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Analyzing the Influence of Harmonic Parameters on Accuracy Indices When Applying Wavelet Transform

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Abstract **- The paper deals with the evaluation of root mean square deviations and maximum absolute relative errors associated to the decomposition followed by recomposition based on Wavelet Packet Transform (WPT) of signals polluted with harmonics. Subtrees associated to sets of harmonics presenting practical interest for industrial applications are addressed. The study uses artificial signals generated through the superposition of perfect sinusoids with pairs of harmonics which proved to be related in an almost exclusive manner to pairs of nodes from the bottom level of a WPT tree. 4 parameters had to be considered when determining the maximum and minimum values of errors for each set: the clustered harmonics' magnitudes and their phase-shifts relative to the component of fundamental frequency. The decomposition/recomposition are time-efficient due to an original system of flags labeling each node from the WPT tree. For each analyzed set of harmonics, 3d graphical representation of minimum and maximum errors along with the associated 3d graphical representation of the phase-shifts are provided. At the same time, per set limits of errors ranges were established and discussed while specific patterns were deduced for the context in which extreme errors appear (phase-shifts and harmonic magnitudes). The results were commented, and conclusions were drawn.**

Cuvinte cheie: *calitatea puterii, analiză Wavelet cu arbori binari, estimarea erorilor. analiza asistata de calculator.*

Keywords: *power quality, wavelet analysis with binary trees, errors estimation, computer aided analysis.*

I. Introduction

Defining the best signal analysis method is a never-ending battle. Each of the methods has its own virtues and flaws which make their applicability limited to certain cases. Fast Fourier Transform (FFT) is a very popular method because it gives full harmonic spectrum, has a short runtime, and has low computational effort. But this method is more precise when analyzing signals that have stationary nature. Because FFT assumes the asymmetry between half-periods of a signal, it shows poor results when computing signals of non-stationary nature.

Short-Time Fourier Transform (STFT) is another popular and fast method that uses a window for diving a signal into smaller parts, thus making the analysis more precise. On the other hand, the window length is fixed so the resolution will be constant for all frequencies. Therefore the analysis with STFT will provide good results for either low-frequency (LF) or high-frequency (HF) spectrum, but not for both.

Wigner-Ville Distribution (WVD) and Pseudo-WVD are methods that are both bilinear in nature and artificial cross terms appear in the decomposition results rendering the feature interpretation problematic.

For all the above-mentioned methods the common flaw is that they are non-reversible [1]. Many of these problems can be solved using Wavelet transform (WT). Wavelet transform is considered to be a significant breakthrough in mathematical analysis. It can be applied to various fields. For example, signal processing, image processing, pattern recognition, speech analysis and many applications could introduce wavelet analysis [2]. It is a timescale transform that uses a variable-length window so it provides good resolution for both LF and HF spectrum, while preserving both time and frequency information. The authors have previously studied a specific type of WT called Wavelet Packet Transform (WPT) that was proposed in 1992 [3]. WPT provides full harmonic spectrum but suffers from decimation phenomenon [4]-[7]. Detailed analysis of wavelet binary tree shown the best parameters (number of levels, wavelet mother, filter length) when applying WT.

The authors extensively studied a specific case of a wavelet tree with seven levels (T7) that uses a wavelet mother from Daubechies family and filter length of 28 ("db14") [6]. One of the important conclusions was that nodes from the bottom level of the wavelet tree exhibits cluster patterns [7]-[10]. This means that nodes can be grouped in clusters of 2, 4 or 8 nodes where each group is affected by 2, 4 and 8 harmonics respectively.

II. Nodes-Harmonics Pairing Patterns and Run-time Saving Decompositions and recompositions Relying on WPT Tress

The Wavelet binary tree (T7) used in this paper was tested in many different operational contexts [6…9]. It has the following characteristics: 7 levels of decomposition, Daubechies wavelet mother with filters of 28 components (called "db14" in Matlab) and 512 components in the signal hosted by the root node.

The artificial test signals used for the decomposition/recomposition relying on T7 were generated by

TABLE I.
CHARACTERISTICS OF HARMONIC MAGNITUDES USED FOR ANALYSIS

Parameters for harmonic magnitudes variation	Harmonic order			
	3	5	7	≥9
Step [%]	2	1	1	0.5
Max [%]	20	10	10	5

superposing a perfect sinusoid with the maxim magnitude of 800 and frequency of 50 Hz with 2 harmonic signals, characterized by their harmonic orders (H1 and H2), magnitudes (M1 and M2) and phase-shifts (phi1 and phi2).

phi1 and phi2 were cycled within the range [-π,π] with the step π/6 whilst M1 and M2 were cycled considering 11 equidistant values such as to cover ranges from 0 to the maximum value (weight from the perfect sinusoid magnitude) as mentioned in Table I.

Clustering properties of T7 trees [8] allowed the authors to reconstruct certain harmonic components associated to clustered harmonics polluting the decomposed signal. The goal is to isolate only certain clusters of harmonics that affect the associated clustered of nodes in an almost exclusive manner [10] and to analyze how those clusters affect the accuracy of wavelet decomposition (WD) and recomposition by varying their phase shifts and magnitudes. Table II shows the 4 sets of harmonic orders (HO) that were used for this analysis as well as the nodes that were affected by those sets of harmonics. The dominant harmonics and nodes [7] are represented with bold fonts.

