ANALYSIS AND SIMULATION OF AN ELECTRIC DRIVING INVERTER USED IN ELECTRIC URBAN TRACTION

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***Abstract* – The asynchronous motors have some advantages by comparison with the d.c. motors. That is why a modernization of the electric urban vehicles was proposed, by replacement of the d.c. motors with inverter and asynchronous motor into electric driving system. The purpose of this paper is to analyse the electric quantities supplied by the invertor, especially the harmonics.**

Keywords: *asynchronous motor, power inverter, harmonic component.*

1. INTRODUCTION

The cage induction motors are used on a large scale in industry because they have a reduced cost price and they are robust by comparison with the direct current motors or the synchronous motors. These are the reasons that determined the use of the asynchronous motors into the electric driving systems; moreover, these motors can be synchronized and closed directly at the three-phase system without being necessary any intermediate equipment and they don't have any sensitive components (for example the commutator). These motors are safe in operation and their speed of rotation is practically independent of its load. Their main drawback is the speed control that is made very hard, with energy losses or using expensive auxiliary installations. But this problem was solved in the long run, when a considerable progress in the domain of the semiconductor convertors was registered.

The driving equipments of the asynchronous motors that are based on power electronics are some of the numerous possible sources of harmonics that have a negative influence on the electrical and electronic equipments. A parasitic signal transmitted by a static convertor to an asynchronous motor causes supplementary loss in the stator windings. At the same time, the harmonic magnetic fields penetrate the ferromagnetic cores of the stator and of the rotor, causing also an increase of the core loss unlike the sinusoidal steady state. Thus the superheating both of the windings and of the ferromagnetic cores can appear. Also, each harmonic of the current with each harmonic of the field produces various direct and inverse harmonic rotating fields. These harmonic fields produce, two by two, elliptical fields that develop parasitic torques, accompanied by pendulous effect and unpleasant mechanical vibrations [1].

Actually, the rotor superheating is the main problem associated with the supply voltage distortion. The increase of the motor operating temperature because of the voltage harmonic distortion will cause the reduction of its operating time.

D.P. Connors a.o. [2] emphasize the fact that the harmonic losses are also dependent on the motor characteristics, defining some loss factors that can be used to estimate the weight of the motor losses variation.

Considering all the advantages of the asynchronous motor, it was proposed the replacement of the d.c. motors that are placed on the electric urban vehicles from Romania with asynchronous motors supplied through inverters. But knowing that these inverters produce harmonics, it is necessary to analyse first their level.

2. THE SIMULATION OF THE INVERTER OPERATION

The purpose of the inverters is to transform an input direct voltage into a symmetrical output alternating voltage, having the required amplitude and frequency. If the input voltage remains constant, the inverter is a voltage inverter and if the input current remains constant, the inverter is a current one.

In the electric urban traction the supply voltage is constant; therefore a three-phase voltage inverter will be used. Its electrical diagram is shown in Fig.1 [3].

2.1. The PWM Control of the Inverter

The technological progress of the last years in the domain of the semiconductor elements with reduced switching time allowed the development of the Pulse Width Modulation techniques (PWM). These techniques are important for the current inverters and the voltage inverters that supply the a.c. motors because they permit the voltage control and the current control at the inverter output.

For the PWM control of the inverter, the control methods can be divided in the following categories:

- the modulation using variable control signals;

- the modulation using pre-established switching times;
- the modulation using local control.

The PWM control allows:

- the amplitude control of the output voltage fundamental;
- the improvement of the output quantities waveforms.

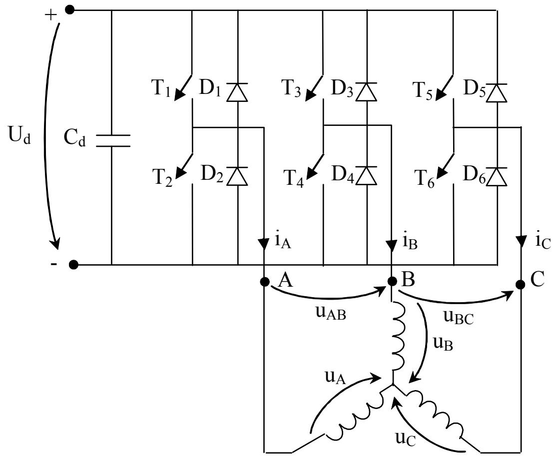

Fig. 1. The electrical diagram of the three-phase voltage inverter

For the harmonic analysis it was chosen to study a three-phase voltage inverter and the PWM control.

