Aircraft State Estimation by using an Adaptive Observer for MIMO Linear Time Varying Systems

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Abstract— Most important adaptive observers study the SISO (single-input-single-output) systems; the generalization of these algorithms for the case of MIMO (multi-input-multi-output) systems is difficult to be made. One of the most important algorithms for state estimation in the case of MIMO linear time varying systems (LTV) was designed by Qinghua Zhang [1]; the algorithm is simple, computationally efficient, viable, and easy to implement software. The Zhang adaptive observer is used in this paper to estimate the state of the aircrafts longitudinal and lateral motions. Two examples are considered: the first one represents the longitudinal motion of a Charlie aircraft, while the second example is associated to the lateral motion of a Boeing 747 airplane.

Keywords- adaptive observer; algorithm; aircraft; longitudinal motion; lateral motion

I. INTRODUCTION

An adaptive observer is both a state estimator and a method to identify the parameters of a system. Thus, an adaptive observer uses a parameters’ identification algorithm or a state estimation algorithm. The parameters’ identification algorithms use the system outputs and the estimated state. There exist a lot of adaptive observers which are used in the case of nonlinear systems [2-8]. For example, in order to design an adaptive observer for a nonlinear system with one input and one output, the initial system must be brought to the canonical form if the system input and states are bounded [2]. In the case of multiple inputs and unique output, the system must be brought to the canonical form by means of coordinates transformation [3,4].

Other adaptive observers for the nonlinear Lipschitz systems are presented in [5] and [6]. The designed observers may not give good results in the case of bounded disturbances. These adaptive observers assure the convergence of the parameters to their desired values, but they may be characterized by small deviations in the case of small random disturbances although the estimation errors remain small [9,10]. To avoid this, some techniques for the modification of the adaptive observers’ structure [9-11] have been introduced. Here we can mention: the usage of the design operators [9-11] or the usage of a “link term” in the parametric control law to make a Lyapunov function time derivative remain negative when a parameter exceeds its normal limit [10,12]. Some existing adaptive observers are characterized by errors even when the estimation errors remain small. This is due to the exterior disturbances and this may be avoided by using robust adaptive observers. In many cases only the input (inputs) and the output (outputs) of a system are measurable and, therefore, the estimation of the state variables plays an important role in the control process [13,14]. In recent years many nonlinear observers have been designed: high – gain observers, sliding mode observers [15-17] etc.; these are complex observers and they can be used for the systems with known structure. The use of neural networks (NN) for the identification and control of dynamic systems is presented in detail in lots of scientific published papers [18-20]. The good results of NNs are due to their capacity of nonlinear functions’ approximation [21,22]. The linear systems state estimation is made by using observers and the Kalman filter. Joint estimation of state and some unknown parameters are made by means of several known solutions, with so-called adaptive observers [7,23,24].

Another use of the adaptive observers is the adaptive control. Because of the possibility of parameters and state vector on-line estimation, the adaptive observers may be integrated into controllers. Another use is the fault detection and isolation (FDI). The faults are modeled as parameter changes. Adaptive observers for linear systems are studied since 1970 [1]. In [25] one designed an observer which supposes the integrating of the state estimation error equation. Other adaptive observers with exponential convergence have been designed several years later [23]; these algorithms are based on minimization of a particular criterion.

II. PROBLEM STATEMENT

One of the most important algorithms for state estimation in the case of MIMO linear time varying systems (LTV) was designed by Q. Zhang [1]; the algorithm is simple, computationally efficient, viable, and easy to implement software. We consider the system described by the state equations [1]:

\[
\begin{align*}
\dot{x}(t) &= A(t)x(t) + B(t)u(t) + \psi(t)\theta, \\
y(t) &= C(t)x(t),
\end{align*}
\]  

