THE MAGNETIC FIELD AND MAXWELL' EQUATIONS RESULT OF THE COULOMB LAW, RELATIVITY AND QUANTUM MECHANICS

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***Abstract* - The aim of this paper is to present the way to obtain the magnetic field, law of the electromagnetic induction and the law of the magnetic circuit starting with the Coulomb law, relativistic kinematics and quantum mechanics.**

Keywords: *magnetic field, Coulomb law, quantum mechanics*

1. INTRODUCTION

The literature [1, 2, 4] presents a lot of possibilities to obtain the magnetic field starting with Coulomb law and the kinematics of the restrain relativity.
The unsolved problem is that of the connection between the induction electromagnetic law and the general principle of restrain and general relativity.
In this work, we are starting with the following physical fundamental concepts for resolving this problem:
A1) Electric charge is a relativistic invariant;
A2) Electric charge has preserved in space-time;
A3) The electric field around the electric charge has a value given by Coulomb law;
A4) The evolution both in space and in time of the charge and of the electric field is compatible with restrain relativity postulates.
A5) The evolution in space and time of the electric charge is compatible with quantum mechanics.

2. THE MAGNETIC VECTOR POTENTIAL

We consider an infinite cylinder of radius r_1 and positive charged with density ρ_0. Other cylinder coaxial of radius r_2 is negative charged, but this has same density charge ρ_0, fig.1. The two cylinders start in opposite sense with the velocity v_0 given of the fixed coordinate system of the laboratory.
In the fixed referential, the electric potential is zero. Let consider a test charge that displaces to right with velocity. Reporting to the mobile cylinder, the test charge has the following values for its velocity:

$$\begin{cases} v'_+ = \dfrac{v_0 - u}{1 - \dfrac{v_0 u}{c_0^2}} \\ v'_- = \dfrac{v_0 + u}{1 - \dfrac{v_0 u}{c_0^2}} \end{cases} \tag{1}$$

We introduce the notations:

$$\beta = \frac{u}{c_0}, \beta_0 = \frac{v_0}{c_0}, \ \beta'_+ = \frac{v'_+}{c_0}, \ \beta'_- = \frac{v'_-}{c_0} \tag{2}$$

We substitute (1) in (2) and we obtain:

$$\beta'_+ = \frac{\beta_0 - \beta}{1 - \beta_0 \beta}, \ \beta'_- = \frac{\beta_0 + \beta}{1 + \beta_0 \beta} \tag{3}$$

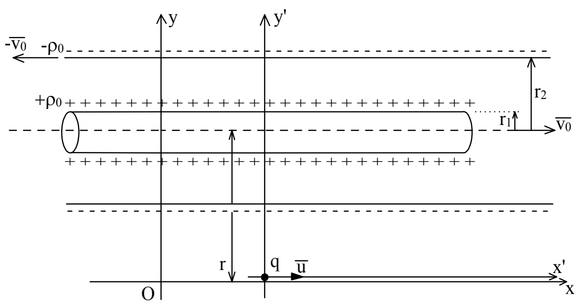

Fig. 1. An explicative to calculate the magnetic vector potential

The density of the electric charge corresponding to the mobile wall is equal:

$$\rho_{-am} = \rho_- \sqrt{1 - \beta_0^2}, \ \rho_{+am} = \rho_+ \sqrt{1 - \beta_0^2} \tag{4}$$

The density of the electric charge of the mobile wall given the test charge is equal:

$$\rho'_- = \frac{\rho_- \sqrt{1 - \beta_0^2}}{\sqrt{1 - \beta'^2_-}}, \ \rho'_+ = \frac{\rho_+ \sqrt{1 - \beta_0^2}}{\sqrt{1 - \beta'^2_+}} \tag{5}$$

In the electric charge test referential, the capacitor is "viewed" not neutral as in repos state, but load with the density of electric charge:

$$\rho_{rez} = \rho'_{+} - \rho'_{-} \tag{6}$$

In the mobile referential of the electric charge $x'Oy'$, the potential is equal:

$$V' = \frac{\rho_{rez}}{2\pi\varepsilon_0} \ln r \tag{7}$$

We substitute (5) and (6) in (7) and we obtain:

$$V' = -\frac{\rho_0 v_0 u \ln r}{\pi\varepsilon_0 c_0^2 \sqrt{1-\beta_0^2}\sqrt{1-\beta_0^2}} \tag{8}$$

In quantum mechanics [2] has presented that in presence of an electric potential, the amplitude phase of probability is:

$$\exp\left(-\frac{i}{\hbar} q V' dt'\right) \tag{9}$$

The relation between the time intervals dt' in the system $x'O'y'$ and laboratory system dt is:

$$dt' = dt_{lab}\sqrt{1-\beta^2} \tag{10}$$

Let us substitute (8) and (10) in (9):

$$\exp\left(\frac{iq}{\hbar} \frac{(\rho_0 v_0)(u t_{lab})\ln r}{\pi\varepsilon_0 c_0^2 \sqrt{1-\beta_0^2}}\right) \tag{11}$$

The term udt_{lab} represents the element of length dl_{lab} in xOy system.