The control using variable control signals (the PWM classical control) consists in the determination of the semiconductor elements switching moments by comparing some reference voltages that are usually triangular, having the frequency f_r and the amplitude $U_{r\,max}$, with some control signals that are usually sine-wave signals, having the frequency f_c and the amplitude $U_{c\,max}$.

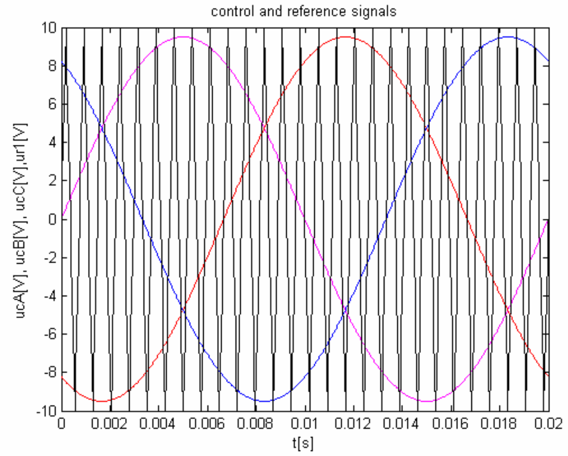

Fig. 2. The control signals and the reference signal (for $U_{r\,max}$=10 V and U_{cmax}=9.5V)

The sine-wave control signals (Fig.2), having the angular frequency ω equal to the output required angular frequency, have the expressions:

$$\begin{aligned} u_{cA}(t) &= U_{c\max}\sin\omega t \\ u_{cB}(t) &= U_{c\max}\sin(\omega t-\frac{2\pi}{3}) \\ u_{cC}(t) &= U_{c\max}\sin(\omega t-\frac{4\pi}{3}) \end{aligned} \qquad (1)$$

As reference signal a triangular voltage u_r is chosen, having the frequency $f_r = m_f \cdot f$, where m_f – the frequency modulation factor – must be an odd multiple of three:

$$m_f = 3\cdot(2k-1), \quad k = 1,2,3,... \qquad (2)$$

Thus, m_f being an odd number, the two half-waves of the output voltage are symmetrical and the even numbers of the harmonics are eliminated.

The triangular reference voltage will be bidirectional. Its time variation can be approximated by the function:

$$u_r(t) = \begin{cases} \frac{4U_{r\max}}{T_r}(t-\left[\frac{t}{T_r}\right]\cdot T_r) & if\ rem\left(\frac{t}{T_r}\right)\in\left[0,\frac{T_r}{4}\right] \\ U_{r\max}-\frac{4U_{r\max}}{T_r}(t-\left[\frac{t}{T_r}\right]\cdot T_r-\frac{T_r}{4}) & if\ rem\left(\frac{t}{T_r}\right)\in\left(\frac{T_r}{4},\frac{T_r}{2}\right] \\ -\frac{4U_{r\max}}{T_r}(t-\left[\frac{t}{T_r}\right]\cdot T_r-\frac{T_r}{2}) & if\ rem\left(\frac{t}{T_r}\right)\in\left(\frac{T_r}{2},\frac{3T_r}{4}\right] \\ -U_{r\max}+\frac{4U_{r\max}}{T_r}(t-\left[\frac{t}{T_r}\right]\cdot T_r-\frac{3T_r}{4}) & if\ rem\left(\frac{t}{T_r}\right)\in\left(\frac{3T_r}{4},T_r\right) \end{cases} \qquad (3)$$

where: $T_r = \frac{1}{f_r}$ is the period of the reference voltage, $\left[\frac{t}{T_r}\right]$ means the greatest integer less than or equal to the quotient $\frac{t}{T_r}$ and $rem\left(\frac{t}{T_r}\right)$ is the remainder of the division.

