THE UNITARY THEORY OF THE ELECTRIC POWERS

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Abstract **– General physics approach is applied to analysis of power components in electrical systems under sinusoidal and non-sinusoidal conditions. Physical essence of active, reactive and distortion are determinate. It is shown that the all three powers are the different aspects of the same physical phenomenon: mechanical action per time of Coulomb forces or inertial forces.**

Keywords**:** *electric power, general physics.*

1. INTRODUCTION

In 1927, C. Budeanu has introduced for the first time the reactive and distorting powers, not based by physical phenomenon, only based by definitions. Many studies have analyzed the physical significances, but the problem has not resolved. Thus, there are many definitions in literature [1], [2], [3] and different approach. In this work, we are starting with following physical fundamental concepts for resolving the electrical powers under sinusoidal and non-sinusoidal conditions:

A1) We can associate a mass to any energy form, based by Einstein formula: $E = mc_0^2$

A2) Force is the variation of the impulse under time;

A3) The instantaneous power is the multiplied under force and velocity, at a moment.

2. THE ACTIVE ELECTRICAL POWER, *P*

Let us consider an electrical current $i(t)$ that pass throw a material conductor. The current density $\bar{J}(t)$ and the mean velocity of electrical charges $\bar{v}(t)$ at a moment time are in relation:

$$\bar{J}(t) = r\bar{v}(t) \tag{1}$$

The force exercised on the electrical density of the electrical field $\bar{E}(t)$ is:

$$\bar{F}(t) = r\bar{E}(t) \tag{2}$$

An instantaneous power is determined as:

$$p(t) = \bar{F}(t) \cdot \bar{v}(t) \tag{3}$$

By substituting ***F(t)*** from (2) in (3) will obtain:

$$p(t) = r\bar{E}(t)\bar{v}(t) \tag{4}$$

Using (1) will obtain:

$$p(t) = \bar{J}(t)\bar{E}(t) \tag{5}$$

This relation (5) represents the power density in a point situated in Ω domain, covering of the electrical current. The instantaneous total power in the Ω domain is equal:

$$P_{tot}(t) = \int_{\Omega} p(t)dv \tag{6}$$

The instantaneous total power has obtained by substituting (5) and the elemental volume $dv = d\bar{S}d\bar{l}$ in (6):

$$P_{tot}(t) = \int_1^2 \int_S \left(\bar{J}(t)d\bar{S}\right)\left(\bar{E}(t)d\bar{l}\right) = u(t)\cdot i(t) \tag{7}$$

The mean power averaged over time interval is:

$$\begin{aligned} P_{med} &= \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} u(t)i(t)dt = \\ &= \langle u(t), i_{//}(t) + i_{\perp}(t)\rangle = \|u(t)\| \cdot \|i_{//}(t)\| \end{aligned} \tag{8}$$

Where $i_{//}(t)$ and $i_{\perp}(t)$ are the orthogonal components of the $i(t)$ in relation with $u(t)$ [5].

Under sinusoidal conditions:

$$\begin{cases} u(t) = U\sqrt{2}\sin wt \\ i(t) = I\sqrt{2}\sin(wt - j) \end{cases} \tag{9}$$

Using (8) and (9) the mean power averaged over a period is equal:

$$P_{med} = UI\cos j \tag{10}$$

Using non-sinusoidal conditions:

$$\begin{cases} u(t) = U_0 + \sum_i U_k\sqrt{2}\sin\left(kwt + a_k\right) \\ i(t) = I_0 + \sum_i I_k\sqrt{2}\sin\left(kwt + a_k - j_k\right) \end{cases} \tag{11}$$

The mean power over a period is equal:

$$P_{med} = U_0 I_0 + \sum_i U_k I_k \cos j_k \tag{12}$$

Analysing (10) and (11) the conclusion drawn is that the mean active power is the mean work done by the coulombian electrical forces over a period.

