# DYNAMIC ELECTROMECHANIC CHARACTERISTIC OF DC ELECTROMAGNETS WITH NON-NULL STARTING FLUX

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*Abstract* – **The electromagnets' behaviour for different values of the gap usually is estimated from the electromechanic characteristic into steady state conditions, although during movable armature displacement, the attraction dynamic force has less values. Considering the DC electromagnet with non-null starting flux, where the initial position of the movable armature is not agree with the mechanical equilibrium, like an electromechanic energy conversion system with two freedom degrees , it can define the non-linear differential equations system which describes its behaviour. Therefore, it can establish the time evolution of the dynamic attraction force, the normalized values and the attraction force into steady state conditions, also using normalized values, which allow a comparison between these two types of characteristics.**

***Keywords***: *electromagnets, steady-state dynamic electromechanic characteristics.*

## 1. INTRODUCTION

For the electromagnets which are operating with non-null starting flux (NNSF), the initial position of the movable armature, at maximum gap, $\delta_0$, is not agree with the mechanical equilibrium. There is an initial resistance force, $F_R(\delta_0)$. The acting of these electromagnets is possible only if the total magnetic flux because of the power supply for the coil electric circuit, exceeds the starting magnetic flux, $\Psi_p$:

$$\psi_p = \sqrt{2L' F_R(\delta_0)} \quad (1)$$

where L' means a constructive constant of the electromagnet which defines the variation of the coil inductance against gap values:

$$L(\delta) = \frac{L'}{\delta + a} \quad (2)$$

where a is an equivalent gap for the magnetic circuit of the electromagnet. The coil inductance varies from the value $L_0 = L(\delta_0)$ at the value $L_1 = L(0)$, $L_1 > L_0$, see relation (2).

The driving transient regime for a DC electromagnet with NNSF, Fig.1, has three steps:

- first step (starting step), from the coil supplied with a DC voltage till the moment when the total magnetic flux has the starting value, $\Psi_p$; the movable armature is still in the position of maximum gap, $\delta_0$, and the coil inductance is $L_0$;
- second step, when the movable armature goes from the initial position of maximum gap, $\delta_0$, to final position of the gap $\delta = 0$, where the coil inductance increases from $L_0$ to $L_1$; at the end of this step the mechanic action of the electromagnet is finished, the acting time, $t_a$, being the sum between starting time, $t_p$, and moving time, $t_m$:

$$t_a = t_p + t_m \quad (3)$$

- third step, when there is the steady-state electric conditions for the electromagnet coil with attracted movable armature and the coil inductance at the value L1.

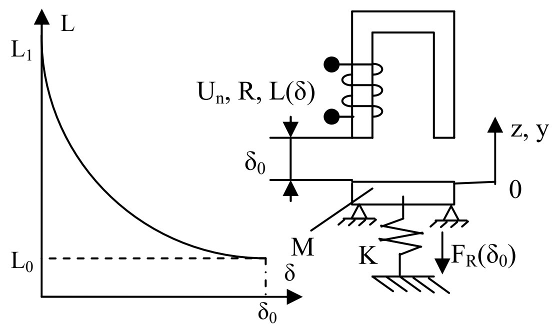


Figure 1: The transient regime for a DC electromagnet with NNSF.

Considering the electric current into steady-state conditions which flows through the coil turns of the electromagnet with NNSF, with the resistance R, supplied from a DC voltage, U,

$$I_n = \frac{U}{R} \quad (4),$$

the total magnetic flux, Ψ, into steady-state conditions, depends by the gap:

$$\psi(\delta) = L(\delta) I_n = \frac{L'}{\delta + a} I_n, \psi(\delta_0) = \psi_0 = L_0 I_n, \quad (5)$$
$$\psi(0) = \psi_1 = L_1 I_n$$

The normalized value of the total magnetic flux into steady-state conditions, $\Psi^*(\delta)$, is defined by the relation, see Fig.1,

$$\psi^*(\delta) = \frac{\psi(\delta)}{\psi_0} = \frac{\delta_0 + a}{\delta + a} = \frac{\delta_0 + a}{\delta_0 - y + a} \geq 1, \qquad (6)$$

hence, for the normalized electromechanic characteristic into steady state conditions, there is the relation:

$$F^*(\delta) = \psi^*(\delta)^2 = k\frac{1}{\left(1 - z^*\right)^2}, \; z^* = \frac{y}{\delta_0 + a}, \qquad (7)$$

where $z^*$ means the normalized displacement of the movable armature, see Fig.1, and y is the movable armature displacement.