By the correlation between the reference signal and the control one, the output voltage waveform may have some symmetries. Thus if u_r has a maximum or a minimum in the middle of the half-waves of u_c, the output voltage half-waves are symmetrical in relation to their middle and the correlation is optimal. Besides, if m_f is integer the modulation is synchronous. The reference signal is shown in Fig.2.

For each element the switching moments are determined by the crossing points between the control voltage and the reference voltage. A switching element from a side of the bridge is closed in the time interval when the control voltage is bigger then the reference one; otherwise the corresponding element from the other side of the bridge is closed. In any moment only one of the elements corresponding to a phase is in conduction.

2.2. The Simulation of the Inverter Output Quantities

We created a software package in MATLAB to simulate the inverter control. It gives us the time variations of the output quantities and analyses the harmonics spectrum.

As input data we consider: the input voltage, U_d=825V, the amplitude of the reference voltage, U_{rmax}=10V, the amplitude of the control voltage, U_{cmax}=9.5V, and the frequency of the output voltage f=50Hz (the supply voltage frequency of the asynchronous motor).

In Fig. 3 the semiconductor elements control is represented for the case when k=7 (m_f =39). The phase voltages and the phase-to-phase voltages are represented in Fig. 4 and Fig. 5.

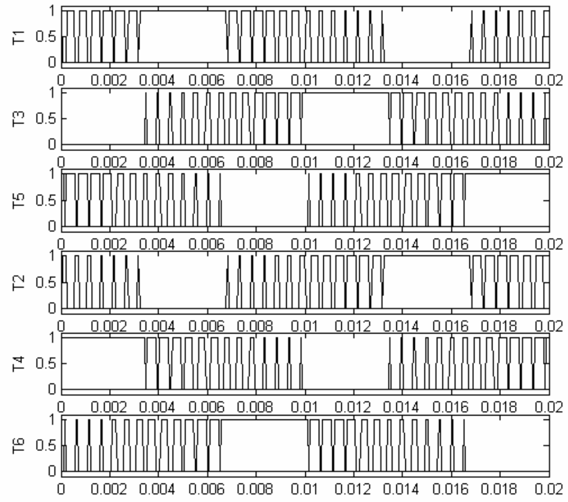

Fig.3. The semiconductor elements control (for m_f=39)

Through the inverter the traction motor is supplied (the asynchronous motor) which has the equivalent resistance of a phase R = 0.5 Ω and the equivalent reactance corresponding to the fundamental X = 1.15 Ω.

The phase current can be calculated as a function of the phase voltage and its harmonics. The phase voltage, using the Fourier series, is written as:

$$u(t) = U_0 + \sum_{\nu=1}^{\infty} U_\nu \sqrt{2} \sin(\nu\omega t + \varphi_\nu) \qquad (4)$$

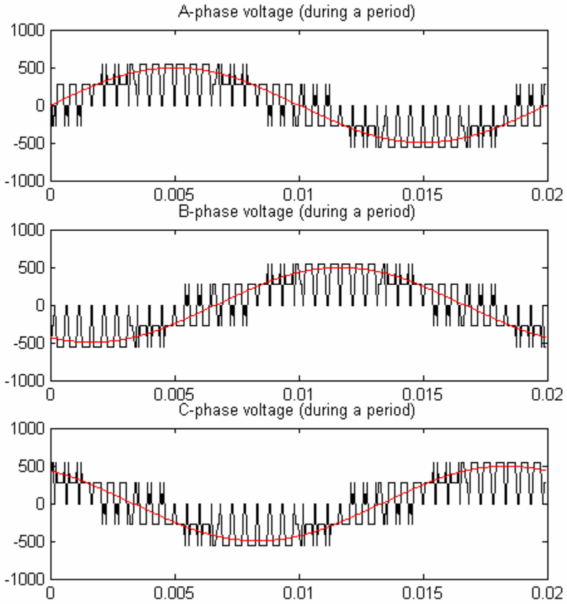

Fig.4. The phase voltages (for m_f=39)