The element of length in the laboratory referential can write function of the element of length in the referential of mobile cylinders:

$$dl_{lab} = dl\sqrt{1-\beta_0^2} \tag{12}$$

We substitute (12) in (11) and we obtain:

$$\exp\left[\frac{iq}{\hbar}\left(\frac{\rho_0 v_0}{\pi\varepsilon_0 c_0^2} \ln r\right) dl\right] \tag{13}$$

We have obtained the amplitude phase of the probability in function of the element of length dl and not in function of the time dt.

We can write the amplitude phase of the probability in function of the space if we introduced a new physical quantity. This is the magnetic potential vector:

$$A_B = \frac{\rho_0 v_0}{\pi\varepsilon_0 c_0^2} \ln r = \frac{\mu_0 i}{2\pi} \ln r \tag{14}$$

Here $i = \frac{\rho_0 v_0}{2}$ because in the conductor, the speed of the negative charges is in relationship to the positive charges and these are immobile.

Let' consider the scalar potential for an infinite conductor charged with the density of the charges ρ_0:

$$V = \frac{\rho_0}{2\pi\varepsilon_0} \ln r \tag{15}$$

From (14) and (15) we obtain:

$$\overline{A}_B = \frac{\overline{v}}{c_0^2} V \tag{16}$$

In conclusion, we demonstrate that the magnetic potential vector is the result of Coulomb law, kinematics of the restrain relativity and quantum mechanics.

In order to find the magnetic field starting with the Coulomb law, we start from the electric potential generated by a charge $dq = \rho dv$, where dv is the elementary volume:

$$dV = \frac{1}{4\pi\varepsilon_0} \frac{\rho}{r} dv \tag{17}$$

We substitute (17) in (16):

$$d\overline{A}_B = \frac{1}{4\pi\varepsilon_0 c_0^2} \frac{\rho\overline{v}}{r} dv = \frac{\mu_0}{4\pi} \frac{\overline{J}}{r} dv \tag{18}$$

The magnetic field $\overline{B}$ is given by (18):

$$d\overline{B} = curl\overline{A}_B = \frac{1}{4\pi\varepsilon_0 c_0^2}\left(\overline{J} \times \frac{\overline{r}}{r^3}\right) dv =$$

$$= \frac{1}{4\pi\varepsilon_0 c_0^2} \overline{v} \times \frac{\rho\overline{r}}{r^3 dv} = \frac{\overline{v}}{c_0^2} \times d\overline{E}$$

Finally:

$$\overline{B} = \frac{\overline{v}}{c_0^2} \times \overline{E} \tag{19}$$

This relation between the magnetic field and electric field is known from relativistic electrodynamics [2],[3].

Finally, also the magnetic field is a consequence of the Coulomb law and kinematics of the restrains relativity.

The magnetic potential vector and the magnetic induction, generated by an electric conduction current are the result of the relativistic combination of two electric convective current.

3. FARADAY'S LAW OF INDUCTION

In order to determine the Faraday's law of induction, we reconsider (16) and the electric potential generated by a charge q:

$$\overline{A}_B = \frac{\overline{v}}{c_0^2}\frac{q}{4\pi\varepsilon_0 r} \tag{20}$$

We multiply this relation, both right and left side, with the test charge q_0 there are at r distance from the electric charge q :

$$q_0\overline{A}_B = \frac{\overline{v}}{c_0^2}\frac{qq_0}{4\pi\varepsilon_0 r} \tag{21}$$

In above relation, $\overline{v}$ is the velocity of electric charge q that appropriate or remove before the test charge q_0. The second term from left relation represents the interaction energy between the two electric charges. We introduce the "field mass" corresponding to the energy of interaction [2]:

$$m_{field}(t) = \frac{qq_0}{4\pi\varepsilon_0 r c_0^2} \tag{22}$$

We substitute (22) in (21):

$$\overline{v}m_{field} = q_0\overline{A}_B \tag{23}$$

The left term is the impulse of the "field mass". This is variable in the time, because the electric charges move one given to another.