Assuming defined the values of the total magnetic flux into dynamic conditions during movable armature displacement, $\Psi_d(\delta)$ or $\Psi_d(t)$ and considering its normalized values:

$$\psi_d^*(\delta) = \frac{\psi_d(\delta)}{\psi_0} \text{ or } \psi_d^*(t) = \frac{\psi_d(t)}{\psi_0}, \qquad (8)$$

the normalized electromechanic characteristic into dynamic conditions, $F_d^*(\delta)$, is estimated by the expression:

$$F_d^*(\delta) = \psi_d^*(\delta)^2 \qquad (9)$$

and allow the comparison with the normalized electromechanic characteristic into steady state conditions, $F^*(\delta)$, see relation (7).

## 2. THE DC ELECTROMAGNET OPERATING WITH NON-NULL STARTING FLUX INTO DYNAMIC CONDITIONS

According to the first step of the acting transient regime of DC electromagnet with non-null starting flux, the time evolution of the electric current which flows through coil turns is given by the relation:

$$i_1(t) = I_n(1 - e^{-\frac{t}{T_0}}), \quad T_0 = \frac{R}{L_0} \qquad (10)$$

hence, the starting time, $t_p$, can be defined by the equation:

$$\psi(t_p) = L_0 i_1(t_p) = L_0 I_n (1 - e^{-\frac{t_p}{T_0}}) = \psi_p \qquad (11)$$

Assuming the variable exchange:

$$\psi_d(t) = \psi_p + \varphi(t), \qquad (12)$$

and considering the electromagnet like an electromechanic energy conversion system with two freedom degrees, the displacement y, of the movable armature and the total magnetic flux $\Psi_d$, respectively, φ, the non-linear differential equations system which describes its behaviour during movable armature displacement, is written like, [1]:

$$\begin{cases} \ddot{y} + \Omega_0^2 y = \frac{1}{2}B_0\varphi^2 + B_0\psi_p\varphi \\ \dot{\varphi} + A_0\varphi = \varepsilon E_0 + G_0 y + C_0 y\varphi \\ \dot{y}(0) = y(0) = 0, \quad \dot{\varphi}(0) = 0 \end{cases} \qquad (13)$$

where $\Omega_0^2$, $A_0$, $B_0$, $C_0$, $E_0$, $G_0$ are constants respect to electromagnet and its coil construction:

$$\begin{aligned} &\Omega_0^2 = \frac{K}{M}, A_0 = \frac{R}{L_0}, B_0 = \frac{1}{ML'}, C_0 = \frac{A_0}{\delta_0 + a}, \\ &E_0 = \left.\frac{d\psi}{dt}\right|_{t=t_p}, G_0 = \frac{R\psi_p}{L'}, \varepsilon = \frac{U'}{U} \end{aligned} \qquad (14)$$

the parameters have the significance mentioned before.

With the notations:

$$\begin{aligned} &\psi_p = \eta\psi_0 = \eta\frac{U}{A_0}, z = \frac{y}{\delta_0 + a}, Q_0 = \frac{1}{2}\frac{B_0}{\delta_0 + a}\psi_0^2, \\ &\varphi^* = \frac{\varphi}{\psi_0}, E_0 = U(1-\eta), G_0 = A_0\eta \end{aligned} \qquad (15)$$

the non-linear differential equations system (14) can be written like:

$$\begin{cases} \ddot{z} + \Omega_0^2 z = Q_0\varphi^{*2} + 2\eta Q_0\varphi^* \\ \dot{\varphi}^* + A_0\varphi^* = A_0[\varepsilon(1-\eta) + z\varphi^* + \eta z] \\ \dot{z}(0) = z(0) = 0, \quad \dot{\varphi}^*(0) = 0 \end{cases} \qquad (16)$$

with the normalized values of dynamic total magnetic flux, $\Psi_d^*$:

$$\psi_d^* = \psi_p^* + \varphi^*, \quad \psi_p^* = \frac{\psi_p}{\psi_0} \qquad (17)$$

where z means the normalized movable armature displacement, see relation (15).