The WPT recomposition was made in the following way [7]:

- Only the first 32 “terminal nodes” (nodes from the 7-th level) were considered because only the nodes from that range are affected by HOs from the studied sets;

- Those nodes were associated with flags whose values are 0 or 1 whether the nodes were affected by the specific HO or not, respectively;

- In case the terminal node has a value of 1 for a certain set of HOs it is considered in the decomposition, and opposite in case it has 0;

- Flags were given to the nodes from the other levels in upwards direction of the wavelet tree depending on the flags of the nodes in the adjacent lower level. For example, if in level j:

- o Both nodes 2 X (k-1) and 2 X k have a value 1, then the node k from level j-1 that decomposes into nodes 2 X (k-1) and 2 X k in level 7 will have a flag value of 3 - full decomposition;
- o Node 2 X (k-1) has a value of 1 and node 2 X k has a value of 0, then the flag value is 2 - left decomposition;

TABLE II.
PROPERTIES OF ANALYZED CLUSTERS OF NODES AND HARMONICS

Properties	Set ID			
	1	2	3	4
Harmonic orders	**3**,5	**7**,9	**15**,17	31,**33**
Nodes	**2**,4	**3**,7	**5**,13	9,**25**
Weight of dominant harmonic energy in the dominant node energy	0.9824	0.8487	0.6964	0.6012

- o Node 2 X (k-1) has a value of 0 and node 2 X k has a value of 1, then the flag value is 1 - right decomposition;
- o Both nodes 2 X (k-1) and 2 X k have a value of 0, then there will be no decomposition.

During recompositions a similar technique (flags and simplified recomposition functions) was successfully used to save runtime.

The goal of the decomposition/recomposition is to inspect spectral leakages. In other words, even though the WD revealed clustering pattern of nodes that contain certain HOs, not all the weights of harmonics are distributed between those clusters. The analysis will reveal how much do parameters of HOs affect the spectral leakages and to determine for which parameters does WD perform the best or the worst.

The difference D between the synthetically generated harmonic signal *yh* and the harmonic signal (*yhr*) obtained through WPT recomposition was computed for every set and every combination of the 4 parameters (harmonic weights and phase-shifts). Afterward maximum absolute relative percentage errors (MAR) were computed as: *max(abs(D))*/*max(abs(yh))**100.

Another index of accuracy that was used was the Root Mean Square Deviation (RMSd) , computed with the formula:

$$RMSd = \sqrt{\frac{\sum_{i=1}^{512}(yh_i - yhr_i)^2}{512}} \quad (1)$$

In the following sections the phase-shifts may be expressed as IDs (e.g. 1 for - π, 2 for - π + π/6 a.s.o.). Also, the harmonic magnitudes may be expressed as IDs (e.g. 1 for M1=0, 2 for M1=step, …, 11 for the maximum value of M1), according to Table I.

Examples of artificial signals, results of decomposition/recomposition and curves of differences (that are actually instantaneous errors) obtained as *yh-yhr*, are depicted by Figs. 1 and 2.

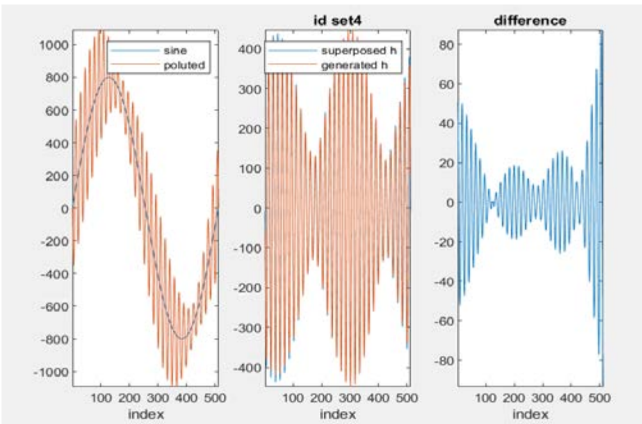

Fig. 1. Example of polluted signal before decomposition (left), harmonic signals before and after recomposition (middle) and the difference between the initial and recovered harmonic signals (right) for the 4-th set.

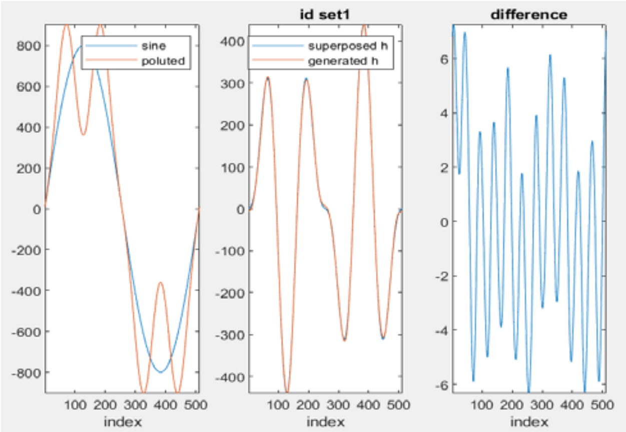

Fig. 2. Example of polluted signal before decomposition (left), harmonic signals before and after recomposition (middle) and the difference between the initial and recovered harmonic signals (right) for the 1-st set.

III. Study of Root Mean Square Deviations

The minimum and maximum values of RMSd were computed, graphically represented and interpreted for each of the 4 sets of 2 harmonics. Surfaces (S1 and S2) were built considering on the horizontal axis Ox the 11 values of the dominant harmonic magnitudes (M1) and on Oy the 11 values of the paired harmonic magnitudes (M2). M1 was allways associated to the dominant harmonic. S1 corresponds to minimum values whilst S2 corresponds to maximum values of RMSd, the difference between the same (x,y) values from them being provided by different phase-shifts phi1 and phi2.

The phase shifts corresponding to S1 and S2 were als represented and interpreted.

A. *Study of extreme values of RMSd*

When none of the paired harmonics is polluting the test signal, constant values were obtained for RMSd , as follows: 1.7 for the 1-st set, 0.65 for the 2-nd, 0.27 for the 3-rd and 0.13 for the 4-th set respectively. These values correspond to residual errors, are specific to the non-ideal feature of the wavelet filter and are highly acceptable as related to the maximum value of the fundamental component (they represent at most 0.2% from it).

