APPROXIMATING EXPRESSIONS FOR THE INDUCTANCE AND THE FORCE OF A CYLINDRICAL COIL WITH STRAIGHT FERROMAGNETIC CORE

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Abstract **– A system consisting of a cylindrical coil with straight cylindrical ferromagnetic core is considered and the coil inductance and the electromagnetic force acting on the core are studied. First, the finite element method has been employed for the magnetic field analysis and determination of the coil inductance and the electromagnetic force for a series of geometric dimensions. Then the inductance and the force are transformed in relative units. Finally, approximating expressions are obtained for often encountered ratios of the geometric dimensions of the system. The approximating expressions are polynomials of different order. Approach for practical implementation of the obtained results is also given and can be employed for fast determination of the inductance and the force.**

Keywords**:** *cylindrical coil, ferromagnetic core, inductance, electromagnetic force, approximation.*

1. INTRODUCTION

One of the first electromagnetic systems that was widely used in the engineering practice is the system consisting of cylindrical coil with straight cylindrical ferromagnetic core. It has been used in a series of electrical devices such as instrument transformers, some kinds of chokes, sensors for linear displacement etc.

This system together with its main parameters – the coil inductance and the electromagnetic force on the core – has been subject of research during the past hundred years. Despite the simplicity of the construction, the exact distribution of the electromagnetic field of the system cannot be obtained analytically. Different attempts for such a solution have been made [1] [2] [3] but with significant simplifying assumptions, as well as for solution based on experimental results [4]. The accuracy of the results obtained in these works is low; the range of the significant geometric parameters is limited thus leading to limited applicability of these solutions.

Numerical methods for field problems are well developed in recent years and are dominating in the area of low-frequency electrical apparatus. Utilizing such numerical methods it is possible to obtain accurate enough solution. Such a solution is obtained in [5] with the help of the boundary integral equation method for two systems and extended in [6] for coil inductance approximation for symmetrical position of the core.

In the present paper, the coil inductance and the electromagnetic force of the system are obtained using the finite element method for a series of geometric dimensions and approximate expressions are proposed as a function of the displacement of the core and the geometric dimensions.

2. COIL INDUCTANCE AND ELECTROMAGNETIC FORCE

Fig. 1 shows the principal construction of the system. The following symbols are used:

- *d* is the average diameter of the coil;
- *l* is the coil length;
- *de* is the diameter of the ferromagnetic core;
- *le* is the length of the ferromagnetic core.

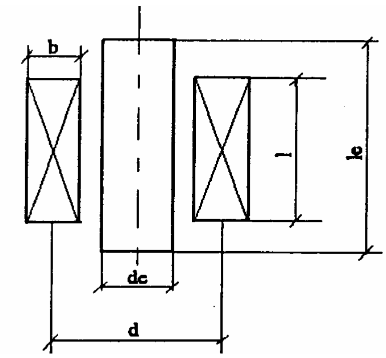

Figure 1: Principal geometry of the system.

The study is carried out assuming constant current density in the coil. The ferromagnetic core is considered unsaturated (i.e. $\mu \gg \mu_o$). The power losses in the system are not taken into account.

Axisymmetric magnetostatic field problem has been solved using the finite element method implemented in the freeware program FEMM [7]. With the help of the Lua Scripting® language the process of the magnetic field analysis and the calculation of the coil inductance L and the electromagnetic force F are

obtained automatically for different ratios of the basic geometric dimensions of the coil and the core. The script allows obtaining results in wide range of the geometric parameter ratios and different positions of the core.
The typical number of the finite elements is above 10 000. The study shows that further increasing the number of elements does not lead to significant change of the results – less than 0.5%.
For inductance determination the postprocessing features of the program are employed.
The electromagnetic force is computed with the help of Maxwell stress tensor. In the zone of the integration contour finer mesh is generated in order to keep the accuracy of the force computation.
The finite element analysis is carried out for different geometric ratios. The results for the inductance and the force are obtained as a function of the relative displacement of the core with respect to its symmetry position:

$$x^* = \frac{2x}{l_e + l}. \qquad (1)$$

In order to present the results in more general way, all the results for the coil inductance are presented later in the paper in relative form L*, which is the inductance of a single-turn winding of the same geometric dimensions divided by the core diameter (taken as a basic dimension). The real value of the inductance can be obtained from L* using the expression

$$L = \left(\frac{\mu_0}{4\pi}\right).d_e.N^2.L^*. \qquad (2)$$

The electromagnetic force is also presented in relative form F*, which is the force of a coil of MMF *NI*=1A, divided by the core diameter. The real value of the electromagnetic force can be obtained from

$$F = \left(\frac{\mu_0}{4\pi}\right).d_e.(NI)^2.F^* \qquad (3)$$

3. RESULTS

The inductance and the force are computed for electromagnetic systems of different ratios of the geometric dimensions often encountered in practice. These ratios are the following:

l/de = 1; 2; 3; 4; 5.
(le-l)/de = 0; 1; 2; 3; 4.
d/de = 1,2; 1,4; 1,6; 1,8; 2,0.
*x** = 0; 0,1; 0,2; 0,3; 0,4; 0,5; 0,6; 0,7; 0,8; 0,9; 1,0.