3. THE REACTIVE ELECTRICAL POWER, *Q*

The process in the circuit with **R** and **L** elements connected in series will analyzed. The instantaneous magnetically energy of the field is equal:

$$w_m(t) = \frac{Li^2(t)}{2} \tag{13}$$

The magnetic energy is a function of the velocity of the electrical charges, from the current density ***i(t)***. The literature [4] show that the Poynting vector that depends to electrical charges velocity does not radiated energy. According (13), the magnetic energy stored in the inductor is non-radiate but is only variable in time.
According to A1) condition, the "field mass" associated to (13) is moving with the speed of light.

$$m_{camp}(t) = \frac{Li^2(t)}{2c_0^2} \tag{14}$$

The associated impuls to the "field mass" is equal:

$$\overline{p}_{camp}(t) = m_{camp}(t)\overline{c}_0 = \frac{Li^2(t)}{2c_0^2}\overline{c}_0 \tag{15}$$

According to A2), the rate of the impuls is equal to inertial force:

$$\overline{F}_{in}(t) = \frac{d\overline{p}}{dt} = \overline{c}_0 \frac{dm_{camp}(t)}{dt} \tag{16}$$

The instantaneous power corresponding to the deplasment of the "field mass" is equal:

$$Q(t) = \overline{c}_0 \overline{F}_{in}(t) \tag{17}$$

The inertial force may be both positive and negative. We join the positive sign of the inertial force to the inductor loading with magnetic energy and the minus sign then the inductor unload. The necessary power for load and unload the inductor id deliberated by the electric generator. We consider the effective value of the inertial force both for loading and for unloading of the inductor:

$$F_{ef} = \sqrt{\frac{1}{T}\int_0^T F_{in}^2(t)dt} \tag{18}$$

The reactive power is the multiplied between the speed of light and the effective inertial force:

$$Q = c_0 F_{in.ef}. \tag{19}$$

3.1. The reactive power for an inductor

Starting with the sinusoidal current:

$$i(t) = I\sqrt{2}\sin wt \tag{20}$$

Then, the "field mass" stored into magnetic field of the inductor is:

$$m_{camp}(t) = \frac{LI^2}{c_0^2}\sin^2 wt \tag{21}$$

In addition, the corresponding impulse is equal:

$$p(t) = \frac{LI^2}{c_0}\sin^2 wt \tag{22}$$

The inertial force at a moment generated by the "field mass" is:

$$F_{in}(t) = \frac{dp}{dt} = \frac{wLI^2}{c_0}\sin 2wt \tag{23}$$

Let consider the interval (0, T/4) when is produced the energy loading of the inductor, and calculate the effective value of the inertial force:

$$F_{ef} = \sqrt{\frac{2}{T}\int_0^{T/4}\left(\frac{wLI^2}{c_0}\sin 2wt\right)^2 dt} = \frac{wLI^2}{2c_0} \tag{24}$$

The reactive power necessary to load the inductor is equal:

$$Q_{incarcare} = c_0 F_{ef} = \frac{wLI^2}{2} \tag{25}$$

Similarly, the reactive power necessary to unload the inductor is equal:

$$Q_{descarcare} = c_0\sqrt{\frac{2}{T}\int_{T/4}^{T/2}\left(\frac{wLI^2}{c_0}\sin 2wt\right)^2 dt} = \frac{wLI^2}{2} \tag{26}$$

The total reactive power is the sum between (25) and (26):

$$Q = wLI^2 \tag{27}$$

Considering the electric parameters of the circuit, the above relation can write:

$$Q = UI\sin j \tag{28}$$

3.2. The reactive power for a capacitor

Starting with the same electrical current (20), we consider a capacitor circuit.
The tension for the capacitor is equal:

$$u_C(t) = \frac{1}{C}\int i dt = -\frac{I\sqrt{2}}{wC}\cos wt \qquad (29)$$

The "field mass" stored into electric field of the capacitor is:

$$m_{camp}(t) = \frac{Cu_C^2(t)}{2c_0^2} = \frac{I^2}{c_0^2 w^2 C}\cos^2 wt \qquad (30)$$

The inertial force at a moment generated by the "field mass" is:

$$F_{in}(t) = -\frac{I^2}{c_0 wC}\sin 2wt \qquad (31)$$

Let consider the interval (0, T/4) when is produced the energy loading of the capacitor, and calculate the effective value of the inertial force:

$$F_{ef} = \sqrt{\frac{2}{T}\int_0^{T/4}\left(-\frac{I^2}{c_0 wC}\sin 2wt\right)^2 dt} = \frac{I^2}{2c_0 wC} \qquad (32)$$