The values computed for RMSd when only the dominant harmonic is 0 whilst its pair in the set is non-zero revealed the influence of phase-shifts. Table III gathers the extreme values on each surface of extreme values.

Table IV gathers the counterpart of the data from Table III, but for the case when only the dominant harmonic within the analyzed sets is non-zero . The influence of phase-shifts was revealed again.

Table III.
Extreme Values of RMSd when the Dominant Harmonic is Zero and the Secondary Harmonic is Non-Zero

Set ID	Minimum RMSd		Maximum RMSd	
	S1	S2	S1	S2
1	1.708	1.71	2.33	2.43
2	0.6567	0.6584	0.86	0.99
3	0.28	0.3	0.9	1.33
4	0.21	0.26	1.71	2.27

Table IV.
Extreme Values of RMSd when the Dominant Harmonic is Non-Zero and the Secondary Harmonic is Zero

Set ID	Minimum RMSd		Maximum RMSd	
	S1	S2	S1	S2
1	1.7027	1.7028	1.88	1.9
2	0.66	0.68	0.98	1.8
3	0.27	0.33	0.79	2.02
4	0.22	0.3	1.76	2.67

For both Tables III and IV, the indices of magnitudes within the analyzed set of harmonics were as follows: 2 for the minimum values and 11 for the maximum values. Therefore one can conclude that when a single harmonic from an analyzed set is non-zero, the RMSd is increasing with the value of that harmonic magnitude.

The extreme values reached by RMSd when both harmonics within the analyzed sets were non-zero are gathered by Table V.

The minimum values for the RMSd were reached for all sets for the combination of indices associated to harmonic magnitudes equal to (2,2) within each set. It means that the lowest values for RMSd in this case are associated to the lowest non-zero magnitudes of paired harmonics.

As for the maximum values for the RMSd, two possible combinations of indices associated to harmonic magnitudes were identified. The 1-st combination, corresponding to values marked with star is (11,2) and its meaning is „highest magnitude for the dominant harmonic combined with lowest magnitude for the paired harmonic order”. The 2-nd one is (11,11) and it means „highest magnitudes for both harmonic orders”.

The maximum RMSd as compared to the highest value of the harmonic magnitude (11-th from the set) is equal to 5.72% and is recorded for the 4-th set, which is known as having the worst filtering propertie of all sets (lowest weight of energy in the dominant node). Again one can consider that highly acceptable errors are generated by the analyzed original algorithm.

B. *Study of phase-shifts associated to extreme values of RMSd*

Table VI gathers the values of phi1 and phi2 associated to the cases when one of the harmonics in the set is zero (when the associated harmonic of a phase-shift was zero, the symbol „-" was used).

The analysis of these results revealed that:

- identical values but with oppposite signs were obtained for the sets with IDs 1 and 4;
- at the second set, 2 identical values but with oppposite signs can appear for S1 for different phase-shifts;
- at the 3-rd set, the differences (S1 vs S2) between the counterpart phase-shifts are always **π/2.**

Table V.
Extreme Values For RMSd When Both Harmonics Are Non-Zero

Set ID	Minimum RMSd		Maximum RMSd	
	S1	S2	S1	S2
1	1.708	1.7115	2.34	2.76
2	0.66	0.68	0.98*	1.8
3	0.27	0.33	0.79*	2.02
4	0.16	0.47	1.62*	4.58

TABLE VI.
VALUES OF PHASE-SHIFTS ASSOCIATED TO EXTREME VALUES OF RMSD WHEN ONLY ONE OF THE PAIRED HARMONICS IS ZERO

Set ID	Condition	phi1		phi2	
		S1	S2	S1	S2
1	M1=0	-	-	- π/2	0
	M2=0	- π/2	0	-	-
2	M1=0	-	-	- π or π	- π
	M2=0	- π or π	- π/2	-	-
3	M1=0	-	-	−2π/3	- π/6
	M2=0	5 π/6	π/3	-	-
4	M1=0	-	-	π/2	0
	M2=0	π/2	0	-	-

A more detailed study was needed for the case when both harmonic orders are non-zero and its results are presented below.

1) First set

Fig. 3 depicts S1 (left), S2 (middle) and S1-S2 (rigth) for the 1-st set.

Fig. 4 depicts the associated phase-shifts for S1 (top) and S2 (bottom) . Left – phi1, middle – phi2 and right , phi1-phi2 for the 1-st set.

phi1 for S1 is usually equal to 2 when M1>=M2, with few exceptions and 3 otherwise. A sort of separation „above and below" the main diagonal of the matrix in which the magnitudes of dominant harmonic determine the raws and those of the paired harmonic determine the columns can be noticed, as in Table VII.

Phi2 for S1 is usually equal to 8 below the main diagonal (M1>M2) , equal to 9 on the main diagonal of the matrix similar to that from Table VII or in its strict vicinity and is 10 over the main diagonal , with few exceptions.

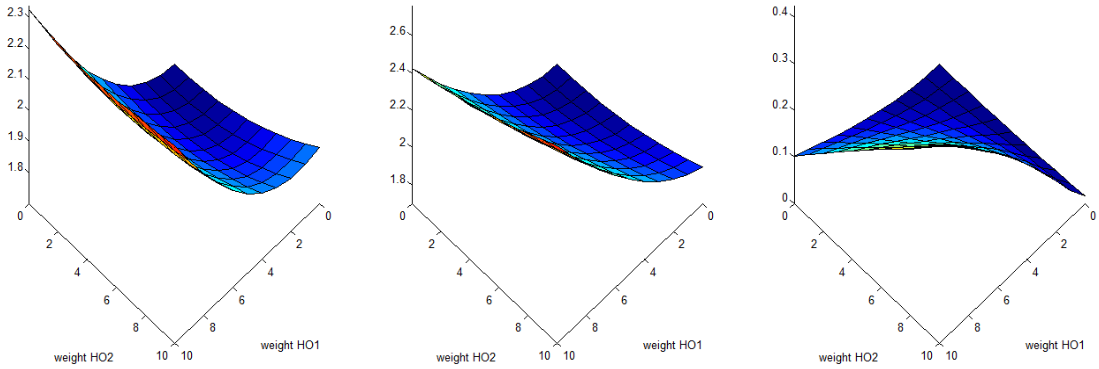

Fig. 3 RMSd deviation of S1 (left), S2 (middle) and S1-S2 (rigth) for the 1-st set.