The results obtained from the finite element analysis for the relative inductance and relative force are shown in Figs 2-4 and Figs. 5-7, respectively. The families of curves are given as a function of the relative displacement when varying one parameter while keeping the rest at constant values.
The computed results are compared with experiment in [8] and good agreement is obtained.
It should be noted that the theoretically derived approximate formula (22) from [3] gives about 10% larger than Fig. 2 inductances for x*=0.

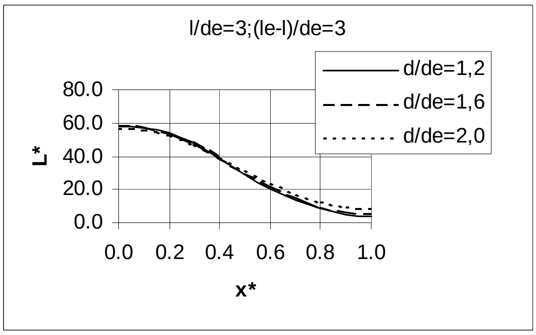

Figure 2: Inductance for different d/d_e.

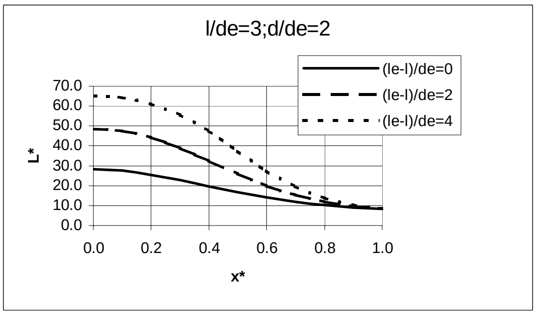

Figure 3: Inductance for different *(le-l)/de*.

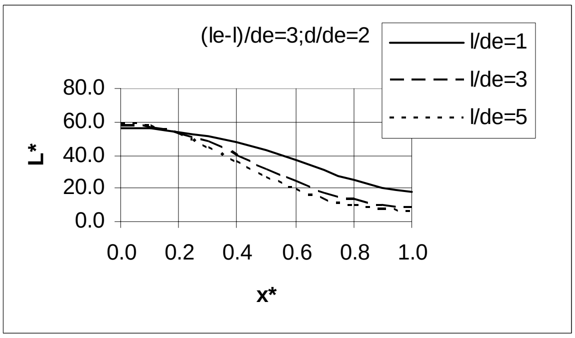

Figure 4: Inductance for different l/d_e.

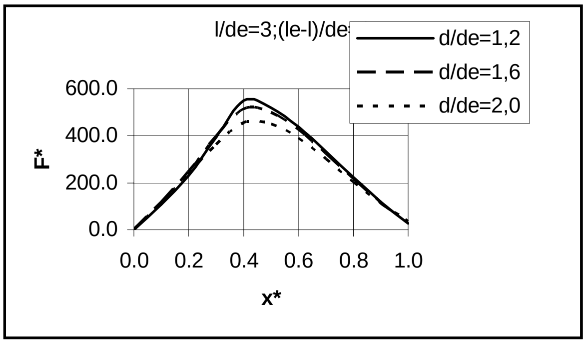

Figure 5: Electromagnetic force for different d/d_e.

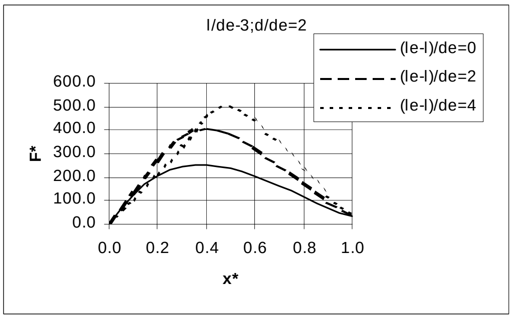

Figure 6: Electromagnetic force for different *(le-l)/de*.

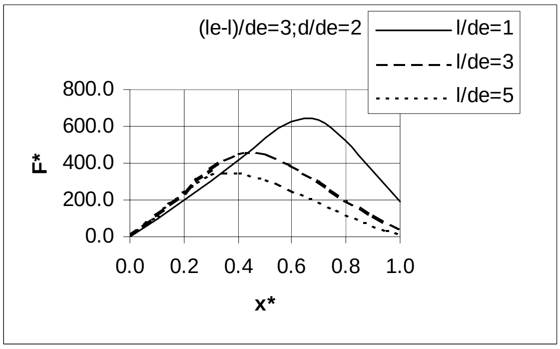

Figure 7: Electromagnetic force for different l/d_e.