Using the finite differences method, the equations system (16) becomes:

$$\begin{cases} \frac{z_{n+2} - 2z_{n+1} + z_n}{\tau^2} + \Omega_0^2 z_n = Q_0\varphi_n^{*2} + 2\eta Q_0\varphi_n^* \\ \frac{\varphi_{n+1}^* - \varphi_n^*}{\tau} + A_0\varphi_n^* = A_0[(1-\eta)\varepsilon + z_n\varphi_n^* + \eta z_n] \end{cases} \qquad (18)$$

and for an imposed integration step, $\tau = 10^{-3}$[s], the solution is:

$$\begin{cases} z_{n+2} = \tau^2 (Q_0 \varphi_n^{*2} - +2\eta Q_0 \varphi_n^* - \Omega_0^2 z_n) + 2z_{n+1} - z_n \\ \varphi_{n+1}^* = A_0 \tau[\varepsilon(1-\eta) + z_n \varphi_n^* + \eta z_n - \varphi_n^*] \\ z_0 = z_1 = 0, \quad \varphi_0^* = 0, \quad n = 0, 1, 2... \end{cases} \tag{19}$$

The moving time of the electromagnet movable armature, $t_m$, defines taking into consideration the condition:

$$z(t_m) = \frac{\delta_0}{\delta_0 + a}, \tag{20}$$

therefore for different operating conditions, (ε, η), the acting time, tact, can be evaluated with the relation (3). Considering the normalized values of the dynamic magnetic flux, $\Psi_d^*$, see relation (17), it can evaluate the normalized dynamic attraction force, $F_d^*$, the relations (8) and (9) and the normalized dynamic electromechanic characteristic, $F_d^*(\delta)$, which compares with the normalized electromechanic characteristic into steady-state conditions, $F^*(\delta)$, relation (7). Also, it can obtain the normalized current evolution, $i^*$, which flows through the coil turns during movable armature displacement:

$$i^* = \psi_d^* (1 - z) \tag{21}$$

and the time evolution of the gap, δ.

## 3. ANALYSIS OF THE TRANSIENT REGIME BEHAVIOUR OF THE REAL DC ELECTROMAGNET WITH NNSF

It considers a real DC electromagnet with NNSF, where:

$$\Omega_0^2 = 3 \cdot 10^5 [rad/s], Q_0 = 3 \cdot 10^5 [1/s], A_0 = 160[1/s], \\ \delta_0 = 3[mm], \delta_0 + a = 4.25[mm] \tag{22}$$

and the voltage supply, U':

$$U' = \varepsilon U, \quad \varepsilon \geq 1 \tag{23}$$

U means the normal voltage supply.