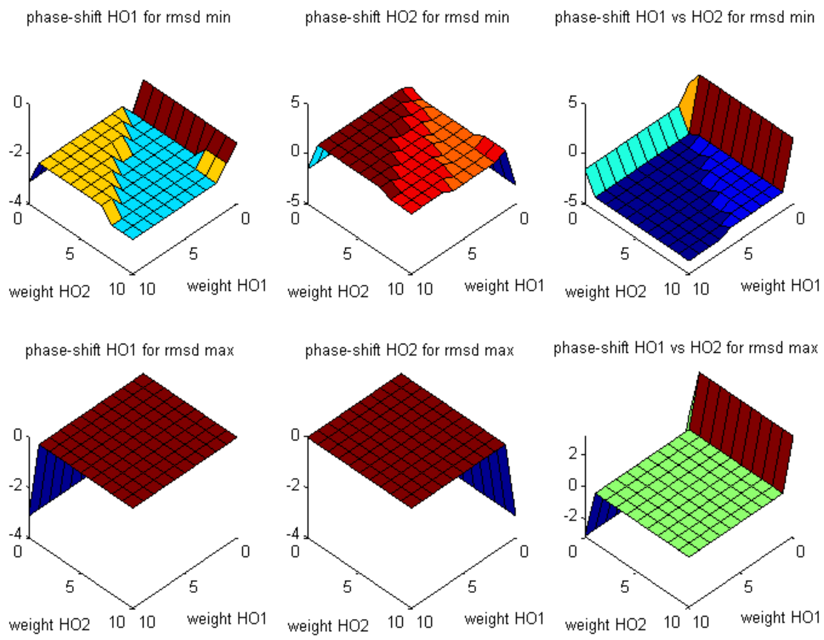

Fig. 4. Associated phase-shifts for S1 (top) and S2 (bottom) . Left – phi1, middle phi2 and right , phi1-phi2. 1-st set.

TABLE VII.
IDENTIFIERS OF PHI1 FOR S1, 1-ST SET

Dominant harmonic / Secondary harmonic	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1	1	1	1	1	1	1	1	1
2	4	2	3	3	3	3	3	3	3	3	3
3	4	2	2	3	3	3	3	3	3	3	3
4	4	2	2	2	3	3	3	3	3	3	3
5	4	2	2	2	2	2	3	3	3	3	3
6	4	2	2	2	2	2	2	3	3	3	3
7	4	2	2	2	2	2	2	2	3	3	3
8	4	2	2	2	2	2	2	2	2	3	3
9	4	3	2	2	2	2	2	2	2	2	2
10	4	3	2	2	2	2	2	2	2	2	2
11	4	3	2	2	2	2	2	2	2	2	2

As for the difference (phi1-phi2) associated to S1, only 2 values were noticed: 1 below the main diagonal and -1 (corresponding to the value - π - π /6) over it.

Usually when M1>M2, the phase-shift between H1 and H2 (phi1-phi2) is - π - π /6 and otherwise is - π + π /6.

Both phase-shifts had the value 7 over all S2 and therefore (ph1-phi2) is always 0.

2) *Second set*

Fig. 5 depicts the S1, S2 and S1-S2 surfaces whilst Fig. 6 depicts the corresponding surfaces with phase-shifts for the 2-nd set.

For S1, phi1 is usually 7 above the main diagonal (fewer values) and 1 or 13 nearby and below it. On the contrary, phi1 is usually 7 below and nearby the main diagonal and 1 or 13 above it (fewer values).

It is why (phi1-phi2) is mapped either in - π or in π for S1.

Both phase-shifts had the value 4 over all S2 and therefore the phase-shift (ph1-phi2) is allways 0.

3) *Third set*

Fig. 7 depicts the S1, S2 and S1-S2 surfaces whilst Fig. 8 depicts the corresponding surfaces with phase-shifts for the 3-rd set.

phi1 for S1 can take 3 values: 9 above the main diagonal, 11 on it and close to it and 12 below it.

phi2 for S1 can take 3 values: 3 above the main diagonal, 4 on it and close to it and 6 below it.

Under these circumstances, most of the values of (phi1-phi2) for S1 are equal to π and a small number of them is π + π /6 .

For S2, phi1 takes the value 6 over the main diagonal, more values 7 nearby it, one value 8 near the main diagonal and close to it and few values 9 under the main diagonal.

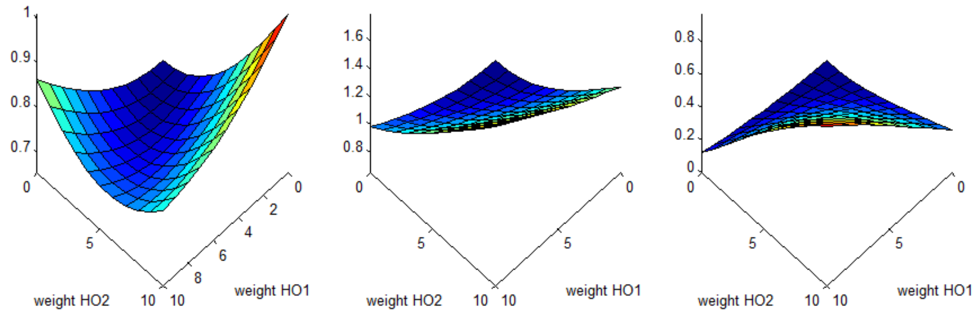

Fig. 5. RMSd deviation of S1 (left), S2 (middle) and S1-S2 (rigth) for the 2-nd set.