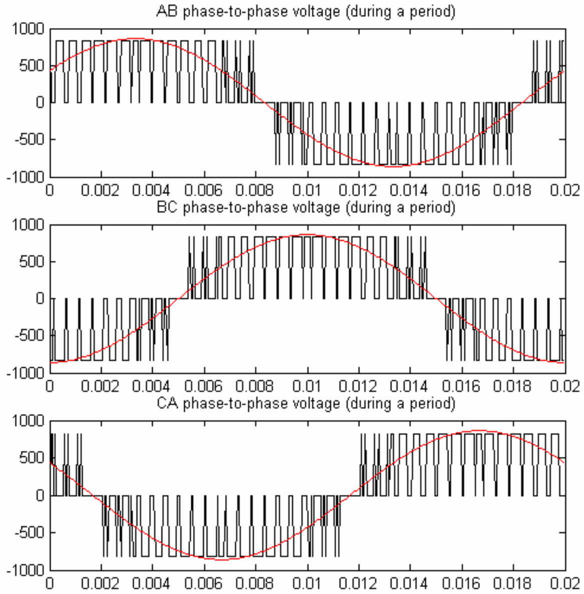

Fig.5. The line voltages to the inverter output (for m_f=39)

The RMS values of the phase voltages harmonics U_ν and the phase angles φ_ν can be calculated, until a certain harmonic number, by solving an equations system of (4) type, if the time variation of the voltage $u(t)$ is known. Then the phase current variation is determined with the formula:

$$i(t) = \frac{U_0}{R} + \sum_{\nu=1}^{\infty} \frac{U_\nu \sqrt{2}}{\sqrt{R^2 + (\nu X)^2}} \sin(\nu\omega t + \varphi_\nu - arctg\frac{\nu X}{R}) \qquad (5)$$

Doing so, in the same case conditions (m_f =39), the current of a phase can be determined. The current of the phase A was represented in Fig. 6; it has the RMS value of 229.57A and it was approximated afterwards by a sinusoid.

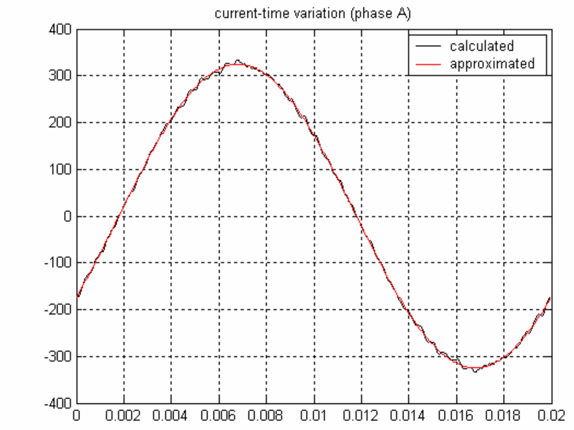

Fig.6 The current corresponding to the phase A (for m_f=39)

2.3. The Harmonic Analysis

In order to draw some conclusions concerning the behaviour of the inverter depending on the control mode, three cases were analysed.

(a) In the first case we choose k=7 (m_f =39). The analysis of the phase voltages harmonics in this case (Fig.7) shows us that the harmonics having an even harmonic number or a harmonic number that is a multiple of three are null; those that have the harmonic order 5, 7, 11, 17, 19 are insignificant (less than 3.5% of fundamental), more important being the harmonics of order 13 (8.26%), 37 (33.49%) and 41 (34.18%).

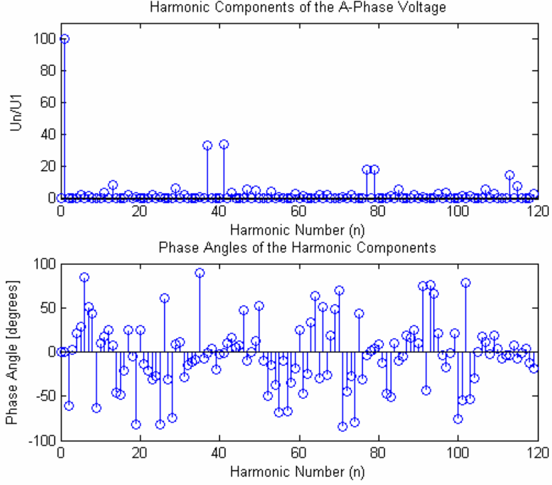

Fig. 7. The phase voltage harmonics (for m_f=39)

The analysis of the line voltages (Fig.8) shows that the same harmonics appear and they have the same weight as the phase voltages; but – as we expected – the line voltages have different phase angles.

