VERTICAL DIAMAGNETIC LEVITATION ARRAY WITH EXTENDED STABILITY AREA

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Abstract – **Stable static levitation of one permanent magnet by another with no energy input and without superconductors is forbidden by Earnshaw's theorem. Adding a diamagnetic material nearby the suspended magnet the intrinsically unstable equilibrium can be stabilized. For a vertical symmetric configuration this paper establishes the equilibrium and stability equation using an analytical procedure of magnetic field computation and suggests methods of enlarging the stability area by alternating the geometry of magnetic fields sources. The applications of these levitation devices concern the designing of ultra-sensitive measurement instruments used where sensitivity to minor variation of the gravitational field is required.**

Keywords: *levitation, stability, diamagnetic body*

1. INTRODUCTION

A classical result given by Earnshaw's theorem [1, 2] shows that free stable static levitation in static or stationary field is not allowed. An exogenous intervention over the levitation system or the usage of superconductor materials can escape the configuration from theorem incidence by violating its terms. Thus the stable static equilibrium could be reached [3]. A much more simple an inexpensive way of stabilizing the equilibrium can be obtained by placing in the proximity of the floater a diamagnetic body. The diamagnetic material, due to its negative magnetic susceptibility, acts as a very weak servo-system keeping the suspended magnet in a very small stability area [4]. This paper suggests how this area can be enlarged using different magnetic field sources and evaluates these enlargements. There is also analyzed the possibility of getting an extended zone with minimum energy assumption.

2. EQUILIBRIUM AND STABILITY

Stable static levitation of permanent magnets requires a minimum of the total energy along with the satisfied equilibrium condition. If an $\boldsymbol{M}$ dipole magnet is imbedded in a field $\boldsymbol{B}$, and $\boldsymbol{M}$ and $\boldsymbol{B}$ are parallel, then the potential energy U of the system is:

$$U = -\boldsymbol{MB} + mgz, \tag{1}$$

where mgz is the gravitational energy.

The magnet will align with the local field direction because of the torque and therefore the energy is only dependent on the magnitude B of the magnetic field. For a circularly symmetric field $B(r, z)$, the equilibrium points will be on z-axis of the symmetry. Then the condition that $(z_0, 0)$ to be an equilibrium point is:

$$\boldsymbol{F} = -\nabla U\Big|_{\substack{r=0\\ z=z_0}} = 0 \Rightarrow \nabla B\Big|_{\substack{r=0\\ z=z_0}} = -\frac{mg}{M}. \tag{2}$$

The stability conditions in $(z_0, 0)$ ask for a minimum value of energy, which means positive curvature function in every direction:

$$\begin{aligned} \frac{\partial^2 U}{\partial z^2}\bigg|_{\substack{r=0\\ z=z_0}} &> 0; \quad \text{vertical stability;} \\ \frac{\partial^2 U}{\partial r^2}\bigg|_{\substack{r=0\\ z=z_0}} &> 0; \quad \text{radial stability.} \end{aligned} \tag{3}$$

To complete the problem, we express the magnitude of the magnetic field B in terms of its z-component $B_z(r, z)$ only. Taking into account that $\nabla \boldsymbol{B} = 0$ and $\nabla\times\boldsymbol{B} = 0$, the following extension of $B(r, z)$ around $(z_0, 0)$ derive [4]:

$$\begin{aligned} B(r,z) &= B_0 + B_0^{'}(z-z_0) + \frac{1}{2}B_0^{''}(z-z_0)^2 + \\ &+ \frac{1}{4}\left[\frac{B_0^{'2}}{2B_0} - B_0^{''}\right] r^2 + \cdots. \quad \text{Here } B_0 = B_z(0,z)\Big|_{\substack{z=z_0\\ r=0}}; \\ B_0^{'} &= \frac{\partial B_z(0,z)}{\partial z}\bigg|_{\substack{z=z_0\\ r=0}}; \quad B_0^{''} = \frac{\partial^2 B_z(0,z)}{\partial z^2}\bigg|_{\substack{z=z_0\\ r=0}}. \end{aligned} \tag{4}$$