By derivate (23) in function of the time, we obtain the inertial force that is orientated inverse from the variation of the velocity:

$$\frac{d}{dt}\left(\overline{v}m_{field}\right) = -\overline{F}_{in} = q_0\frac{\partial\overline{A}_B}{\partial t} \tag{24}$$

The above relation can write:

$$\frac{\overline{F}_{in}}{q_0} = -\frac{\partial\overline{A}_B}{\partial t} \tag{25}$$

The first term is the induced electric field $\overline{E}_s$:

$$\overline{E}_s = -\frac{\partial\overline{A}_B}{\partial t} \tag{26}$$

The total electric field that acting on a electric test charge is compound from the solenoidal electric field (26) and potential electric field generated by the electric charge:

$$\overline{E} = -\frac{\partial\overline{A}_B}{\partial t} - \nabla V \tag{27}$$

This relation represents the electromagnetic law of the induction, which can write in the following form:

$$curl\overline{E} = -\frac{\partial\overline{B}}{\partial t} \tag{28}$$

4. LORENTZ' CONDITION

In literature [2],[4], Lorentz' condition is gives because of the arbitrary decision to choose the divergence of the magnetic potential vector $\overline{A}_B$, in order to separate the equations for $\overline{A}_B$ and V in general equation:

$$-\nabla^2\overline{A}_B + \nabla\left(\nabla\overline{A}_B\right) + \frac{1}{c_0^2}\frac{\partial}{\partial t}\nabla V + \frac{1}{c_0^2}\frac{\partial^2\overline{A}_B}{\partial t^2} = \frac{\overline{J}}{\varepsilon_0}$$

In order to separate the equations for $\overline{A}_B$ and V , we make the following condition:

$$div\overline{A}_B + \frac{1}{c_0^2}\frac{\partial V}{\partial t} = 0 \tag{29}$$

This is the well-known relation of "Lorentz' condition".

The same result may be obtaining much intuitive if we applied the divergence operator to the relation:

$$\overline{A}_B = \frac{\overline{v}}{c_0^2}V \tag{30}$$

Using the following relations:

$$\begin{cases} \nabla\left(\overline{v}V\right) = \overline{v}\nabla V + V\nabla\overline{v} \\ \overline{v} = const \\ \dfrac{\partial V}{\partial x} = \dfrac{\partial V}{\partial t}\dfrac{dt}{dx} = -\dfrac{1}{v}\dfrac{\partial V}{\partial t} \end{cases} \tag{31}$$

We can write (30) as:

$$div\overline{A}_B = -\frac{1}{c_0^2}\frac{\partial V}{\partial t}$$

This equation represents the Lorentz' condition.

5. THE LAW OF THE MAGNETIC FLUX

We start to following relation:

$$\overline{B} = \frac{\overline{v}}{c_0^2} \times \overline{E}$$

We apply the divergence operator both right and left expression:

$$div\overline{B} = \frac{1}{c_0^2}div\left(\overline{v} \times \overline{E}\right) \tag{32}$$

Starting to:

$$\begin{cases} \nabla\left(\overline{v} \times \overline{E}\right) = -\overline{v}\nabla \times \overline{E} + \overline{E}\nabla \times \overline{v} \\ \overline{v} = const \\ \nabla \times \overline{E} = 0 \end{cases} \tag{33}$$

We obtain the law of the magnetic flux in differential form:

$$div\overline{B} = 0$$

6. THE LAW OF THE MAGNETIC CIRCUIT

Starting with the following relation:

$$\overline{B} = \frac{\overline{v}}{c_0^2} \times \overline{E}$$

Where we applied the rotation operator both of the terms:

$$curl\overline{B} = \frac{1}{c_0^2} curl\left(\overline{v} \times \overline{E}\right) \quad (34)$$

According to the following relations:

$$curl\left(\overline{v} \times \overline{E}\right) = \left(\overline{E}\nabla\right)\overline{v} - \left(\overline{v}\nabla\right)\overline{E} + \left(\nabla\overline{E}\right)\overline{v} - \left(\nabla\overline{v}\right)\overline{E}$$

$$\overline{v} = const$$

$$\nabla\overline{E} = \frac{\rho}{\varepsilon_0}$$

We obtain:

$$curl\overline{B} = \frac{1}{c_0^2}\left[-\left(\overline{v}\nabla\right)\overline{E} + \frac{\rho}{\varepsilon_0}\overline{v}\right] \quad (35)$$