The reactive power necessary to load the capacitor is equal:

$$Q_{incarcare} = c_0 F_{ef} = \frac{I^2}{2wC} \qquad (33)$$

Similarly, the reactive power necessary to unload the inductor is equal:

$$Q_{descarcare} = c_0\sqrt{\frac{2}{T}\int_{T/4}^{T/2}\left(-\frac{I^2}{c_0 wC}\sin 2wt\right)^2 dt} = \frac{I^2}{2wC} \qquad (34)$$

The total reactive power is the sum between (33) and (34):

$$Q = \frac{I^2}{wC} \qquad (35)$$

3.3. The reactive power in LC circuit

Comparing (23) and (31) we observe that the inertial force produced into inductor has opposite sign compare that the capacitor.
The sum of (23) and (31) is equal:

$$F_{in.rez.}(t) = \frac{I^2}{c_0}\left(wL - \frac{1}{wC}\right)\sin 2wt \qquad (36)$$

The reactive power is:

$$Q = 2c_0\sqrt{\frac{2}{T}\int_0^{T/4}\left[\frac{I^2}{c_0}\left(wL - \frac{1}{wC}\right)\sin 2wt\right]^2 dt} = I^2\left(wL - \frac{1}{wC}\right) \qquad (37)$$

We can write the above relation as:

$$Q = UI\sin j_1 - UI\sin j_2 \qquad (38)$$

4. THE DISTORTING POWER

Let consider an electrical circuit operated under non-sinusoidal conditions:

$$u(t) = U_0 + \sum_{k=1}^{\infty} U_k \sin(kwt + a_k) \qquad (39)$$

$$i(t) = I_0 + \sum_{k=1}^{\infty} I_k\sqrt{2}\sin(kwt + a_k - j_k) \qquad (40)$$

According with C. Budeanu theory, the distorting power is equal:

$$D^2 = \sum_m \sum_{n\neq m}\left[(U_m I_n)^2 + (U_n I_m)^2\right] - \sum_m \sum_{n\neq m}\left(2U_m U_n I_m I_n \cos(j_m - j_n)\right) \qquad (41)$$

Starting with the following relations:

$$\cos(j_m - j_n) = \cos j_m \cos j_n + \sin j_m \sin j_n \qquad (42)$$

$$\begin{cases}\cos j_m = \dfrac{R}{Z_m}\\ \cos j_n = \dfrac{R}{Z_n}\end{cases} \qquad \begin{cases}\sin j_m = \dfrac{mwL}{Z_m}\\ \sin j_n = \dfrac{nwL}{Z_n}\end{cases} \qquad (43)$$

$$\begin{cases}U_m = I_m Z_m\\ U_n = I_n Z_n\end{cases} \qquad (44)$$

We can rewrite (41):

$$D^2 = \sum_m \sum_{n\neq m}\left[(m-n)wLI_m I_n\right]^2 \qquad (45)$$

We calculate the "field mass" using (40):

$$m_{camp}(t) = \frac{L}{c_0^2}\sum_m \sum_{n\neq m}\cos\left[(m-n)wt + a_{m-n} - j_{m-n}\right] - \frac{L}{c_0^2}\sum_m \sum_{n\neq m}\cos\left[(m+n)wt + a_{m+n} - j_{m+n}\right] \qquad (46)$$

where:

$$a_{m-n} = a_m - a_n;$$
$$a_{m+n} = a_m + a_n$$

and:

$$j_{m-n} = j_m - j_n;$$
$$j_{m+n} = j_m + j_n$$

Starting to the derivative function of time for (46), we obtain the inertial force:

$$F_{in}(t)=\frac{L}{c_0}\sum_m\sum_{n\neq m}\frac{I_mI_n}{2}\begin{bmatrix}-(m-n)\sin[(m-n)\omega t+\alpha_{m-n}-\varphi_{m-n}]+\\+(m+n)\omega\sin[(m+n)\omega t+\alpha_{m+n}-\varphi_{m+n}]\end{bmatrix}\quad(47)$$

