_2} = \frac{Z_G - Z_0}{Z_G + Z_0} \Rightarrow \quad (15)$$

$$\Rightarrow Z_G = Z_0 \frac{1 + \Gamma_G}{1 - \Gamma_G}$$

The reflection coefficient from the output  $\Gamma_{out}$ :

$$\Gamma_{out} = \left. \frac{b_2}{a_2} \right|_{V_G=0, V_1=Z_{in} I_1, V_2=Z_{out} I_2} = \frac{Z_{out} - Z_0}{Z_{out} + Z_0} \Rightarrow \quad (16)$$

$$\Rightarrow Z_{out} = Z_0 \frac{1 + \Gamma_{out}}{1 - \Gamma_{out}}$$

The reflection coefficient from the load  $\Gamma_L$  has:

$$\Gamma_L = \left. \frac{a_2}{b_2} \right|_{V_G=0, V_G=-Z_G I_1, V_2=-Z_L I_2} = \frac{Z_L - Z_0}{Z_L + Z_0} \Rightarrow \quad (17)$$

$$\Rightarrow Z_L = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L}$$

Taking into consideration equations from (4) and the definition relations corresponding to the four reflection coefficients  $\Gamma_{in}$ ,  $\Gamma_G$ ,  $\Gamma_{out}$  and  $\Gamma_L$ , after a few simple computations,  $\Gamma_{in}$  and  $\Gamma_{out}$  can be expressed function of  $\underline{\mathbf{S}}$  parameters, as follows:

$$\Gamma_{in} = S_{11} + \frac{S_{12} S_{21} \Gamma_L}{1 - S_{22} \Gamma_L} \quad (18)$$

$$\Gamma_{out} = S_{22} + \frac{S_{12} S_{21} \Gamma_G}{1 - S_{11} \Gamma_G}$$

$\underline{\mathbf{Z}}_{in}$  and  $\underline{\mathbf{Z}}_{out}$  allow substitution, according to the Thévenin theorem, of the 2-port network form Figure 2 with the equivalent circuits presented in Figure 3 [12–14].

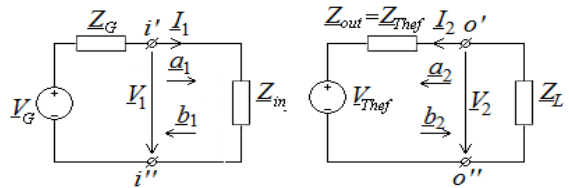


Fig. 3. Input and output equivalent circuits.

The voltage:

$$\underline{V}_{Thef} = \underline{V}_2 \Big|_{I_2=0, \underline{V}_1=\underline{V}_G-\underline{Z}_G I_1} = \frac{\underline{Z}_{21}}{\underline{Z}_G + \underline{Z}_{11}} \underline{V}_G$$

and

$$\underline{Z}_{Thef} = \frac{\underline{V}_{Thef}}{\underline{I}_2} = \underline{Z}_{out}$$

The system consisting of two magnetically coupled resonators used in WPTS can be considered as a linear passive 2-port network.

The received active power, at input gate  $i' - i''$  (Fig. 1), has the expression:

$$P_1 = P_{in} = \text{Re}(\underline{V}_1 \cdot \underline{I}_1^*) = |a_1|^2 - |b_1|^2 \quad (19)$$

The given active power, at output gate  $o' - o''$  (Fig. 1) has the expression:

$$P_2 = P_L = -\text{Re}(\underline{V}_2 \cdot \underline{I}_2^*) = -|a_2|^2 + |b_2|^2 \quad (20)$$

if we take into account  $\mathbf{b} = \underline{\mathbf{S}} \cdot \mathbf{a}$  with  $\underline{\mathbf{S}} = \begin{bmatrix} \underline{S}_{11} & \underline{S}_{12} \\ \underline{S}_{21} & \underline{S}_{22} \end{bmatrix}$ .

The transmission efficiency of the active power from the transmitter coil (from input gate  $i' - i''$ ) to a load (output gate  $o' - o''$ ) can be computed using the relations:

$$\begin{aligned} \eta_{21} &= \frac{P_2}{P_1} \cdot 100 = \frac{R_L}{R_{in}} \cdot \left| \frac{\underline{Z}_{21}}{\underline{Z}_{22} + \underline{Z}_L} \right|^2 \cdot 100 = \\ &= \frac{|S_{21}|^2 \cdot (1 - |\Gamma_L|^2)}{\left(1 - \left|1 - |\Gamma_L|^2\right|^2\right)} \cdot |1 - \underline{S}_{22} \Gamma_L|^2 \cdot 100 \end{aligned} \quad (21)$$

where  $R_{in} = \text{Re}(\underline{Z}_{in})$  and  $R_L = \text{Re}(\underline{Z}_L)$ .