| nτ [s] | z | φ | Ψ*d | Fd* | F* | δ [mm] | i* |
|---|---|---|---|---|---|---|---|
| 0,002 | 0 | 0,13248 | 0,83248 | 0,693023 | 1 | 3 | 0,83248 |
| 0,003 | 0,010598 | 0,183283 | 0,883283 | 0,780189 | 1,021539 | 2,954957 | 0,873922 |
| 0,004 | 0,041499 | 0,227456 | 0,927456 | 0,860174 | 1,088466 | 2,823629 | 0,888967 |
| 0,005 | 0,098239 | 0,269221 | 0,969221 | 0,939389 | 1,229751 | 2,582483 | 0,874005 |
| 0,006 | 0,179547 | 0,31338 | 1,01338 | 1,026939 | 1,485568 | 2,236925 | 0,831431 |
| 0,007 | 0,276322 | 0,364351 | 1,064351 | 1,132843 | 1,909454 | 1,825632 | 0,770248 |
| 0,008 | 0,372927 | 0,425112 | 1,125112 | 1,265876 | 2,5431 | 1,415062 | 0,705527 |
| 0,009 | 0,450919 | 0,496227 | 1,196227 | 1,430959 | 3,316863 | 1,083593 | 0,656825 |
| 0,01 | 0,494621 | 0,575135 | 1,275135 | 1,62597 | 3,915311 | 0,897859 | 0,644426 |
| 0,011 | 0,497144 | 0,656027 | 1,356027 | 1,838809 | 3,954688 | 0,887139 | 0,681887 |
| 0,012 | 0,464877 | 0,730925 | 1,430925 | 2,047547 | 3,492143 | 1,024275 | 0,765722 |
| 0,013 | 0,418347 | 0,79241 | 1,49241 | 2,227286 | 2,955782 | 1,222024 | 0,868064 |
| 0,014 | 0,38811 | 0,837519 | 1,537519 | 2,363966 | 2,670869 | 1,350534 | 0,940793 |
| 0,015 | 0,406096 | 0,870992 | 1,570992 | 2,468017 | 2,835098 | 1,27409 | 0,933018 |
| 0,016 | 0,495047 | 0,905709 | 1,605709 | 2,578303 | 3,921913 | 0,896051 | 0,810808 |
| 0,017 | 0,65997 | 0,95998 | 1,65998 | 2,755534 | 8,649007 | 0,195126 | 0,564443 |

Table 1: Parameter values @ coefficients ε = 1.5 and η = 0.7

| nτ [s] | z | φ | Ψ*d | Fd* | F* | δ [mm] | i* |
|---|---|---|---|---|---|---|---|
| 0,002 | 0 | 0,17664 | 0,87664 | 0,768498 | 1 | 3 | 0,87664 |
| 0,003 | 0,014362 | 0,244378 | 0,944378 | 0,891849 | 1,029354 | 2,938963 | 0,930815 |
| 0,004 | 0,056573 | 0,303447 | 1,003447 | 1,006906 | 1,123527 | 2,759565 | 0,946679 |
| 0,005 | 0,134661 | 0,359979 | 1,059979 | 1,123555 | 1,335449 | 2,427692 | 0,917241 |
| 0,006 | 0,247467 | 0,42122 | 1,12122 | 1,257134 | 1,765831 | 1,948264 | 0,843755 |
| 0,007 | 0,383231 | 0,494219 | 1,194219 | 1,42616 | 2,628785 | 1,371268 | 0,736557 |
| 0,008 | 0,521468 | 0,58437 | 1,28437 | 1,649606 | 4,366949 | 0,783761 | 0,614612 |
| 0,009 | 0,638352 | 0,694032 | 1,394032 | 1,943326 | 7,645874 | 0,287005 | 0,504149 |

Table 2: Parameter values @ coefficients ε = 2 and η = 0.7

| nτ [s] | z | φ | Ψ*d | Fd* | F* | δ [mm] | i* |
|---|---|---|---|---|---|---|---|
| 0.002 | 0 | 0.08832 | 0.88832 | 0.0078 | 1.004012 | 3 | 0.88832 |
| 0.003 | 0.00791 | 0.122189 | 0.922189 | 0.01493 | 1.006027 | 2.964403 | 0.914894 |
| 0.004 | 0.030732 | 0.151806 | 0.951806 | 0.023045 | 1.008048 | 2.861706 | 0.922555 |
| 0.005 | 0.072224 | 0.180197 | 0.980197 | 0.032471 | 1.010076 | 2.674993 | 0.909403 |
| 0.006 | 0.131089 | 0.210692 | 1.010692 | 0.044391 | 1.012109 | 2.410098 | 0.878201 |
| 0.007 | 0.200366 | 0.24618 | 1.04618 | 0.060605 | 1.014148 | 2.098351 | 0.836561 |
| 0.008 | 0.268467 | 0.28833 | 1.08833 | 0.083134 | 1.016194 | 1.791901 | 0.79615 |
| 0.009 | 0.321906 | 0.336946 | 1.136946 | 0.113533 | 1.018246 | 1.551423 | 0.770956 |
| 0.01 | 0.349252 | 0.389593 | 1.189593 | 0.151783 | 1.020304 | 1.428366 | 0.774126 |
| 0.011 | 0.345291 | 0.441733 | 1.241733 | 0.195128 | 1.022368 | 1.446191 | 0.812974 |
| 0.012 | 0.314067 | 0.487657 | 1.287657 | 0.23781 | 1.024439 | 1.586697 | 0.883246 |
| 0.013 | 0.269447 | 0.522338 | 1.322338 | 0.272837 | 1.026516 | 1.78749 | 0.966038 |
| 0.014 | 0.232412 | 0.543772 | 1.343772 | 0.295688 | 1.028599 | 1.954146 | 1.031463 |
| 0.015 | 0.225401 | 0.554738 | 1.354738 | 0.307734 | 1.030689 | 1.985695 | 1.049378 |
| 0.016 | 0.265239 | 0.562837 | 1.362837 | 0.316786 | 1.032785 | 1.806424 | 1.00136 |
| 0.017 | 0.356988 | 0.57862 | 1.37862 | 0.334801 | 1.034887 | 1.393554 | 0.886469 |
| 0.018 | 0.490898 | 0.612785 | 1.412785 | 0.375505 | 1.036996 | 0.79096 | 0.719252 |
| 0.019 | 0.64377 | 0.673704 | 1.473704 | 0.453878 | 1.039111 | 0.103034 | 0.524977 |