Using (18) for each harmonic, we calculated the effective value of the inertial force:

$$\boldsymbol{F_{in}}=\mathbf{2}\left[\begin{array}{l}\sum\limits_{m=0}^{\infty}\dfrac{\mathbf{1}}{\boldsymbol{T}}\left[\underbrace{\int_0^{T/2m}\boldsymbol{F_{in}^2(t)dt}+\ldots+\int_{(2m-1)\frac{T}{2m}}^{2m\frac{T}{2m}}\boldsymbol{F_{in}^2(t)dt}}_{m}\right]+\\ \sum\limits_{n\neq m}\dfrac{\mathbf{1}}{\boldsymbol{T}}\left[\underbrace{\int_0^{T/2n}\boldsymbol{F_{in}^2(t)dt}+\ldots+\int_{(2n-1)\frac{T}{2n}}^{2n\frac{T}{2n}}\boldsymbol{F_{in}^2(t)dt}}_{n}\right]\end{array}\right]^{1/2}$$

Taking under consideration the orthogonally of the trigonometric functions, we obtain:

$$F_{ef.in.}^2c_0^2=\sum_{k=1}^{\infty}\left(k\omega LI_k^2\right)^2+\sum_m\sum_{n\neq m}\left(m^2+n^2\right)\omega^2L^2\left(I_mI_n\right)^2\quad(48)$$

The above relation is the multiplication between the square power produced by the inertial force and the square of speed of light.

We calculate the following expression, starting with the reactive and distorting powers given by C. Budeanu:

$$Q=\sum_{k=1}^{\infty}U_kI_k\sin\varphi_k=\sum_{k=1}^{\infty}\left(k\omega LI_k^2\right)\quad(49)$$

$$D^2=\sum_m\sum_{n\neq m}\left[(m-n)\omega LI_mI_n\right]^2\quad(50)$$

$$Q^2+D^2=\sum_{k=1}^{\infty}\left(k\omega LI_k^2\right)^2+\sum_m\sum_{n\neq m}\left(m^2+n^2\right)(\omega L)^2I_m^2I_n^2\quad(51)$$

We can observe that (48) and (51) are identically.

In conclusion, the square of the power given by the effective value of the inertial force is equal to the sum of the reactive and distorting powers, according C. Budeanu and S. Fruze [2], [3].

We decompose (48) using physical criteria.

4.1. The decomposition under physics criteria

Based by superposition principle, the electrical current into a circuit with non-sinusoidal periodic voltage, is equal to the sum of the currents generated by each harmonic of voltage, if only itself operation in the circuit.

Based to rel. (27), the reactive power generated by the *k* harmonic is equal:

$$Q_k=k\omega LI_k^2$$

The reactive power generated by all harmonics is equal:

$$Q_{total}=\sum_{k=1}^{\infty}k\omega LI_k^2\quad(52)$$

Therefore, we obtain an identically expression that of C. Budeanu.

The distorting power proposes by C. Budeanu result from rel.(51).

The decomposing of the reactive power after S. Fruze, in two orthogonal components: reactive power and distorting power from C. Budeanu, has based on the superposition principle using in the electric circuit theory.

Reactive power after C. Budeanu represents the interaction between the current harmonics of the same order, $m=n$.

Distorting power is the interaction between the current harmonics of different order, $m\neq n$.

5. THE APPARENT POWER

In an electrical circuit two forces work the Coulomb force, that is displace with very small velocity and the inertial force of the "field mass" that is displace with speed of light.

Our scope is to determine a Coulomb equivalent force that is displacing with the same velocity with the inertial force of the "field mass". Adding these two forces, we obtain a resulting force that is displacing with speed of light. His effective value determines the apparent power.

The expression that gives the equivalent electric force must have the same form that the tension applies:

$$F_{echiv}(t)=k\sin\omega t\quad(53)$$

The constant k has determined from the following relation:

$$c_0^2F_{ef.echiv}^2=c_0^2\frac{1}{T}\int_0^Tk^2\sin^2\omega t=(UI\cos\varphi)^2$$

$$k=\frac{\sqrt{2}UI\cos\varphi}{c_0}\quad(54)$$

We substitute (54) to (53) and we obtain the equivalent electric force:

$$F_{echiv}(t)=\frac{\sqrt{2}UI\cos\varphi}{c_0}\sin\omega t\quad(55)$$

Using rel.(23), (27), (28) and rel.(55) we obtain the resultant force from the electric circuit:

$$F_{rez}(t)=\frac{\sqrt{2}UI\cos\varphi}{c_0}\sin\omega t+\frac{UI\sin\varphi}{c_0}\sin2\omega t\quad(56)$$