The signal transmission efficiency from the transmitter coil to the load (from  $i' - i''$  to  $o' - o''$ ) is:

$$\eta_{21\_S21} = \underline{S}_{21} \cdot \underline{S}_{21}^* \cdot 100 = |S_{21}|^2 \cdot 100 \quad (22)$$

And the signal transmission efficiency from the receiver coil to the generator (from  $o' - o''$  to  $i' - i''$ ) is:

$$\eta_{12\_S12} = \underline{S}_{12} \cdot \underline{S}_{12}^* \cdot 100 = |S_{12}|^2 \cdot 100 \quad (23)$$

According to the maximum power transfer theorem, the load receives maximum power if  $\underline{Z}_L = \underline{Z}_{out}^*$  and it has the expression:

$$P_{L\_max} = \frac{V_{Thef}^2}{4 \cdot \text{Re}(\underline{Z}_{out})} \quad (24)$$

Three definitions widely used for the power gain of a 2-port network are: the transducer power gain  $G_t$ , the available power gain  $G_a$  and the power gain  $G_p$ , also called the operating gain [25–27].

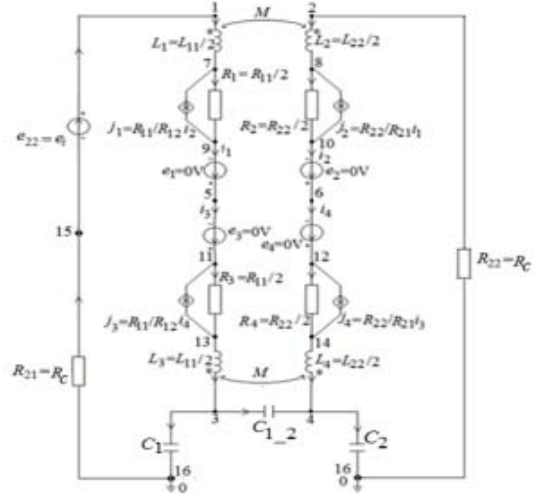
#### IV EXAMPLE

As an example we consider the two magnetically coupled printed coils from Figure 4a. In Figure 4b we

present the equivalent SPICE scheme obtained using ANSYS EXTRACTOR Q3.



(a)



(b)

Fig. 4. a) Two magnetically coupled printed coils b) the SPICE equivalent circuit.

The parameters of the corresponding equivalent circuit (Fig. 4, b) have the following values:  $L_1 = L_3 = 1.78035 \mu\text{H}$ ,  $L_2 = L_4 = 1.78046 \mu\text{H}$ ,  $M = 0.3241 \mu\text{H}$ ,  $k = 0.18203$ ,  $C_1 = C_2 = 2.2123 \text{ pF}$ ,  $C_{12} = 4.2123 \text{ pF}$ ,  $R_1 = R_3 = 0.12469 \Omega$ ,  $R_2 = R_4 = 0.1267 \Omega$ ,  $R_{12} = R_{21} = 0.0568 \Omega$ ,  $B_{1,2} = R_{12}/(2.0 \cdot R_1) = 0.45553$ ,  $B_{2,1} = R_{21}/(2.0 \cdot R_2) = 0.45524$ ,  $R_i = R_L = R_c = Z_0 = 5.0 \Omega$ ,  $E_i = 15 \text{ V}$ ,  $d = 100 \text{ mm}$ .

The analysis was performed for the case when the coupling factor is:  $k = M / \sqrt{L_1 \cdot L_2} = 0.18203$  and the distance between the coils is 100 mm. As follows, we present the results obtained by running the routines implemented in MATLAB.  $\underline{\Gamma}_G$  and  $\underline{\Gamma}_L$  are depicted in Figures 5a and 5b. In Figures 6a and 6b are presented the frequency variations of the modules corresponding to the scattering parameters  $\underline{S}_{22}$  and  $\underline{S}_{21}$  obtained using CSAP (Fig. 6, a) and respectively, ADS program (Fig. 6, b).

The validity of the obtained results is confirmed by the fact that the two variations are identical, regardless the used computation software. Figure 7 presents the variation of the maximum power (defined using relation (24)) with respect to frequency, and Fig. 8 shows the variations with respect to frequency of the efficiencies:  $\eta_{21}$ ,  $\eta_{21\_S21}$ ,  $C_p \cdot 100$ ,  $\eta_2$  and  $\eta_{12\_S12}$ . From Figure 8, the variations of active power transmission efficiency and of the signals from input to output are identical to those transmitted from output to input, because the 2-port net-

work of the two magnetically coupled coils form Figure 4 is a symmetric 2-port network.