Table 3: Parameter values @ coefficients ε = 1.5 and η = 0.8

The computation results for the solutions (19), considering different starting conditions (different values for η), and also different values of the voltage supply, U', (different values for ε), are reported in the tables 1, 2 and 3.

The curves δ(t) and $F_d^*(t)$ shown in Fig.2 and Fig.3, for different values of the ε, outlines the influence of the voltage supply upon dynamic behaviour of the DC electromagnet with NNSF, the oscillatory effect of the movable armature displacement for ε = 1.5 is changed in the case of ε = 2.

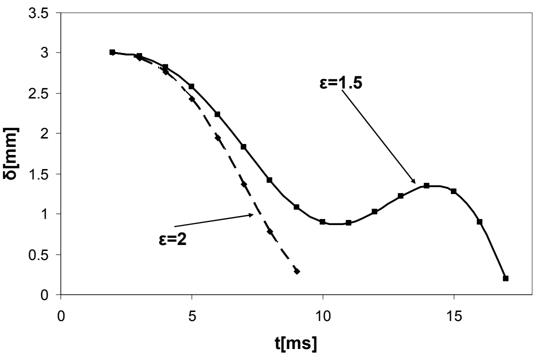


Figure 2: Gap evolution vs. time. Comparison between characteristics @ coeffcient ε = 1.5 (solid line) and coeffcient ε = 2 (dash line).

If it considers the dynamic safety acting coefficient, $K_d$, defined by the relation:

$$K_d(\delta) = \frac{F^*(\delta)}{F_d^*(\delta)} \tag{24}$$

and it represents the curves $K_d(\delta)$, Fig.4, it notes that its super-unit values are smaller than the case of the same electromagnet operating with null starting flux.

If it estimates the starting time $t_p$, moving time, $t_m$ and acting time, $t_a$, see the relations (3), (11) and (20), for different dynamic operating conditions of the DC electromagnets with NNSF, it obtains the values reported into tables 4 and 5, and the curves are shown in Fig.5 and Fig.6.

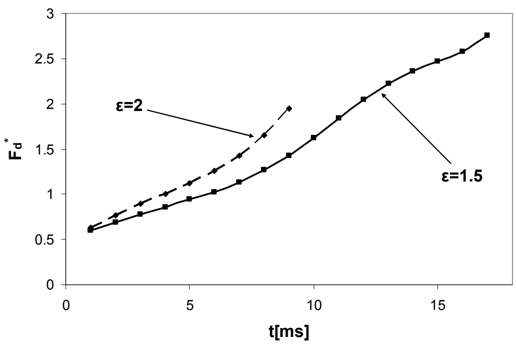


Figure 3: Normalized dynamic characteristic vs. time. Comparison between characteristics @ coeffcient ε = 1.5 (solid line) and coeffcient ε = 2 (dash line).