Our approach toward stabilizing the equilibrium is to place a diamagnetic material beneath the levitated magnet. In this case, in the relation of potential energy (1) a new term Cz^2 is added, which represent the diamagnetic material influence. According to the magnitude of the magnetic flux density extension for the external magnetic field (4) and taking into account the influence of the diamagnetic material presence, the whole potential energy U become:

$$U = -M\left[B_0 + \left\{ B_0^{'} - \frac{mg}{M} \right\}(z - z_0) + \frac{1}{2}B_0^{''}(z - z_0)^2 + \frac{1}{4}\left\{ \frac{B_0^{'}}{2B_0} - B_0^{''} \right\} r^2 + \cdots \right] + C(z - z_0)^2. \quad (5)$$

From the equilibrium condition required by (2), we obtain the magnetic field gradient $B^{'}$ equal to $-mg/M$. This means that the quantity in the first curly branches of relation (5) must go to zero. Vertical and horizontal stability conditions given by (3) can be now rewritten:

$$\begin{cases} \left.\dfrac{\partial^2 U}{\partial z^2}\right|_{\substack{z=z_0\\ r=0}} > 0 \Rightarrow D_v = C - \dfrac{MB_0^{''}}{2} > 0; \\ \left.\dfrac{\partial^2 U}{\partial r^2}\right|_{\substack{z=z_0\\ r=0}} > 0 \Rightarrow D_h = \dfrac{M}{4}\left\{ B_0^{''} - \dfrac{B_0^{'2}}{2B_0} \right\} > 0. \end{cases} \quad (6)$$

For achieving a stable static levitation of the floating magnet, the above condition, called discriminants of stability D_v and D_h, must be simultaneous satisfied. Without diamagnetic materials, setting $C = 0$, we see that if the curvature of flux density of the field is positive and large enough to create horizontal stability: $B^{''} > (mg)^2/(2M^2B_0)$ $D_h > 0$, the magnet is unstable vertically: $D_v < 0$.

3. TECHNICAL IMPLEMENTATION

If we consider now the case when $B^{''} > 0$, and it is large enough to create horizontal stability ($D_h > 0$), adding a diamagnetic material in vertical direction a vertical stabilised configuration can be obtained. In Fig. 1 a simple vertical configuration that uses a coil as the field source is shown. Fig. 2 magnifies the cylindrical levitated magnet of height L and radius R near the diamagnetic material in two different cases: one with one diamagnetic material and the other with two diamagnetic sheets placed symmetrically to the floater. The lifter is a cylindrical symmetric bar magnet of a common ceramic material. The piece of diamagnetic material can be made from graphite, bismuth or pyrolytic graphite (materials with great absolute values for magnetic susceptibility). For the floating magnet one uses rare-earth materials such as mixture of $Nd_2Fe_{14}B$, which could reach 1.2 T for remanent flux density. All this restrictions are necessary to satisfy the equilibrium condition (2) and to fulfill the stability requirement (6) in the equilibrium point as well. Combining conditions for vertical stability using (6) we can write:

$$\frac{2C}{M} > B_0^{''} > \frac{m^2 g^2}{2B_0 M^2}. \quad (7)$$

The C term is proportional to the diamagnetic susceptibility (χ) and gets smaller if the gap d between the levitated magnet and diamagnetic body increases (Fig. 2). Its value can be determined by using the dipole approximation [6]: $C = 3|\chi|\mu_0 M^2/(\pi D^5)$, where $D = 2d + L$.

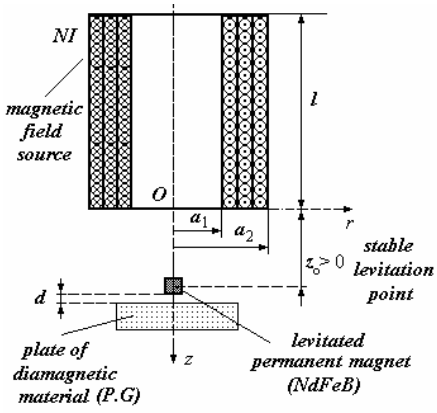

Fig. 1: Vertically stabilized levitation system.