(1)
where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^r$, $y(t) \in \mathbb{R}^m$, $A, B, C$ - known matrices but time varying, $\theta \in \mathbb{R}^p$ - the vector of unknown parameters (considered to be constant), $\psi(\theta) \in \mathbb{R}^{m \times r}$ - the matrix of the unknown parameters' vector. The aim of the Zhang algorithm is the estimation of the vectors $x(t)$ and $\theta$, by measuring $y(t)$, and considering that the system input is the measurable vector $u(t)$. As long as the matrices $A, B, C$ in (1) are not constant matrices, the state estimation by means of Kalman filter is difficult or maybe impossible. A solution is the obtaining of an extended system by putting the vector $\theta$ inside the state vector $x(t)$. The system remains a time varying one and, for the estimation of the new state vector, the observability and uniformity of the system are needed [26]. This is why the Kalman filter usage in the case of extended systems is not a simple problem [1,27,28]. The design of an adaptive observer for MIMO systems comes from the necessity of systems’ online identification, the aim in this case being the estimation of the unknown parameters’ vector. For the system (1) one desires the components of the unknown parameters vector to be the coefficients in some measurable signals. This condition is not a restrictive one and the system may be brought to desired canonical form by using some nonlinear transformations. For example, for the SISO systems [1]:

$$y(t) = a_1y(t-1) + a_2y(t-2) + \ldots + a_ny(t-n) + b_1u(t-1) + b_2u(t-2) + \ldots + b_mu(t-m),$$

(2)

where $\theta = [a_1, a_2, \ldots, a_n, b_1, b_2, \ldots, b_m]^T$ – the unknown parameters’ vector. System (1) may be written under the form [24]:

$$\begin{align*}
\dot{x} &= A(t)x + Bu + \gamma \xi^T(t)\theta, \\
y &= Cx.
\end{align*}$$

(3)

Most recent designed observers are based on a dynamical transformation, putting the original system into some canonical form [7,24]. The systems with only one output, after transformation, have the form [1]:

$$\dot{z}(t) = A_0z(t) + Bu + \gamma \xi^T(t)\theta, \quad y(t) = c_0z(t),$$

(4)

where the matrix $A_0$ and the vector $c_0$ have special forms, $\gamma \in \mathbb{R}^r$ - constant column vector, $\xi(t) \in \mathbb{R}^p$ - vector of signals which is obtained by filtering the signals $u(t)$ and $y(t)$. The unknown parameters’ vector $\theta$ affects the state equation through the scalar product $\xi^T(t)\theta$ and the column vector $\gamma$. The novelty of the Zhang algorithm is the design of the adaptive observer for the MIMO linear time varying systems. The Zhang algorithm [1], until 1991, was only the second algorithm which estimates the state and the unknown parameters’ vector for the MIMO systems. The first algorithm belongs to Besancon [7], but the Zhang algorithm is characterized by a better convergence and simplicity. Also, Zhang succeeded a unified formulation for some adaptive observers which are based on dynamical transformations [1].

III. THE DESIGN OF THE ZHANG ADAPTIVE OBSERVER

For the design of the Zhang adaptive observer, the first equation (1) is written:

$$\dot{x}(t) = [A(t) - K(t)C(t)]x(t) + B(t)u(t) + K(t)y(t) + \psi(\theta) \theta,$$

(5)

where $K(t)$ is the observer gain matrix. The state vector $x(t)$ is influenced by two “exogenous excitations” $B(t)u(t) + K(t)y(t)$ and $\psi(\theta) \theta$. The state vector $x(t)$ is divided into two parts: $x_n(t)$ and $x_o(t)$ so that [1]:

$$x(t) = x_n(t) + x_o(t),$$

(6)

with

$$\begin{align*}
\dot{x}_n(t) &= [A(t) - K(t)C(t)]x_n(t) + B(t)u(t) + K(t)y(t), \\
\dot{x}_o(t) &= [A(t) - K(t)C(t)]x_o(t) + \psi(\theta) \theta; \quad x_o(t) = \hat{\theta}(t) \theta + \omega(t),
\end{align*}$$

(7)

where $\hat{\theta}(t)$ is the estimation of $\theta$, while the role and the significance of $\omega(t)$ are to be presented later.