We consider that the displacement of the charge is making upon Ox axis, which is tangent to the trajectory of the electric charge:

$$\frac{\partial\overline{E}}{\partial x} = \frac{\partial\overline{E}}{\partial t}\frac{\partial t}{\partial x} = -\frac{1}{v}\frac{\partial\overline{E}}{\partial t} \quad (36)$$

From (29) and (30) we obtain:

$$curl\overline{B} = \frac{\rho\overline{v}}{\varepsilon_0 c_0^2} + \frac{1}{\varepsilon_0 c_0^2}\frac{\partial\overline{D}}{\partial t} \quad (37)$$

We introduce the quantities:

$$\mu_0 = \frac{1}{\varepsilon_0 c_0^2}$$

$$\overline{J} = \rho\overline{v}$$

Finally, we are determinate the relation known as the law of the magnetic circuit:

$$curl\overline{H} = \overline{J} + \frac{\partial\overline{D}}{\partial t} \quad (38)$$

The law of the magnetic circuit is a consequence of the displacement of the electric Coulomb field, with a velocity in the space.

The law of the magnetic circuit is a consequence of the law of the electric charge conservation, because according to A3), the electric field is joint to electric charge that is conserved (A2).

The law of the conservation of electric charge is o consequence of the quantum mechanics. From Schrödinger equation, results the conservation of the probability current [2]:

$$div\overline{J}_p = -\frac{\partial\rho_p}{\partial t}, \quad (39)$$

where:

$$\rho_p = \Psi\Psi^* \quad (40)$$

$$\overline{J}_p = \frac{1}{2m}\left[\Psi^*\left(-i\hbar\nabla - q\overline{A}_B\right)\Psi + \Psi\left(-i\hbar\nabla - q\overline{A}_B\right)\Psi^*\right]$$

From above relations, we obtain the density of electric charge and the density of the electric current:

$$\rho = q\rho_p$$

$$\overline{J} = q\overline{J}_p \quad (41)$$

From (35) and (33) we determine the law of the conservation of electric charge:

$$div\overline{J} = -\frac{\partial\rho}{\partial t} \quad (42)$$

The relationship between (32) and (40) is evidently, if in (36) we introduce:

$$\rho = div\overline{D} \quad (43)$$

After this substitution:

$$div\left(\overline{J} + \frac{\partial\overline{D}}{\partial t}\right) = 0 = div\left(curl\overline{H}\right) \quad (44)$$

The above relation is identically to (32) and this proves the causal relationship between quantum mechanics and the law of the magnetic circuit.

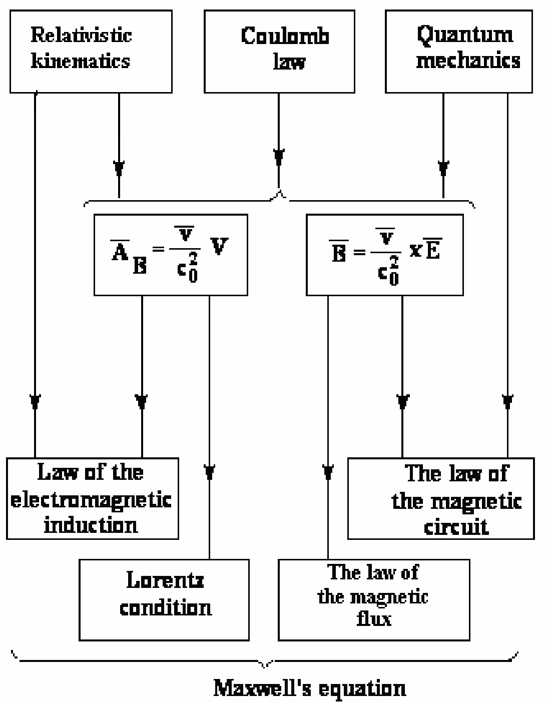

Fig. 2. The diagram of the causal relationships

7. DIAGRAM OF THE CAUSAL RELATIONSHIPS

The all results that we obtained in 1-6 paragraphs may synthesized in a diagram (Fig.2), which presents the logical relationships between the fundamental elements of the theory presented and the consequences immediate.

8. CONCLUSIONS

(1)The electrodynamics equations are the consequence of the fundamental theories: restrain relativistic kinematics, quantum mechanics and Coulomb's law.
(2) Starting with the phase of the amplitude of the probability from quantum mechanics, Lorentz' transformations and Coulomb's law, we obtained the magnetic potential vector and the magnetic induction.
(3) The law of the electromagnetic induction has obtained starting to the inertial principle from the relativity theory.
(4) The rapport between the inertial force and electric charge represent a solenoid electric field.

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