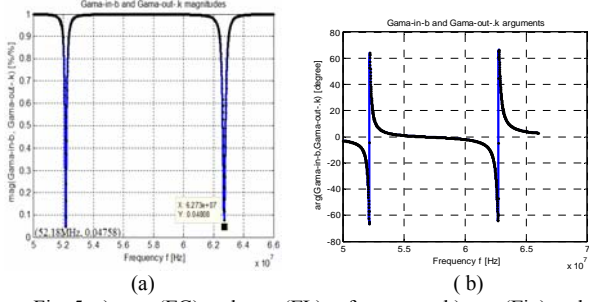


Fig. 5. a)  $\text{mag}(\Gamma)$  and  $\text{mag}(\Gamma_L)$  vs frequency; b)  $\text{arg}(\Gamma_{in})$  and  $\text{arg}(\Gamma_{out})$  vs frequency.

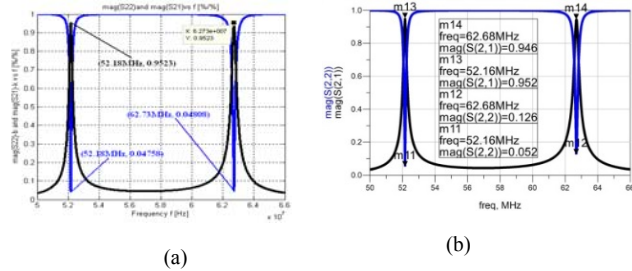


Fig. 6. a)  $\text{mag}(S_{22})$  and  $\text{mag}(S_{21})$  vs frequency - CSAP; b)  $\text{mag}(S_{22})$  and  $\text{mag}(S_{21})$  vs frequency - ADS.

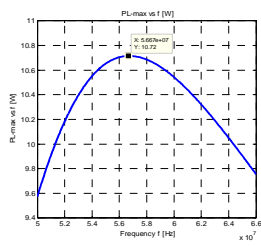


Fig. 7. Maximum load power vs frequency.

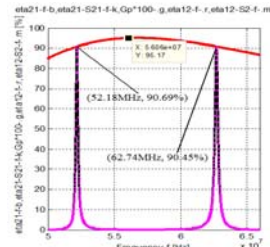


Fig. 8. Efficiencies vs frequency.

For  $k_1 = 0.18203$ ,  $d_1 = 100$  mm;  $k_2 = 0.12489$ ,  $d_2 = 150$  mm;  $k_3 = 0.09458$ ,  $d_3 = 200$  mm and  $k_4 = 0.086278$ ,  $d_4 = 220$  mm, in Figure 9 are presented the variation with frequency of the signals transmission efficiency  $\eta_{21\_S21} = \eta_{12\_S12}$ , obtained using the program CSAP.

Smith chart is one of the most used graphical procedures for high frequency circuit applications. Smith chart offers an intelligent way to visualize the complex functions and continues to be used decades after its discovery. From mathematical point of view, Smith chart is a 4-D representation of all possible complex impedances function of the coordinates defined by the complex reflection coefficient. The definition domain of the reflection coefficient is a circle with unit radius in the complex plane. In general, to a transmission line with losses, the reflection coefficient can have the amplitude bigger than one, due to the characteristic impedance, that requires an extension of Smith chart [1, 27]. In Figures 10a–d are depicted the variations with frequency on Smith chart of  $\underline{S}$  parameters. The system of the two studied magnetically coupled coils, being symmetrical, has the corresponding Smith charts of the parameters  $\underline{S}_{11}$  and  $\underline{S}_{22}$  ( $\underline{S}_{12}$  and  $\underline{S}_{21}$ ) identical.

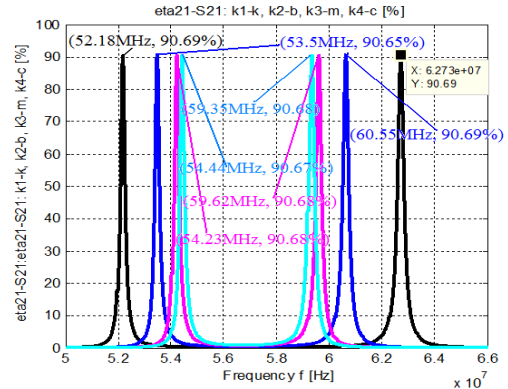


Fig. 9. Efficiencies vs frequency for  $k_1, k_2, k_3$ , and  $k_4$  - CSAP.