We suppose that between $\hat{\theta}(t)$ and $\hat{x}_o(t)$ one can establish the following relationship [1]:

$$\hat{x}_o(t) = \Gamma(t) \cdot \hat{\theta}(t),$$

(10)

where $\Gamma(t) \in \mathbb{R}^{m \times r}$. From (9) and (10) it results:

$$\dot{\hat{x}}_o(t) = \Gamma(t) \dot{\hat{\theta}}(t) = [A(t) - K(t)C(t)]\dot{\hat{x}}_o(t) + \psi(\theta) \dot{\hat{\theta}}(t) + \omega(t).$$

(11)

If we consider $\omega(t) = \Gamma(t) \dot{\hat{\theta}}(t)$, (11) gets the form:

$$\dot{\hat{x}}_o(t) = [A(t) - K(t)C(t)]\dot{\hat{x}}_o(t) + \psi(\theta) \dot{\hat{\theta}}(t);$$

(12)

$\psi(\theta)$ is a known matrix and, by integrating of (12), the matrix $\Gamma(t)$ is determined. The estimated state vector is:

$$\dot{\hat{x}}(t) = \hat{x}_n(t) + \hat{x}_o(t)$$

(13)

and, according to (8) and (9) it yields [1]:

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\[ \dot{x}(t) = [A(t) - K(t)C(t)]x(t) + B(t)u(t) + K(t)v(t) + \psi(t)\hat{\theta}(t) + \Gamma(t)\dot{\theta}(t). \] (14)

The following assumptions are made:

**Assumption 1** [1]

The matrix pair \((A(t), C(t))\) is such that there is a bounded time varying matrix \(K(t) \in \mathbb{R}^{n \times m}\) so that the system:

\[ \hat{\eta}(t) = [A(t) - K(t)C(t)]\eta(t) \] (15)

is globally exponentially stable.

**Assumption 2** [1]

Let us consider \(\Gamma(t) \in \mathbb{R}^{m \times p}\) – the matrix of signals generated by the differential equation (12). Assume that \(\psi(t)\) is persistently exciting so that there are two positive constants \(\delta, \varpi\), and a symmetric and positive defined matrix \(\Sigma(t) \in \mathbb{R}^{n \times n}\) such that for all \(t\) the following inequality holds

\[ \int_{t_0}^{t} \Gamma(t)\Sigma(t)\Gamma(t)\varpi \geq \delta I. \] (16)

Assumption 1 states that, for any given parameters’ vector \(\theta\), an observer may be designed for the system (1) by using the gain matrix \(K(t)\). The assumption 2 is a persistent excitation condition and is necessary for the estimation of the two vectors. The design of the observer is based on the following theorem [1]:

**Theorem 1** [1]

We consider \(M \in \mathbb{R}^{p \times p}\) – any symmetric and positive defined matrix. If assumptions 1 and 2 hold, for any constant vector \(\theta\), the estimation of the state vector \(x(t)\) and of the unknown parameters’ vector \(\theta\) is made by using the equations [1]:

\[ \begin{align*}
    \dot{x}(t) &= M\hat{x}(t) + B(t)u(t) + \psi(t)\hat{\theta}(t) + \\
    &+ \left[ K(t) + \Gamma(t)MT^{-1}(t)C(t)\Sigma(t) \right] [v(t) - C(t)x(t)], \\
    \hat{\theta}(t) &= MT^{-1}(t)C(t)\Sigma(t) [v(t) - C(t)x(t)],
\end{align*} \] (17)

equations which are associated to an exponential adaptive observer for the system (1); for any initial conditions \(x(t_0), \hat{x}(t_0), \hat{\theta}(t_0),\) and for any \(t \in \mathbb{R}^p\), the errors \(\hat{x}(t) - x(t)\) and \(\hat{\theta}(t) - \theta(t)\) exponentially tend to zero when \(t \to \infty\). The equations (17) lead to (14).

To demonstrate the above theorem, two lemmas are used.

**Lemma 1** [1]

Let us consider \(\Phi(t) \in \mathbb{R}^{m \times p}\) – continuous matrix and \(M \in \mathbb{R}^{p \times p}\) – symmetric and positive defined matrix. If there are the positive constants \(\alpha, \beta, \gamma\) so that, at each time moment,

\[ \alpha I \leq \int_{t_0}^{t} \Phi^T(t)\Phi(t)\varpi \leq \beta I, \] (18)

then, the system

\[ \begin{align*}
    \dot{z}(t) &= -\Phi^T(t)\Phi(t)z(t) \quad (19)
\end{align*} \]

is globally exponentially stable.