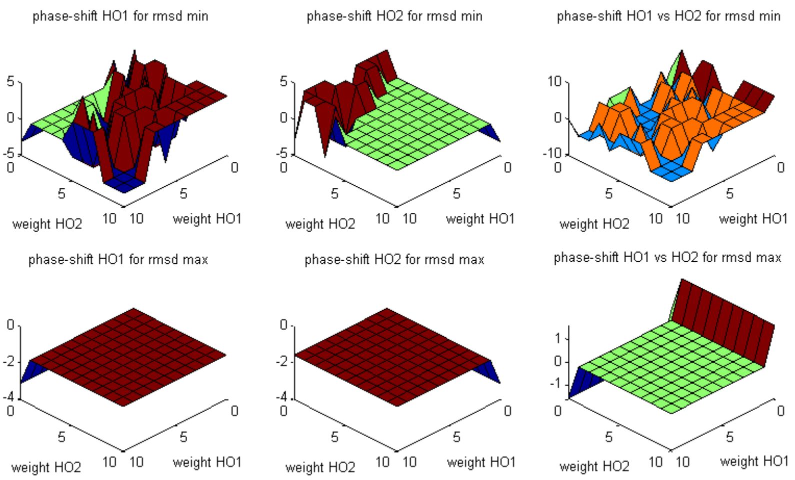

Fig. 6. Associated phase-shifts for S1 for RMSd. (top) and S2 (bottom) . Left – phi1, middle phi2 and right , phi1-phi2. 2-nd set.

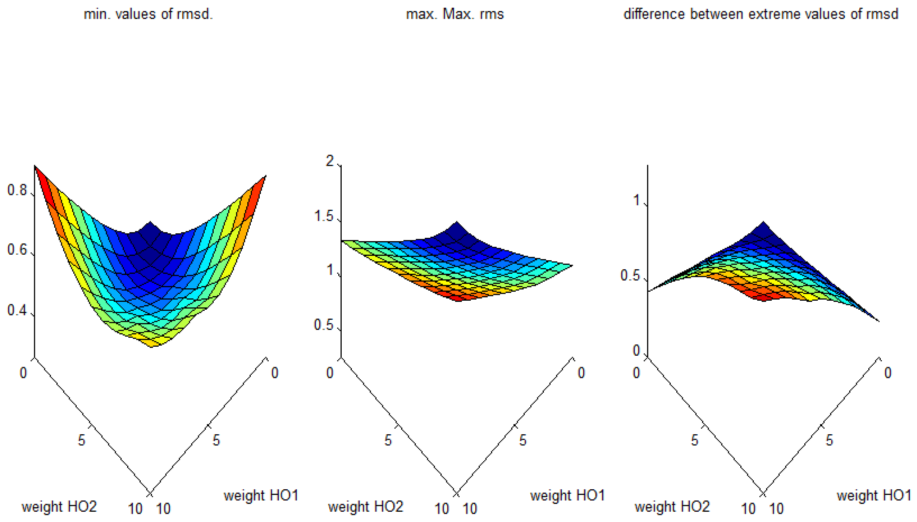

Fig. 7. RMSd deviation of S1 (left), S2 (middle) and S1-S2 (rigth) for the 3-rd set.

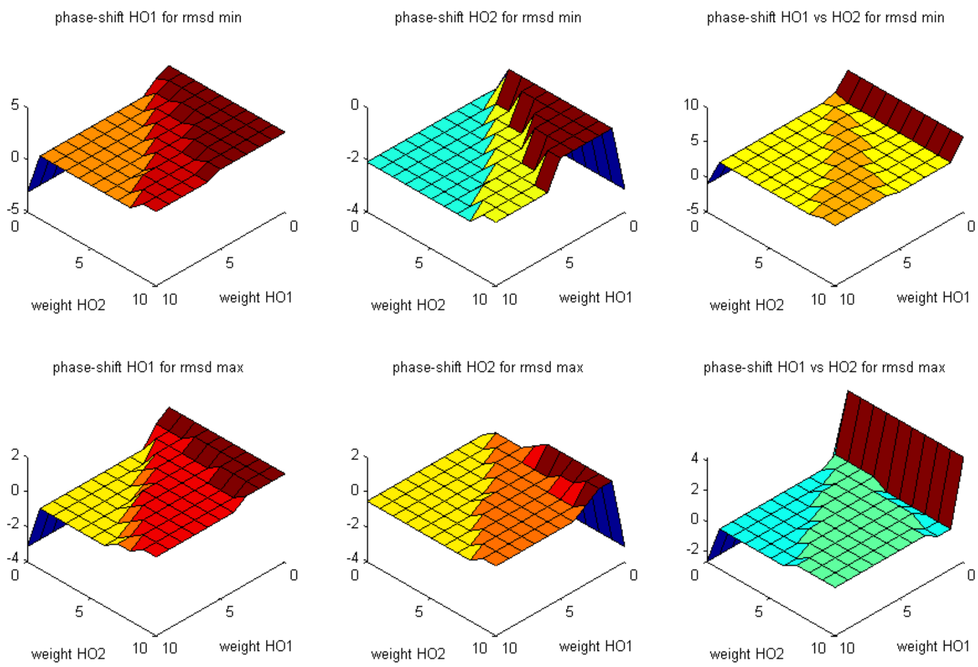

Fig. 8. Associated phase-shifts for RMSd. S1 (top) and S2 (bottom) . Left – phi1, middle phi2 and right , phi1-phi2. 3-rd set.

diagonal, 7 on it and nearby it, 4 values 8 for high values of M1 and small values of M2 and few values of 9 nearby the main diagonal, under it for high values of M1.