We can see that a large gap or a weaker diamagnetic material requires a larger field B in the levitation position. The limits for separation d can be achieved taking into account (7) and the vertical dimension L of the floating magnet:

$$L < D < \left(\frac{12\mu_0 B_0 M^3 |\chi|}{\pi (mg)^2} \right)^{1/5}. \quad (8)$$

The usage of two diamagnetic materials doubles the value of C term which leads to an extension of the above estimated stability interval with 1.1 units– Fig. 2. This interval is also increased by a greater absolute value for the magnetic susceptibility of diamagnetic materials.

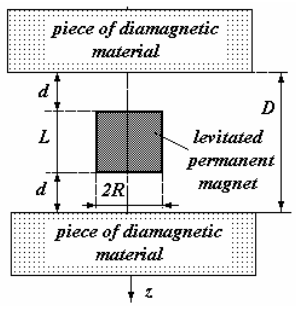

Fig. 2: Two diamagnetic pieces.

4. NUMERICAL APPROACH

In order to achieve a quantitatively analyse over the above studied configuration we set up a very simple model with specifically geometrical data and material properties. For the lifter magnet we used a cylindrical symmetric coil made with $l = 20$ cm, $a_1 = 2$ cm, $a_2 = 8$ cm and $NI = 1500$ Asp. A tiny cylinder (diameter $2R = 4$ mm and high $L = 4$ mm) was chosen as the floater. Its magnetic momentum is $M = 0.044$ Am^2 and mass $m = 0.39$ g. A piece of graphite ($\chi = 160\cdot 10^{-6}$) placed at $d = 0.5$ mm under the levitated magnet stabilises the equilibrium. All these numerical data leads to $C = \dfrac{3|\chi|\mu_0 M^2}{\pi D^5} = 0.118$.

The magnetic field variation outside the lifter magnet on the z-symmetry axis could be accurately determinate from the finite solenoid equation [7]:

$$B(z) = \frac{\mu NI}{2l}\left(\begin{array}{l} \dfrac{l+z}{a_2 - a_1}\ln\dfrac{a_2 + \sqrt{a_2^2 + (l+z)^2}}{a_1 + \sqrt{a_1^2 + (l+z)^2}} - \\ -\dfrac{z}{a_2 - a_1}\ln\dfrac{a_2 + \sqrt{a_2^2 + z^2}}{a_1 + \sqrt{a_1^2 + z^2}} \end{array} \right). \quad (9)$$

The equilibrium equation (2) has one solution namely $z_0 = 1.32$ cm. In this point both stability restriction (6) are satisfied – Fig.3.

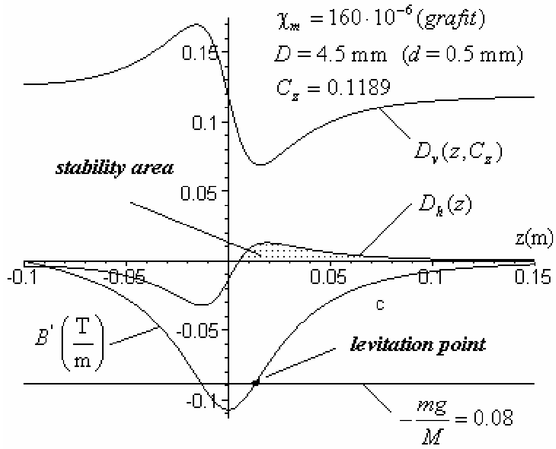

Fig. 3: The stability areas for the source.

The maximum value of the gap spacing is $d_{max} = 0.98$ mm. That corresponds to $C = 0.049$ which gives a positive value of D_v at the levitation point. Thus the stability area is restricted by $d \in (0 – 0.98)$ mm. As we already suggested in the previous section, using two pieces of diamagnetic materials (the bottom of Fig. 2), along with a greater value for the absolute magnetic susceptibility (pyrolitic graphite) can increase this relatively narrow stability zone up to 1.66 mm.

5. EXTENDING THE STABILITY AREA

Using a special geometry for the magnet field source the very narrow stability zone can be enlarged.