**Lemma 2** [1]

If the LTI system:

\[ \dot{\xi}(t) = F(t)\xi(t) \] (20)

is globally exponentially stable, \(u(t) = \) bounded and integrable signal and \(\lim_{t \to \infty} u(t) = 0\), then \(z(t)\), described by the equation:

\[ \dot{z}(t) = F(t)\dot{z}(t) + u(t), \] (21)

converges to zero \(\lim_{t \to \infty} z(t) = 0\).

The demonstrations of the two lemmas are widely presented in [1]. To demonstrate the theorem 1, we eliminate \(MT^{-1}(t)C(t)\Sigma(t) [v(t) - C(t)x(t)]\) between the two equations (17) and we get [1]:

\[ \begin{align*}
    \dot{x} &= A\hat{x} + Bu + \psi\hat{\theta} + K(y - C\hat{x}) + \Gamma\dot{\theta}, \\
    \dot{\theta} &= A\hat{\theta} + Bu + \psi\hat{\theta},
\end{align*} \] (22)

Let us consider:

\[ \begin{align*}
    \tilde{x} &= x - \hat{x}, \\
    \tilde{\theta} &= \hat{\theta} - \theta
\end{align*} \] (23)

and, taking into account \(\hat{\theta} = 0 \iff \dot{\theta} = \dot{\theta}\), it yields:

\[ \begin{align*}
    \dot{\tilde{x}} &= A\tilde{x} + Bu + \psi\tilde{\theta}, \\
    \dot{\tilde{\theta}} &= A\tilde{\theta} + Bu + \psi\tilde{\theta}
\end{align*} \] (24)

or, by the above two equations’ difference,

\[ \begin{align*}
    \dot{\tilde{x}} &= (A - KC)\tilde{x} + \psi\tilde{\theta} + \Gamma\dot{\theta}. \\
    \dot{\tilde{\theta}} &= A\tilde{\theta} + Bu + \psi\tilde{\theta}
\end{align*} \] (25)

We define \(\eta(t)\) as a linear combination of the error vectors \(\tilde{x}(t)\) and \(\tilde{\theta}(t)\) [1]:

\[ \eta(t) = \tilde{x}(t) - \Gamma\tilde{\theta}(t). \] (26)

By the derivation of the equation (26) we get:

\[ \dot{\eta} = (A - KC)\eta + [A - KC]\Gamma + \Psi - \Gamma\dot{\theta}. \] (27)

But, from (12), it results \(\Gamma = (A - KC)\Gamma + \Psi\) and (27) becomes:

\[ \dot{\eta} = (A - KC)\eta. \] (28)

If assumption 1 holds, the system (28) is globally exponentially
stable and \( \lim_{t \to +\infty} \eta(t) = 0 \). Now, we calculate \( \hat{\theta} \) [1]:

\[
\begin{align*}
\dot{\hat{\theta}} &= \dot{\hat{\theta}} - \hat{\theta} = \dot{\theta} = 0 = M^T C^T \Sigma (y - C \hat{x}) = -M^T C^T \Sigma C \hat{x} = -M^T C^T \Sigma C \hat{x} = -M^T C^T \Sigma C (\eta + \Gamma \hat{\theta}).
\end{align*}
\]

(29)

Thus, the homogenous part of the system (29) is:

\[
\begin{align*}
\dot{\hat{\theta}} &= -M^T C^T \Sigma C \hat{\theta}.
\end{align*}
\]

(30)

If \( \psi \) is bounded and \( \Gamma \) is generated by the exponentially stable system (12), it results \( \Gamma \) is bounded. From condition (16) of the persistent excitation and from lemma 3, with \( \Phi = \Sigma^{1/2} \Gamma \), we find that the system (30) is stable [1]. We demonstrated that system (30) is stable and \( \lim_{t \to +\infty} \eta(t) = 0 \); we obtain, by means of lemma 2, with \( F = -M^T C^T \Sigma C \Gamma \), the exponential convergence to zero of \( \hat{\theta}(t) \). Finally, because \( \eta \to 0, \hat{\theta} \to 0 \), according to (26), we get \( \hat{x} \to 0 \) [1]. If the excitation condition (assumption 2) is not fulfilled, the Zhang algorithm doesn’t guarantee the convergences \( \hat{\theta}(t) \to 0 \) and \( \hat{x}(t) \to 0 \). We may only demonstrate that \( C_f \hat{\xi}(t) \to 0 \) [29] (only the prediction error \( C_f \hat{\xi}(t) - y(t) \) is convergent to zero). The matrix \( K(t) \) stabilizes the state estimation process. \( \Sigma(t) \) may be any bounded and positive defined matrix, while \( M \) may be any constant and positive defined matrix. The choice of the two matrices influences the velocity estimation of the state and unknown parameters’ vector 0 [1].