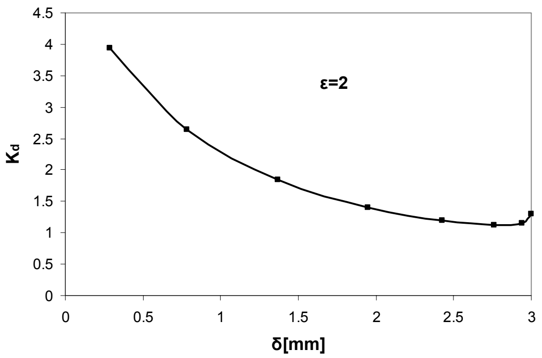


Figure 4: Dynamic safety acting coefficient vs. gap.

| $t_a$[ms] | $t_m$[ms] | $t_p$[ms] | ε |
|---|---|---|---|
| 29.52483 | 22 | 7.52483 | 1 |
| 26.422506 | 20.11 | 6.32250 | 1.1 |
| 24.75618 | 19.2845 | 5.47 | 1.2 |
| 23.38573 | 18.55329 | 4.83224 | 1.3 |
| 22.206483 | 17.87431 | 4.33217 | 1.4 |
| 21.133163 | 17.20435 | 3.928804 | 1.5 |
| 12.577782 | 9.88539 | 2.69239 | 2 |

Table 4: $t_a$, $t_m$, $t_p$= f(ε)

| $t_a$[ms] | $t_m$[ms] | $t_p$[ms] | η |
|---|---|---|---|
| 29.52483 | 22 | 7.52483 | 0.7 |
| 41.07832 | 30.958845 | 10.00589 | 0.8 |
| 58.044812 | 43.653655 | 14.39115 | 0.9 |

Table 5: $t_a$, $t_m$, $t_p$= f(η)

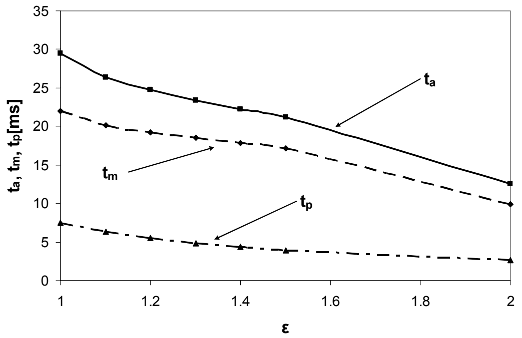


Figure 5: The acting time $t_p$ (solid line), moving time, $t_m$ (dash line) and starting time, $t_a$, (dash dot line) @ different ε coeffcients.

Certainly, there is a decreasing of the starting moving and acting times when the voltage supply is increasing (higher ε values) and also their decreasing at heavy starting conditions (higher η values).

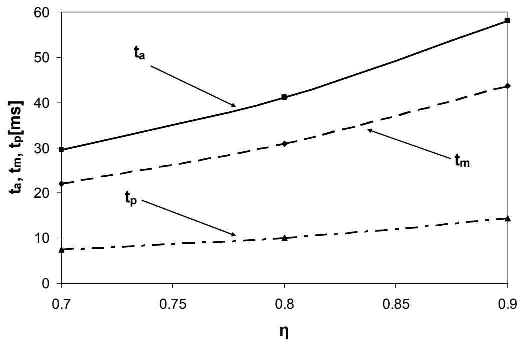


Figure 6: The acting time $t_p$ (solid line), moving time, $t_m$ (dash line) and starting time, $t_a$, (dash dot line) @ different η coefficients.

## 4. CONCLUSIONS

This paper proposes a mathematical model to study the transient regime of the DC electromagnets with NNSF, where they are considered like an electromechanic energy conversion system with two freedom degrees described by a non-linear differential equations system. It defines the time evolution during electromagnets' driving, of some parameters which characterize the assembly behaviour (the movable armature displacement and normalized dynamic magnetic flux) which allow to estimate the normalized dynamic electromechanic characteristic and outlines the voltage supply influence and starting conditions.

It compares the normalized electromechanic characteristic into steady state conditions with normalized dynamic electromechanic characteristic and it defines the dynamic safety acting coefficient, an important parameter when designing these kinds of assemblies for certain imposed dynamic performances.

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