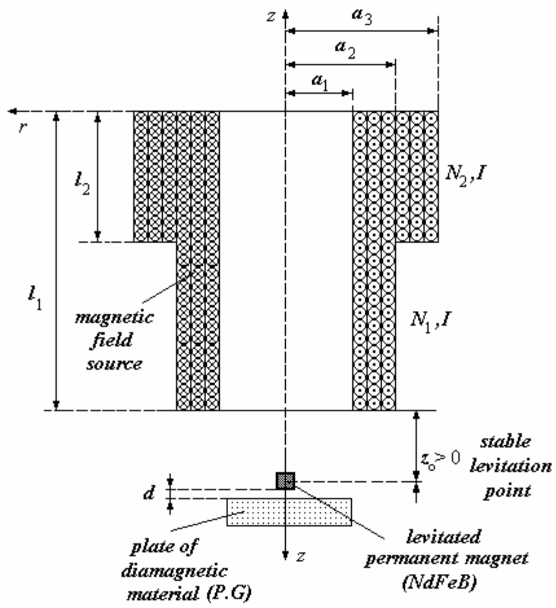

Fig. 4: The model for the field new source.

In Fig. 4 is shown a "multiple winding coil" with 2 steps. The magnetic field on the z -axis is the sum of all magnetic fields generated by every coil:

$$B(z) = B_1(z) + B_2(z) \quad (10)$$

In (10) the expression for flux density of each coil is given by a relation similar with (9). We assume $N_1 = 500$, $N_2 = 500$, $I = 3$ A, $l_1 = 20$ cm, $l_2 = 4$ cm, $a_1 = 2$ cm, $a_2 = 8$ cm, $a_3 = 14$ cm. With this numerical data in Fig. 5 is indicated the extended area where the equilibrium is stable.

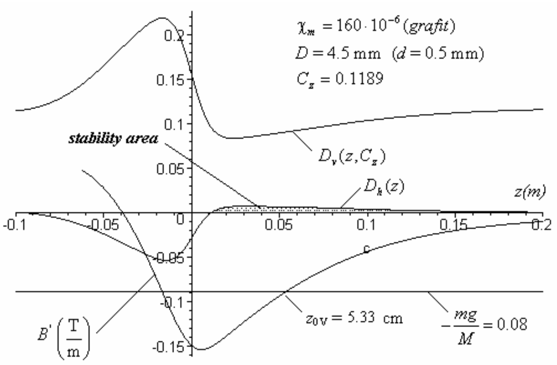

Fig. 5: The extended stability areas for the new magnetic field source.

It is important to notice that in this case the equilibrium point, placed at z_0= 5.33 cm, has a much larger stability area than in the case of a simple coil.

The maximum value of the gap spacing is $d_{max} = 1.33$ mm. That corresponds to $C = 0.028$ which gives a positive value of D_v at the levitation point. If two diamagnetic pieces of highest diamagnetic materials are used (pyrolitic graphite), than the maximum stability interval would reach 2.71 mm

Analyzing all the axial symmetric configurations we can say that the "multiple winding" coil can be a solution for extending the relative feeble stability area of equilibrium. This magnetic field source geometry also provides an easier implementation that can be achieved without any special technologies.

6. CONCLUSIONS

The stabilization problem of permanent magnet levitation in stationary fields by diamagnetic materials is treated over a particular case of a vertical symmetric magnetic field source.

The equilibrium and stability conditions are analyzed for a specified configuration determining the location of equilibrium point and its stability area. We find out that the inflection point of the magnetic field, where the levitation is possible to occur, is fixed by the geometry of the lifter magnet, not by its strength. The instability is related to the curvature of the lifter field and force balance depends on the gradient. That makes it feasible to engineer the location of the stable zones by adjusting the geometry of the field source and to control the gradient by adjusting its strength. For the specific levitation array a numerical simulation was also made presenting the quantitative aspect for these developed devices. From this examination we achieve excessive tiny values for the gap spacing between the floater and diamagnetic materials which can be enlarged by using diamagnetic pieces with higher absolute value for magnetic susceptibility.

The extension of these relative small stability areas is enlarged using special geometry configuration (for the magnetic field sources. The results show an increase of more than 50 % for one of the stability areas of the magnetic field sources.

The application of this permanent magnet levitation can be found in very high-sensitive gravity sensors or in designing frictionless suspension whose parameters (such rigidity) can by controlled by adjusting the field profile. The levitation of diamagnetic materials can be also useful in designing ultra-sensitive geophysical equipment such as gravimeters, seismometers or other devices where high sensitivity to a minor variation of gravitational field is required.

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