In general the phase current harmonics (Fig.9) are not important (less than 0.5%), more significant being the same harmonics of order 37 (0.99%) and 41 (0.91%).

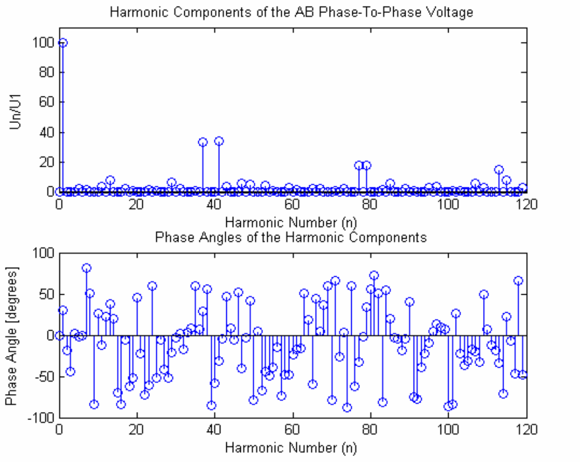

Fig.8. The line voltage harmonics (for m_f=39)

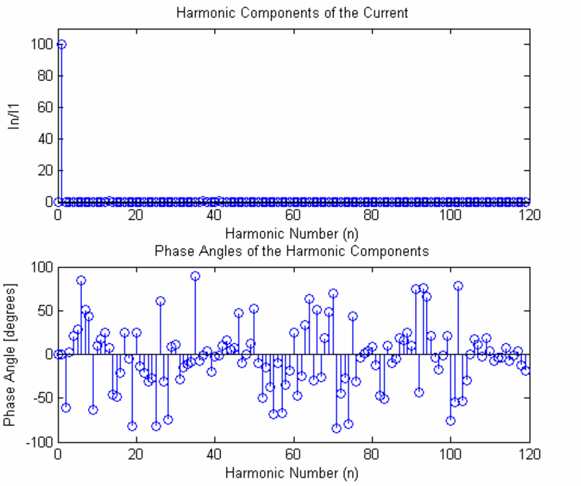

Fig.9. The harmonics of the current (for m_f=39)

Using the formula from literature for the total harmonic distorsion [1]:

$$THD = \frac{\sqrt{Y_2^2 + Y_3^2 + \ldots}}{Y_1} \tag{6}$$

where Y is either the voltage or the current.

The UTHD resulted 0.884 both for the phase and for the line voltages. The ITHD was 0.257 for the phase current. The RMS values of the phase and respectively of the line voltages are 351.2 V and 608.3V. The RMS value of the current is 229.57A. The RMS values of the voltages and of the current were calculated using the formula [4]:

$$Y = \sqrt{Y_0^2 + \sum_{n=1}^{\infty} Y_n^2} \tag{7}$$

(b) In the second case, when k=5 (m_f =27), the analysis of the phase voltage harmonics indicates a significant weight of the harmonics having the order 25 (30.16%), 29 (27.04%), 53 (24.18%) and 55 (28.47%), the percent of the harmonics 5, 7, 11, 13 being less than 5% (Fig.10).

