ESTABLISHING THE EQUIVALENT OVER-TEMPERATURE IN POWER TRANSFORMERS

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Abstract – **The paper presents a method for establishing the equivalent over-temperature of the power transformers, based on the statistical thermal model, which assimilates the transformer with a homogeneous body having as heat source the copper losses. On the basis of the normal density distribution of the load factor, the statistical density distribution of the over-temperature is established. An application is proposed for a power transformer: the mean value of the over-temperature is obtained based on the load curves. This method is useful in prediction of life-time of electrical insulation of transformers, and in load management strategies.**

Keywords: *power transformer, equivalent over temperature, load curves, statistical model*.

1. INTRODUCTION

The power transformer plays a major role in the reliable supplying of the consumers. Its reliability depends on the endurance of the insulation system. His ageing rate is determined by the synergy action of the heat, electrical and environmental stresses [1] – [3].

The calculus of these stresses made the main object for many papers [4] – [7]. For the temperature distribution calculus, various models [8] – [12] have been used based on different hypotheses.

The purpose of the present paper is to determine the mean value of the winding insulation over-temperature for a power transformer, in correlation to the load factor.

The statistic correlation between the winding insulation temperature and load factor could be used to estimate the lifetime curves of the transformer insulation in various functioning conditions.

On the other hand, the energy optimization issue has taken in account, since early 1970s, considering energy, economy and environment as the three principal factors in sustainable development.

To find the economic models, to apply the management theories to reduce energy consumption and to obtain desirable load curves are the main tasks of the researchers [13] – [18]. One of the causes of the high loses in the electric network is losses in transformers and the improvement of the system load factor.

This paper presents a method to obtain the transformer over-temperature – cause of degradation of electrical insulation, and some considerations regarding correcting the consumption strategies.

2. WINDING INSULATION OVER TEMPERATURE AND LOAD FACTOR

Most of distribution transformers operate in variable load-conditions, characterized by load curves variable in time for the active power $P(t)$, the intensity of the electrical current $I(t)$, the active electrical energy $W(t)$, and respectively over-temperature curves of winding insulation $\theta(t)$, variable as well.

Over-temperature is a parameter which describes almost completely the thermal stress. As the actual system of monitoring the functioning parameters of the transformer consist in recording the load curves values and the temperature ones, the authors propose the establishment the correlation between over-temperature and load factor of transformer.

The thermal model assimilates the transformer with a homogeneous body having as heat sources the losses P, which is given by the differential equation [8]:

$$C\frac{\mathrm{d}\theta_i}{\mathrm{d}t}+\Lambda\,\theta_\mathrm{i}= P \tag{1}$$

In this relation the following notations are used:
C – caloric power of winding [J/K];
Λ – coefficient of heat transfer through conduction, convection and radiation [W/K];
θ_i – over-temperature of the winding, respectively, its insulation system.

Having in view the fact that the power transformer functions at various values of electrical current intensity, the load factor is defined as:

$$\beta = \frac{I}{I_n}, \tag{2}$$

where I is the current intensity in various conditions, and I_n is the current intensity in rate functioning conditions.

The power losses P could be expressed in function of the load factor as:

$$P \approx P_k = \frac{P_{kn}}{I_n^2} \cdot I^2 = \left(\frac{I}{I_n}\right)^2 P_{kn} = \beta^2 P_{kn} \tag{3}$$

where P_k represents the copper losses in certain operation conditions, and P_{kn} represent the copper losses in rate ones.

The relation (3) is valid only if the iron losses are neglected, as well as the winding resistance variation with temperature.

Having in view relation (3), relation (1) becomes:

$$\frac{d\theta}{dt} + \frac{\Lambda}{C}\theta = \frac{P_{kn}}{C}\beta^2 \tag{4}$$

The solution of differential equation (4) is:

$$\theta = \frac{P_{kn}}{\Lambda}\beta^2 + C_1 \exp\left(-\frac{\Lambda}{C}t\right) \tag{5}$$

where, the integration constant C_1 can be determined from the initial condition (for $t = 0$ it will result $\theta = \theta_0$):

$$C_1 = \theta_0 - \frac{P_{kn}}{\Lambda}\beta^2 \tag{6}$$

With the relations (6) and (5) the expression for winding over-temperature will become:

$$\theta = \frac{P_{kn}}{\Lambda}\beta^2 + \left(\theta_0 - \frac{P_{kn}}{\Lambda}\beta^2\right)\exp\left(-\frac{\Lambda}{C}t\right) \tag{7}$$

As, for short time, $C \ll \Lambda t$, we can consider the expression $\exp\left(-\frac{\Lambda}{C}t\right) \approx 0$ and in the relation (7) the second term can be neglected. In this case, the equation solution (1) becomes:

$$\theta = \frac{P_{kn}}{\Lambda}\beta^2 \tag{8}$$

The over-temperature depends on the square of the load factor of the current intensity and on the constructive parameters of the transformer. But, these quantities should be treated as random variable, and the statistical approach is necessary.

3. THERMAL STRESSES AS STATISTICAL PROCESS

An estimation of the thermal stress degree of the transformer can be done with the equivalent over-temperature. The equivalent over-temperature of the winding insulation, corresponding to a certain functioning period and load-conditions, represents that over-temperature value, considered as constant, which produces the same degradation effect on the insulation as the exploitation over-temperature. Its calculus can be done having in view the statistical approach.

The thermal stresses of the power transformer can be considered as a statistic process, where the current intensity $I(t_i)$, the load factor $\beta(t_i)$ and the over temperature θ_i, as random quantities, are characterized by specific statistical distributions. We can add, that in the case of rate functioning conditions ($\beta = 1$) the value of random over temperature θ_i coincides with the value of nominal winding over temperature θ_n [4].