\[
\begin{align*}
P(t) &= A(t) P(t) + P(t) A^T(t) - P(t) C^T(t) R^{-1}(t) C(t) P(t) + Q(t),
\end{align*}
\]

(32)

with \( Q \in R^{n \times n}, R \in R^{n \times m} \) – symmetric and positive defined matrices. Another method to obtain the matrix \( K(t) \) is the method of poles’ positioning; a stable matrix \( (A - KC) \) must be obtained [1]. The block diagram of the ensemble system – Zhang adaptive observer (Fig.1) is obtained by using the equations (1), (12), (17), (31), and (32).

IV. SIMULATION RESULTS

In this section we use the Zhang adaptive observer to estimate the state of the aircrafts longitudinal and lateral motions. Two examples are considered: the first one represents the longitudinal motion of a Charlie aircraft, while the second example is associated to a Boeing 747 lateral motion.

**Example 1: Longitudinal motion**

For the beginning, let us consider the longitudinal motion of a Charlie aircraft with the state equation [30]:

\[
\begin{align*}
\dot{\hat{\mathbf{p}}} &= \begin{bmatrix} -0.026 & 0.025 & -0.1 & 0 \\ -0.36 & -3 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0.4212 & -38.49 & 0 & -3.67 \end{bmatrix} \begin{bmatrix} \hat{\mathbf{p}} \\ \hat{\mathbf{a}} \\ \hat{\mathbf{b}} \\ \hat{\mathbf{c}} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix},
\end{align*}
\]

(33)

where

\[
\begin{align*}
\dot{\mathbf{v}} &= \Delta \mathbf{V} \mathbf{V}^T, \quad \dot{\mathbf{a}} = \frac{t}{\tau_a} \mathbf{a}, \quad \mathbf{b} = \mathbf{b}, \quad \dot{\mathbf{c}} = \mathbf{c}, \quad \dot{\mathbf{d}} = \mathbf{d};
\end{align*}
\]

(34)

\[
\tau_a = 2.1 \text{ sec} - \text{the aerodynamic time constant} \ [27]. \quad \mathbf{V} \text{ is the aircraft velocity, } \mathbf{a} - \text{the aircraft attack angle, } \mathbf{b} - \text{the pitch angle, } \mathbf{c} - \text{the elevator deflection, } \dot{t} - \text{the time, } \mathbf{b} - \text{the mean aerodynamic chord, } \mathbf{b} - \text{horizontal flight velocity; } \mathbf{``m''} \text{ is associated to nondimensional variables, while } \mathbf{``m''} \text{ means the deviation of a variable from its initial value.}
\]

The Zhang algorithm for the design of an adaptive observer was implemented in Matlab/Simulink. As the input signal of the system \( u \) we may consider a unitary step signal, a sinusoidal signal or any random signal. In the simulation, we calculated, by using the algorithm ALGLX [30], the matrix \( K \) and we considered, as the input vector of the system, the vector \( u = \delta \mathbf{a} = -K \mathbf{a} \). We obtained the graphic characteristics in Fig. 2 (the time dependencies of the state vector estimation errors \( \mathbf{x}_i, i = 1,4 \), time variation of the unknown parameters’ vector estimation error \( \hat{\theta} \), and the time variation of the vector \( \hat{\theta} \)) and the graphic characteristics in Fig. 3 (the four components of the state vector \( \mathbf{x}_i, i = 1,4 \) – solid blue line and the four components of the estimated state vector \( \mathbf{x}_i, i = 1,4 \) – red dashed line). The state vector estimation errors and the unknown vector estimation error tend to zero.
while the graphics of the state variables $x_i, i = 1, 4$ are superposed over the graphics of the estimated state variables.