Accordingly, (phi1-phi2) for S2 has many values of 0 far from the main diagonal and the remaining ones are associated to the index 8 (which is associated to $\pi/6$).

4) *Fourth Set*

Fig. 9 depicts the S1, S2 and S1-S2 surfaces whilst Fig. 10 depicts the corresponding surfaces with phase-shifts. For S1, phi1 can take 3 values: 4 above the main diagonal, many values of 10 and only few values of 11 under the main diagonal. For S1, phi2 can take 3 values: 10 above the main diagonal, many values of 4 and only few values of 5 under the main diagonal.

Therefore phi1- phi2 can take only 2 values: - π above the main diagonal and π under it.

For S2, phi1 and phi2 have all values equal to 7 and therefore (phi1-phi2) is 0.

IV. Study of Maximum Absolute Relative Errors

The variation of MAR with harmonic magnitudes and phase-shifts is approached in this section.

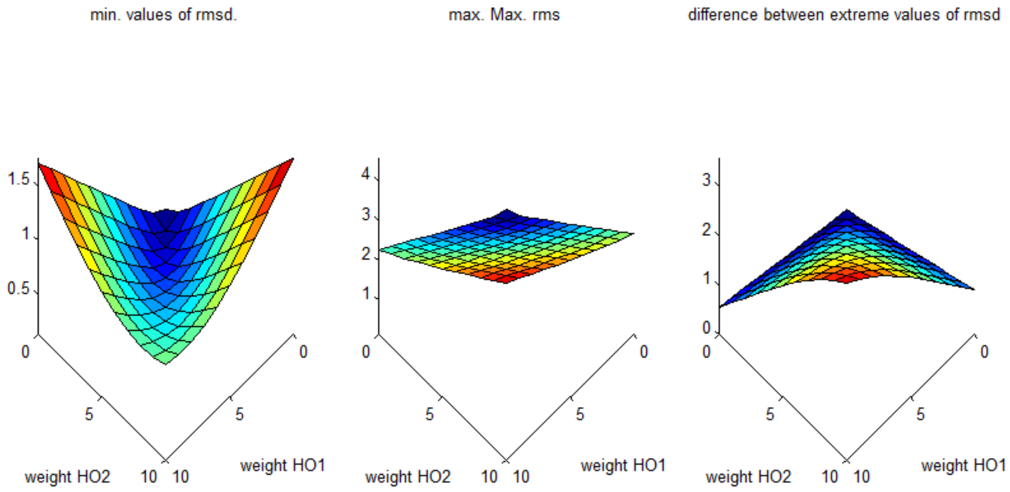

Fig. 9. RMSd deviation of S1 (left), S2 (middle) and S1-S2 (rigth) for the 4-th set.

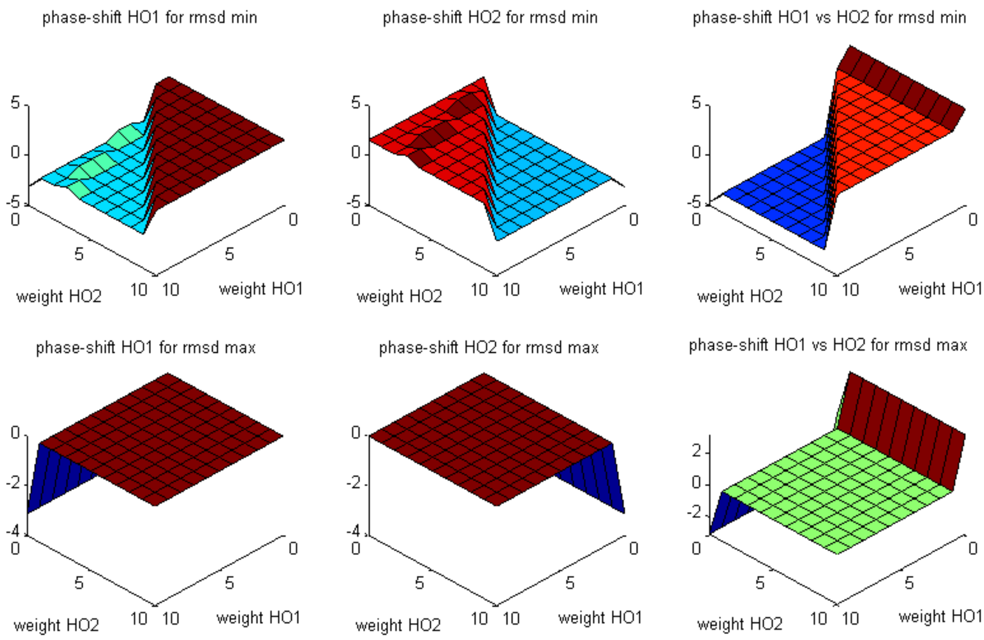

Fig. 10. Associated phase-shifts for RMSd. S1 (top) and S2 (bottom) . Left – phi1, middle phi2 and right , phi1-phi2. 4-th set.

Figs 11...14 depict the surfaces with extreme values of MAR for each of the analyzed sets, following the same rule applied in the previous section. Left – surface S1 with minimum values, middle – surface S2 with maximum values and right – difference between S1 and S2.

The extreme situations (when one of the harmonic from set is 0) were analyzed for MAR as well. In this aim, Table VIII gathers the extreme values for MAR when only the dominant harmonic is 0. In Table VIII, all minimum values were found to be associated to the maximum magnitude of the secondary harmonic, except for the value 17.87% computed for the 4-th set, S1, which is associated to the harmonic with ID 4, despite an usual „descending" trend of the rest of values observed toward the maximum magnitude of the 2nd harmonic.