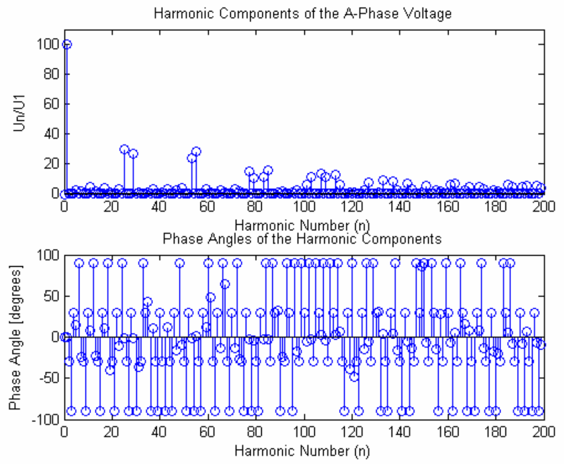

Fig. 10. The phase voltage harmonics (for m_f=27)

The analysis of the line voltages shows that the same important harmonics appear in a percent very close of those of the phase voltages: 25 (29.87%), 29 (27.21%), 53 (24.36%) and 55 (28.64%) (Fig.11). As for the current harmonics, they are insignificant (less then 0.6%).

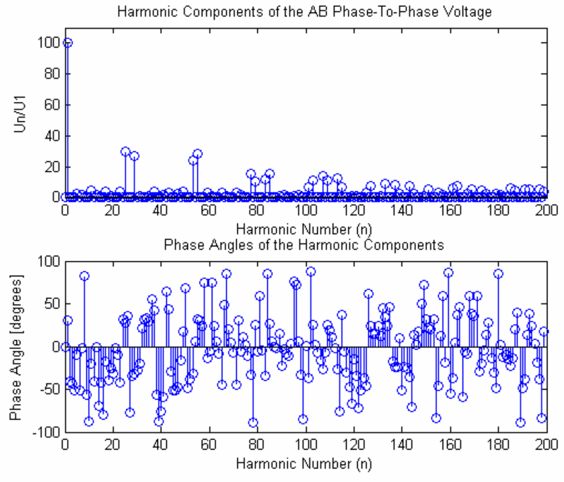

Fig.11. The line voltage harmonics (for m_f=27)

Moreover the RMS values were calculated, obtaining 365.58 V for the phase voltages, 623.29 V for the line voltages and 216.26 A for the current. The UTHD are 0.923 and 0.916 respectively and the ITHD is 0.299.

(c) In the third case, for k=3 (m_f =15), the phase voltage harmonics spectrum indicates an important weight of the harmonics of order 13 (32.63%), 17 (26.28%), 29 (23.94%), 31 (31.77%), the harmonics of order 5, 7, 11, being under 4% (Fig.12). As concerns the line voltages, the same significant harmonics appear, in a comparable percent with those of the phase voltages: 13 (32.07%), 17 (26.6%), 29 (24.27%), 31 (32.07%) (Fig.13). The current harmonics have an insignificant contribution (less than 0.8%), more important being the harmonics of order 13 (2.74%) and 17 (1.69%).

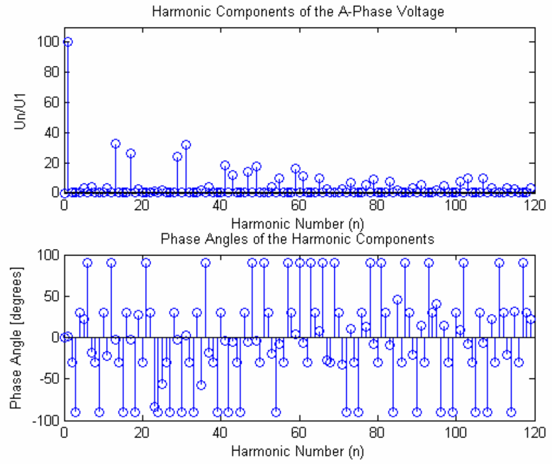

Fig.12. The phase voltage harmonics (for m_f=15)

The RMS values of the phase voltage and of the line voltage are 361.95 V and respectively 616.6 V. The current RMS value is 210.53 A. The UTHD of phase and respectively of line voltages are 0.911 and 0.902. The ITHD has a value of 0.357.

These three cases were analysed for the same maximum value of the control voltage (U_{cmax}=9.5V).