Figure 2. State vector estimation errors, the unknown parameters’ vector estimation error, and the time variation of the vector $\theta$
(Charlie aircraft longitudinal motion)

Figure 3. State variables \{$x_i$\} and the estimated state variables \{$\hat{x}_i$\}.
(Charlie aircraft longitudinal motion)

The Zhang algorithm does not establish a methodology of choosing the matrices $M, \Sigma, Q$, and $R$, this being a disadvantage of the algorithm. The unknown parameters’ vector has, in this case, only one component $\theta = 2$.

**Example 2: Lateral motion**

Let us consider now the lateral motion of a Boeing 747 flying with Mach number $M = 0.8$ at the altitude $H = 40000$ ft; state equation of the lateral motion is [30]:

$$
\begin{pmatrix}
\Delta \hat{\beta} \\
\Delta \hat{\omega}_y \\
\Delta \hat{\omega}_z \\
\Delta \phi
\end{pmatrix}
= 
\begin{bmatrix}
0.0558 & -0.9968 & 0.0802 & 0.0415 \\
0.598 & -0.115 & -0.0318 & 0 \\
0.305 & 0.388 & -0.465 & 0 \\
0.0073 & 0 & 0.0805 & 1 \\
0 & 0 & -0.475 & 0.123 \\
0.153 & 1.063 & 0 & 0
\end{bmatrix}
\begin{pmatrix}
\Delta \beta \\
\Delta \omega_y \\
\Delta \omega_z \\
\Delta \phi
\end{pmatrix}
+ 
\begin{pmatrix}
0.0073 \\
-0.475 & 0.123 \\
0.153 & 1.063
\end{pmatrix}
\begin{pmatrix}
\delta_d \\
\delta_e
\end{pmatrix}

(35)

$\beta$ is the airplane sideslip angle, $\omega_y$ – the airplane yaw angular rate, $\omega_z$ – the airplane roll angular rate, $\phi$ – the airplane roll angle, $\delta_d$ – the rudder deflection, and $\delta_e$ – the ailerons deflection.

Figure 4. State vector estimation errors and the unknown parameters’ vector estimation error (Boeing 747 lateral motion)

Figure 5. State variables \{$x_i$\} and the estimated state variables \{$\hat{x}_i$\}.
(Boeing 747 lateral motion)

We design the Zhang adaptive observer for the lateral motion of the Boeing 747 too. The gain matrix of the system $K$ is again calculated by using the ALGLX algorithm [30] and we considered as the input vector the vector containing the deflection of the rudder and the deflection of the ailerons $u = [\delta_d, \delta_e] = -Kx$. We obtained the graphic characteristics in Fig. 4 (the time dependencies of the state vector estimation errors $\hat{x}_i, i = 1, 4$, and the time variation of the unknown parameters’ vector estimation error $\hat{\theta}$) and the graphic characteristics in Fig. 5 (the four components of the state vector $x_i, i = 1, 4$ – solid blue line and the four components of the estimated state vector $\hat{x}_i, i = 1, 4$ – red dashed line). The unknown parameters’ vector has, in this case, two components and it has been chosen $\theta = \begin{bmatrix} 0.9 & 3 \end{bmatrix}$ . Same remarks regarding the convergence of the adaptive observer can be made.
V. CONCLUSIONS

The linear systems state estimation is made by using observers and the Kalman filter. Joint estimation of state and some unknown parameters are made by means of several known solutions, with so-called adaptive state estimators. One of the most important algorithms for state estimation in the case of MIMO linear time varying systems (LTV) was designed by Q. Zhang [1]; the algorithm is simple, computationally efficient, viable, and easy to implement software. The Zhang adaptive observer is used in this paper to estimate the state of the aircrafts longitudinal and lateral motions. Two examples are considered: the first one represents the longitudinal motion of a Charlie aircraft, while the second example is associated to the lateral motion of a Boeing 747 airplane.

For the aircraft longitudinal and lateral motions we obtained as graphic characteristics: the time dependencies of the state vector estimation errors, the time variation of the unknown parameters’ vector estimation error, the time variation of the vector unknown parameters’ vector, the time variation of the four components of the state vector (solid blue line), and the four components of the estimated state vector (red dashed line). The state vector estimation errors and the unknown vector estimation error tend to zero, while the graphics of the state variables are superposed over the graphics of the estimated state variables. The Zhang algorithm does not establish a methodology of choosing the matrices $M$, $\Sigma$, $Q$, and $R$, this being a disadvantage of the algorithm.

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