On the other hand, the maximum values were found to be associated to the minimum magnitude. It means that a sort of „reversed dependence" is established in this case between the values of MAR and harmonic magnitudes.

TABLE VIII.
EXTREME VALUES FOR MAR WHEN ONLY THE DOMINANT HARMONIC IS 0

Set ID	Minimum MAR [%]		Maximum MAR [%]	
	S1	S2	S1	S2
1	6.85	9.43	50.44	52.55
2	7.79	11.02	48.27	55
3	9.69	14.41	18.53	27.82
4	17.87	22.51	19.96	27.36

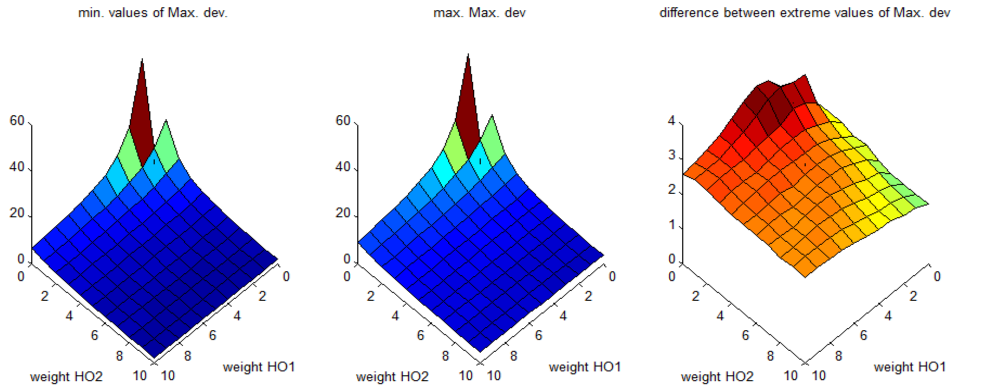

Fig. 11. Surfaces with extreme values of MAR for the 1st set. S1-left, S2- middle, S1-S2 – right.

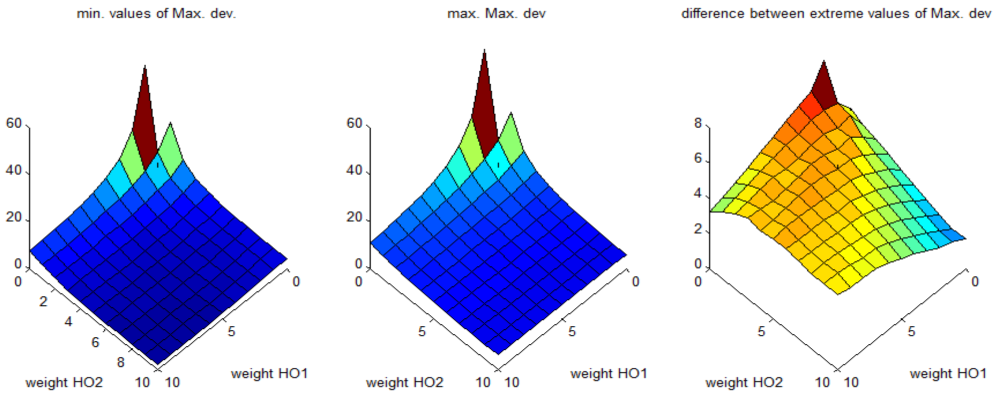

Fig. 12. Surfaces with extreme values of MAR for the 2nd set. S1-left, S2- middle, S1-S2 – right.

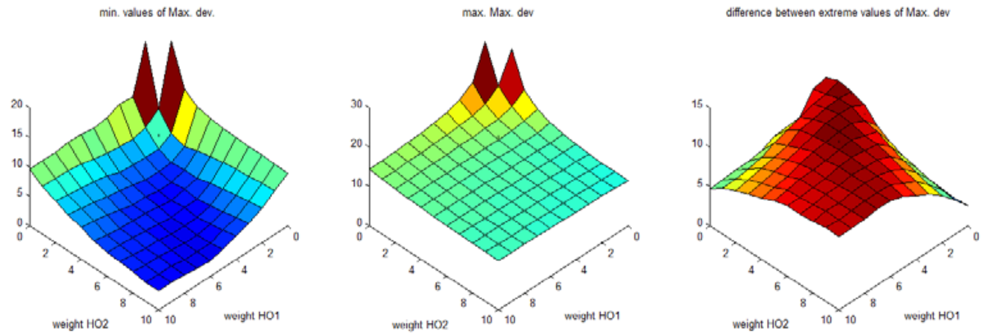

Fig. 13. Surfaces with extreme values of MAR for the 3rd set. S1-left, S2- middle, S1-S2 – right.

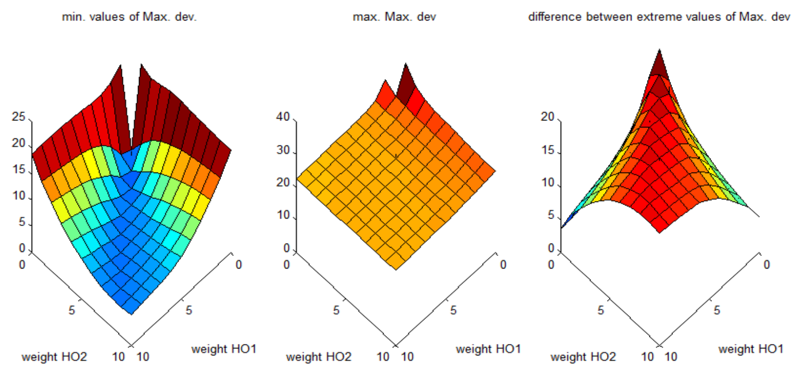

Fig. 14. Surfaces with extreme values of MAR for the 4th set. S1-left, S2- middle, S1-S2 – right.