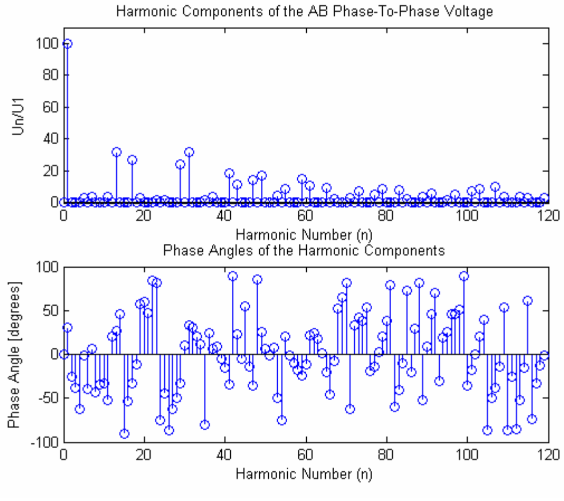

Fig.13. The line voltage harmonics (for m_f=15)

In terms of the first case, k=7 (m_f =39), the influence of the control voltage maximum value on the output voltages was analysed. The other parameters remained unchanged. The results of this study are presented, comparatively, in Table 1.

Because it was observed that the harmonics spectrum and consequently the RMS values for all the phase voltages, for all the line voltages and for the currents are the same, only one case of each it was presented (the voltage of the phase A, u_A, the line voltage u_{AB} and the current i_A).

	$U_{cmax} = 9.5$V	$U_{cmax} = 6$V	$U_{cmax} = 5$V
UTHD	0.884	0.920	0.929
ITHD	0.257	0.318	0.354
The RMS value of the phase voltage	351.2 V	287.27 V	272.09 V
The RMS value of the line voltage	608.3 V	493.39 V	466.23 V
The RMS value of the current	229.57 A	144.78 A	130.55 A
The significant harmonics of the voltage	13 (8.26%) 37 (33.49%) 41 (34.18%)	5 (5.13%), 7 (13.10%) 37 25.64%), 41 (22.7%)	5 (7.69%), 7 (19.62%) 13 (10.29%) 37 27.34%), 41 (24.22%)
The significant harmonics of the current	37 (0.99%), 41 (0.91%)	5 (1,12%), 7 (2.04%), 37 (0.76%), 41 (0.60%)	5 (1,67%), 7 (3.05%), 13 (0.86%), 37 (0.81%), 41 (0.64%)

Table 1: The influence of the control voltage amplitude on the output currents and voltages

As a result of the harmonic analysis it comes out that if the frequency modulation factor m_f is an odd number, the phase and the line voltages will include only the harmonics having an odd harmonic number. We made also the following observations:

- the phase voltages, the line voltages and the currents do not include the harmonics having the order 3 and multiple of three;
- the voltages harmonics of frequencies $m_f \cdot f$, $3m_f \cdot f$, $5m_f \cdot f$ ... are null, but some significant harmonics appear on the both sides of the harmonics having the frequencies multiple of $m_f \cdot f$ (for example in the first case it comes out that the harmonic of order 39 disappears, but the harmonics of order 37 and 41 have an important weight);
- analysing the values from Table 1, we concluded that the voltages RMS values and the current RMS value are proportional to the control voltage;
- also, from Table 1, it comes out that the THD and the number of significant harmonics both for the voltages and for the current are increasing when the amplitude of the control voltage is decreasing;
- the values of the harmonic distortion factors are depending also, in a certain measure, of the number of the harmonics we considered and of the calculus accuracy.

The observations made as a result of this analysis are in accordance with those existing in the technical literature [3], [5], [7], [8], [9].

3. CONCLUSIONS

It comes out that the PWM classical control, using variable control signals, it is efficient considering that the harmonics having an inferior order are generally insignificant and that the current harmonics are less than 1% almost in all the cases. So this control method can be used practically well enough. Besides the latest semiconductor elements have a reduced switching time; this means that practically the transistors conduction will be realized very similar to the simulations result.

The method drawbacks refer first to the UTHD values that are too big and then to the necessity to secure the reference signal and the control signals and to correlate them perfectly.

This control method is viable, but in order to optimize the electric urban vehicles drive we are going to analyse other possibilities to control the inverter, too. These other methods will have in view a lower level of the voltage harmonics and a low energy consumption.

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