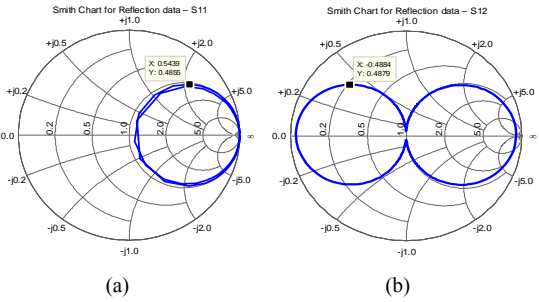


Fig. 10. Smith charts for S parameters corresponding to WPTS form Fig. 4: a)  $S_{11}$ ; b)  $S_{12}$ ; c)  $S_{21}$ ; d)  $S_{22}$ .

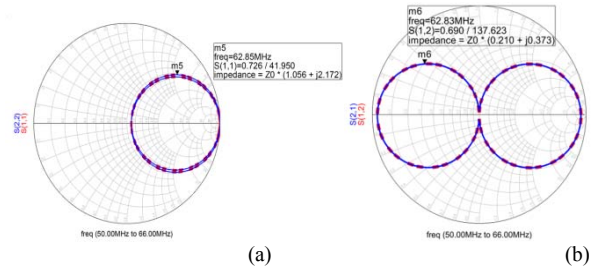


Fig. 11. Smith charts for S parameters corresponding to WPTS form Fig. 4 obtained using ADS: a)  $S_{11}$  and  $S_{22}$ ; b)  $S_{12}$ ; c)  $S_{21}$  and  $S_{21}$ .

Figures 10a and 10b present the variations with frequency on Smith chart of parameters  $\underline{S}_{11}$  and  $\underline{S}_{22}$  and respectively,  $\underline{S}_{12}$  and  $\underline{S}_{21}$ , obtained using the program CSAP. The system being symmetrical, the Smith chart of the parameters  $\underline{S}_{11}$  and  $\underline{S}_{22}$  ( $\underline{S}_{12}$  and  $\underline{S}_{21}$ ) are identical. From Figures 10a and 10b and from Figures 11a and 11b we see the results obtained using MATLAB are identical to the results obtained using ADS.

## CONCLUSIONS

This paper, by extending the considerations made in [27] presents, based on electric circuit theory, the correct definition of  $\underline{S}$  parameters was followed by their automated generation. This automatic generation has been performed using the state equations or the modified nodal equations. After obtaining  $\underline{S}$  parameters, the next step was to generate the: the matrices  $\underline{T}$ ,  $\underline{Z}$  and  $\underline{Y}$ ; the reflection coefficients  $\Gamma_G, \Gamma_L, \Gamma_{in}$  and  $\Gamma_{out}$ ; the active power transmission efficiencies  $\eta_{21}$  and  $\eta_{12}$ ; the signal transmission efficiencies  $\eta_{21\_S_{21}}$  and  $\eta_{12\_S_{12}}$ ; input and output impedances  $Z_{in}$  and  $Z_{out}$ ; Thévenin equivalent generator; the transducer power gain  $G_p$ , the available power gain  $G_a$  and the power gain  $G_p$ , and the Smith charts for  $\underline{S}$  parameters. By considering the 2 magnetically coupled coils (used for wireless power transfer), as a linear passive 2-port network, in harmonic state, function of  $\underline{S}$  parameters, all the parameters described above were determined.

This paper also presents the practical use of these parameters to make the transmission and propagation processes more efficient for electromagnetic energy wireless transfer from transmitter to receiver. By comparing the variations with frequency of the active power transmission efficiency with those corresponding to signals transmission, we conclude the frequencies corresponding to extreme points are not identical. The active power transmission efficiencies have only one maximum, while the signals transmission efficiencies present two maxima, showing in this case the splitting frequency phenomenon.

The linear passive 2-port network used to equivalent the system consisting of the two magnetically coupled coils allows the computation of the maximum active power given to the load. From the mathematical point of view, the Smith chart is a 4-D representation of all complex impedances function of the complex reflection coefficient, for any given frequency.

From Figs. 9 we can see the variation of the signals transmission efficiencies function of the coils' coupling coefficient. The accuracy in defining the scattering parameters  $\underline{S}$ , based on analog circuit theory in harmonic state, in complex domain, is confirmed by the results obtained using commercial software that contains routines specific for generating  $\underline{S}$  parameters.

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