TABLE IX.
EXTREME VALUES FOR MAR WHEN ONLY THE DOMINANT HARMONIC IS NON-ZERO

Set ID	Minimum MAR [%]		Maximum MAR [%]	
	S1	S2	S1	S2
1	2.22	3.94	24.24	26.34
2	4.56	6.25	24.28	28.31
3	8.99	11.57	18.6	25.9
4	19.27	25	20.26	32.5

One can also notice smaller differences between the extreme values of sets with higher IDs (e.g. the difference between the minimum and maximum values of the 1-st set is higher than 46%, whilst its counterpart for the 4-th set is 13%). Actually the MARs associated to the smallest harmonic magnitudes correspond to small absolute values and therefore cannot be considered as significant in these contexts.

Table IX gathers the extreme values for MAR when only the dominant harmonic is non-zero.

Unlike the conclusions drawn for the values of RMSd, in the case of MAR computed when one of the harmonics in the pair is 0, the indices of magnitudes within the set of the polluting harmonic were as follows: 11 for the minimum values (except for the value 19.27% of S1, 4-th set, where the ID is 4) and 2 for the maximum values respectively. Therefore, one can conclude that when a single harmonic from an analyzed set is non-zero, the MAR is usually decreasing with the value of the harmonic magnitude.

Computations were also made for cases when both harmonics in a set are non-zero. The results are gathered by Tables X and XI.

One can conclude based on the data from Tables X and XI that when both harmonics from a set act jointly, usually the minimum values of MAR are associated to the highest magnitudes of the dominant harmonic, except for the 4-ts set. On the other hand, the maximum values of

TABLE X.
MINIMUM VALUES FOR MAR FOR BOTH SURFACES ALONG WITH THE INDICES OF MAGNITUDES IN THE SET WHEN BOTH HARMONICS IN THE SET ARE NON-ZERO

	Minimum of MAR – S1		Minimum of MAR – S2	
Set ID	Value [%]	Combination of IDs	Value [%]	Combination of IDs
1	1.9	(11,9)	3.87	(11,3)
2	1.75	(11,11)	5.97	(11,10)
3	2.64	(11,7)	11.7	(11,2)
2	5.03	(3,3)	22.58	(2,11)

TABLE XI.
MAXIMUM VALUES FOR MAR FOR BOTH SURFACES ALONG WITH THE INDICES OF MAGNITUDES IN THE SET WHEN BOTH HARMONICS IN THE SET ARE NON-ZERO

	Maximum of MAR – 1-st surface		Maximum of MAR – 2-nd surface	
Set ID	Value [%]	Combination of IDs	Value [%]	Combination of IDs
1	16.18	(2,2)	19.39	(2,2)
2	15.89	(2,2)	20.77	(2,2)
3	8.57	(2,2)	18.58	(2,2)
4	16.03	(11,2)	25.22	(2,2)

MAR are ussualy associated to the smallest values for both harmonic magnitudes, except for the 4-th set, 1-st surface. Considering the high and close orders of the harmonics clustered in the 4-th set (33-rd harmonic is dominant and paired with the 31-rd harmonic), one can explain the „special" behavior of the error surfaces for this set.

V. CONCLUSION

The extended study presented in this paper addresses the errors which characterize the determination of 4 types of harmonic signals, generated from the following pairs of harmonic orders: (3,5), (7,9), (15,17) and (31,33).

The main conclusions relative to the values of RMSd are:

- when none of the paired harmonics is polluting the test signal, small values were obtained for RMSd , representing at most 0.2% from the fundamental harmonic;
- when a single harmonic H from the set is non-zero, simulations revealed that the RMSd is increasing with the magnitude of H;
- when the harmonics acted jointly, the lowest values for RMSd were associated to the lowest non-zero magnitudes of paired harmonics. As for the maximum values for the RMSd, two possible combinations of indices associated to harmonic magnitudes were identified. The 1-st combination can be translated into „highest magnitude for the dominant harmonic combined with lowest magnitude for the paired harmonic order" whilst the 2-nd one has the meaning „highest magnitudes for both harmonic orders".

Therefore one can consider that highly acceptable RMSd errors were generated.

The main conclusions relative to the phase-shifts associated to the extreme values of RMSd when one of the harmonics is zero are:

- identical values but with oppposite signs were obtained for the sets with IDs 1 and 4;
- at the second set, 2 identical values but with oppposite signs can appear for S1 for different phase-shifts;
- at the 3-rd set, the differences (S1 vs S2) between the counterpart phase-shifts are always $\pi/2$.

When both harmonics were non-zero, many times a sort of separation „above and below" the main diagonal of the matrix in which the magnitudes of dominant harmonic determine the raws and those of the paired harmonic determine the columns could be noticed with respect to phase-shifts. Behavioral patterns could were deduced for each set, being more obvious for sets where the weigth of the dominant harmonic is closer to 1, thus providing better filtering properties.

The analysis of MAR when a single harmonic is non-zero revealed that:

- a sort of „reversed dependence" is established between the values of MAR and the harmonic magnitude;
- smaller differences were noticed between the extreme values of sets with higher IDs (e.g. the difference between the minimum and maximum values of the 1-st set is higher than 46%, whilst its counterpart for the 4-th set is 13%). Actually the MARs associated to the smallest harmonic magnitudes correspond to small absolute values and therefore cannot be considered as significant in these contexts;
- unlike the conclusions drawn for the values of RMSd, the MAR is usually decreasing with the value of the harmonic magnitude.

The analysis of MAR when both harmonics were non-zero revealed that usually the minimum values of MAR are associated to the highest magnitudes of the dominant harmonic, except for the 4-ts set. On the other hand, the maximum values of MAR are usually associated to the smallest values for both harmonic magnitudes, except for the 4-th set, 1-st surface.

Future work will be concerned with the study of errors accompanying the composition/recomposition focusing on clusters of 4 harmonics.

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