3. INDUCTANCE AND FORCE APPROXIMATION

The obtained curves are approximated by polynomial expressions using the least square method.
The type of the function and the values of the coefficients are given in Table 1 for the inductance approximation and in Table 2 for force approximation.
All the approximating expressions in Table 1 and Table 2 ensure relative error with respect to the finite element results of less than 1%.

Figure	Parame-ter	Approximation	Values of the coefficients
2	d/de=1,2	$y=a+bx+cx^2+dx^3+ex^4+fx^5$	a=57.52 b=1.977 c=-50.10 d=-418.2 e=732.2 f=-319.5
	d/de=1,6	$y=a+bx+cx^2+dx^3+ex^4$	a=58.12 b=26.73 c=-280.0 d=264.8 e=-63.58
	d/de=2,0	$y=a+bx+cx^2+dx^3+ex^4+fx^5$	a=55.74 b=21.47 c=-176.5 d=-34.39 e=289.2 f=-147.0
3	(le-l)/de =0	$y=a+bx+cx^2+dx^3+ex^4$	a=28.2 b=0.0725 c=-89.75 d=104.4 e=-34.56
	(le-l)/de =2	$y=a+bx+cx^2+dx^3+ex^4$	a=48.39 b=13.30 c=-216.0 d=236.3 e=-73.44
	(le-l)/de =4	$y=a+bx+cx^{1,5}+dx^2+ex^3$	a=64.96 b=-1.407 c=81.96 d=-307.4 e=171.0
4	l/de=1	$y=a+bx+cx^2+dx^3+ex^4$	a=56.21 b=-3.5305 c=-13.22 d=-109.4 e=87.55
	l/de=2	$y=a+bx+cx^2+dx^3+ex^4+fx^5$	a=55.74 b=21.47 c=-176.5 d=-34.39 e=289.2 f=-147.0
	l/de=4	$y=a+bx+cx^2+dx^{2,5}+ex^3$	a=54.02 b=96.20 c=-1071.2 d=1474.9 e=-548.3

Table 1: Approximations for the coil inductance.

5. CONCLUSIONS

The obtained computed results and approximating expressions for the coil inductance and electromagnetic force of cylindrical coil with straight ferromagnetic core allow high accuracy in their determination. As the direct results from the finite element analysis are verified experimentally, the accuracy of the approximation is estimated with respect to the finite element results. In general this relative difference does not exceed 1%.
The range of the varying of the parameters of the system is taken in wide limits. The influence of each parameter when other are set to constant values can be seen form the graphs. The average coil diameter, for example, has very weak influence on the inductance and the force while varying the displacement.
The inductance and the force depend on the coil length in a way that longer coil length lowers the values of the inductance and the force, the maximum being shifted to the longer displacements.
The greater core length in turn leads to higher inductance and force, due to the increase of the flux in the core.

The obtained approximations are readily applicable for inductance and force calculation for a wide range of geometric parameters.

Fig.	Parameter	Approximation	Values of the coefficients
5	d/de =1,2	$y=a+bx+cx^2+dx^3+ex^4+fx^5$	a=7.379 b=-343.8 c=13928 d=-36507 e=33526 f=-10582
	d/de =1,6	$y=a+bx+cx^2+dx^{2,5}+ex^3$	a=2.042 b=-41.56 c=21585 d=-42371 e=20868
	d/de =2,0	$y=a+bx+cx^2+dx^3+ex^4+fx^5$	a=4.430 b=127.3 c=10603 d=-28870 e=26215 f=-8044.2
6	(le-l)/de =0	$y=a+bx+cx^2+dx^{2,5}+ex^{0,5}$	a=0.3555 b=1812 c=-4323 d=2621 e=-75.34
	(le-l)/de =2	$y=a+bx+cx^2+dx^3+ex^4+fx^5$	a=0.3170 b=980.2 c=4704 d=-17955 e=18744 f=-6435
	(le-l)/de =4	$y=a+bx+cx^2+dx^3+ex^4+fx^5+gx^6$	a=0.3597 b=1375 c=-7172 d=44216 e=-103496 f=97595 g=-32478
7	l/de=1	$y=a+bx+cx^2+dx^3+ex^4+fx^5$	a=-2.669 b=1573 c=-6788 d=25091 e=-33980 f=14299
	l/de=2	$y=a+bx+cx^2+dx^3+ex^4+fx^5$	a=3.5885 b=422.8 c=7401 d=-20739 e=18264 f=-5310
	l/de=4	$y=a+bx+cx^2+dx^3+ex^4+fx^5+gx^6$	a=0.9280 b=513.0 c=9951 d=-42757 e=67956 f=-49686 g=14038

Table 2: Approximations for the electromagnetic force.

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