The two forces given in rel.(55) and rel.(23) are orthogonal under a period.
The effective value for the resultant force given to rel. (56) is equal:

$$F_{ef.rez} = \frac{1}{c_0} UI \qquad (57)$$

The apparent power is equal:

$$S = F_{ef.rez} c_0 = UI \qquad (58)$$

Under non-sinusoidal condition, rel. (56) becomes:

$$F_{rez}(t) = \sum_{k=0}^{\infty} \frac{\sqrt{2} U_k I_k \cos j_k \sin kwt}{c_0} - \sum_{k=0}^{\infty} \frac{U_k I_k \sin j_k \sin 2kwt}{c_0} \qquad (59)$$

The first term represent the equivalent Coulomb force $\overline{F}_{echiv}$ and the second represent the inertial force of the "field mass":

$$F_{echiv}(t) = \sum_{k=0}^{\infty} \frac{\sqrt{2} U_k I_k \cos j_k}{c_0} \sin kwt \qquad (60)$$

$$F_{in}(t) = \sum_{k=0}^{\infty} \frac{U_k I_k \sin j_k}{c_0} \sin 2kwt \qquad (61)$$

The two forces are orthogonally, that is:

$$\langle \overline{F}_{echiv}(t), \overline{F}_{in}(t) \rangle = 0$$

The effective value to the resultant force given (59) is equal:

$$\left\| \overline{F}_{ef.rez} \right\|^2 = \left\| \overline{F}_{echiv} \right\|^2 + \left\| \overline{F}_{in} \right\|^2 \qquad (62)$$

The effective value of the Coulomb force equivalent is equal:

$$\left\| \overline{F}_{echiv} \right\|^2 = \frac{1}{T} \int_0^T \left(\sum_{k=0\infty} \frac{\sqrt{2} U_k I_k \cos j_k}{c_0} \sin kwt \right)^2 dt =$$

$$= \frac{1}{c_0^2} \left(\sum_{k=0}^{\infty} U_k I_k \cos j_k \right)^2 = \frac{P^2}{c_0^2} \qquad (63)$$

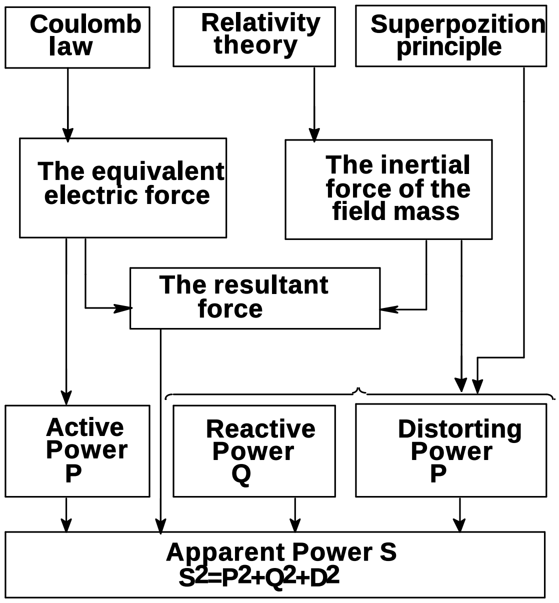
We substitute rel.(49) and rel. (63) into rel.(62), that becomes:

$$c_0^2 \left\| F_{rez} \right\|^2 = P^2 + Q^2 + S^2 = U_{ef}^2 I_{ef}^2$$

where:

$$U_{ef}^2 = \sum_{k=0}^{\infty} U_k^2 \quad \text{and} \quad I_{ef}^2 = \sum_{k=0}^{\infty} I_k^2$$

The all results may synthesize in a diagram, which presents the logical relationships between the fundamental elements of the theory presented and the consequences immediate.



6. CONCLUSIONS

(1)The physical essence of power components is determined as the main aim of the article. This problem we been solving using only the effective values for the Coulomb forces and inertial forces over a period.
(2)The equivalent effective Coulomb forces produce a mechanic work that is the electric active power.
(3)The effective inertial forces of the 'field mass" and superposition principle produce a mechanical work that is reactive and distorting powers.
(4)The effective resultant force that is obtained from equivalent Coulomb forces an inertial forces of the "field mass' produce a mechanical work that is apparent power.

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