Therefore, the relation (8) can be described as:

$$\theta_i = \theta_n \beta_i^2 . \tag{9}$$

In the following considerations, the random quantities θ_i and β_i will be noted θ and β.

In the hypothesis of a normal repartition for the current intensity [10] and, respectively, for the load factor, the expression of the repartition density of the load factor is:

$$f(\beta) = \frac{1}{\sqrt{2\pi}\,\sigma_\beta}\exp\left[-\frac{\left(\beta - \bar{\beta}\right)^2}{2\sigma_\beta^2}\right] \tag{10}$$

where $\sqrt{\sigma^2{}_\beta}$ represents the standard deviation of the load factor and $\bar{\beta}$ is the mean value of the load factor.

The over-temperature repartition density can be established based on the density repartition of the load factor.

Considering the equation (9), the load factor β can be expressed in function of the temperature θ:

$$\beta_1 = \frac{1}{\sqrt{\theta_n}}\sqrt{\theta} \quad ; \quad \beta_2 = -\frac{1}{\sqrt{\theta_n}}\sqrt{\theta} \tag{11}$$

and its variation with temperature will be:

$$\frac{d\beta_1}{d\theta} = \frac{1}{2\sqrt{\theta_n}\sqrt{\theta}} \, ; \, \frac{d\beta_2}{d\theta} = -\frac{1}{2\sqrt{\theta_n}\sqrt{\theta}} \tag{12}$$

Having in view that the repartition function F(x) of a random quantity is defined as the probability of an event β to take lower values than an x_0 one:

$$F(\beta_0) = P(\beta < \beta_0) \tag{13}$$

and that the repartition density function f(x) is defined as:

$$f(\beta_0) = \lim_{\Delta\beta_0 \to 0} \frac{P\{\beta_0 < \beta < \beta_0 + \Delta\beta_0\}}{\Delta\beta_0} = \lim_{\Delta\beta_0 \to 0} \frac{F(\beta_0 + \Delta\beta_0) - F(\beta_0)}{\Delta\beta_0} = \frac{dF}{d\beta_0} \tag{14}$$

it results the probability of an event ξ (in this case, load factor and over-temperature) to take lower values than an a one is:

$$\begin{aligned} P\{\theta \le a'\} &= P\left\{\beta = \beta(\theta) \le a' \,|\, \beta = \frac{\sqrt{\theta}}{\sqrt{\theta_n}}\right\} + \\ &+ P\left\{\beta = \beta(\theta) \le a' \,|\, \beta = -\frac{\sqrt{\theta}}{\sqrt{\theta_0}}\right\} = \\ &= \int_{-\infty}^{a'} f(\beta)d\beta\Big|_{\beta = \frac{\sqrt{\theta}}{\sqrt{\theta_n}}} + \int_{-\infty}^{a'} f(\beta)d\beta\Big|_{\beta = -\frac{\sqrt{\theta}}{\sqrt{\theta_n}}} = \\ &= \int_{-\infty}^{a'} f[\beta_1(\theta)]\left|\frac{d\beta_1}{d\theta}\right|d\theta + \int_{-\infty}^{a'} f[\beta_2(\theta)]\left|\frac{d\beta_2}{d\theta}\right|d\theta = \\ &= \int_{-\infty}^{a'} \left\{f(\beta_1)\left|\frac{d\beta_1}{d\theta}\right| + f(\beta_2)\left|\frac{d\beta_2}{d\theta}\right|\right\}d\theta \end{aligned} \tag{15}$$

On the other hand, the expression of the repartition function of the over-temperature is:

$$G(\theta) = P\{\theta \le a\} = \int_{-\infty}^{a} g(\theta)d\theta \tag{16}$$

Therefore, from relations (15) and (16) it results:

$$g(\theta) = f(\beta_1)\left|\frac{d\beta_1}{d\theta}\right| + f(\beta_2)\left|\frac{d\beta_2}{d\theta}\right| = f[\beta(\theta)]\left|\frac{d\beta}{d\theta}\right| \tag{17}$$

Having in view the relations (10), (11), (12) and (17), it will result the following expression for the repartition density of over-temperature:

$$g(\theta) = \frac{1}{\sqrt{2\pi}\sqrt{\theta_n}\,\sigma_\beta} \exp\left(-\frac{\bar{\beta}^2}{2\sigma_\beta^2}\right) \cdot \exp\left(-\frac{\theta}{2\theta_n\sigma_\beta^2}\right)\frac{1}{\sqrt{\theta}}\operatorname{ch}\left(\frac{\bar{\beta}}{\sqrt{\theta_n}\,\sigma_\beta^2}\sqrt{\theta}\right) \tag{18}$$

We have to emphasize that relation (18) should verify the normality condition:

$$\int_0^\infty g(\theta)d\theta = 1 \tag{19}$$

The relation (18) can be written as:

$$g(\theta) = A\exp\left(e^{-\alpha\theta}\right)\cdot\frac{1}{\sqrt{\theta}}\operatorname{ch}\left(\gamma\sqrt{\theta}\right) \tag{20}$$

where A, α and γ represent:

$$A = \frac{1}{\sqrt{2\pi}\sqrt{\theta_n}\,\sigma_\beta}\exp\left(-\frac{\bar{\beta}^2}{2\sigma_\beta^2}\right); \; \alpha = \frac{1}{2\theta_n\sigma_\beta^2}; \; \gamma = \frac{\bar{\beta}}{\sqrt{\theta_n}\,\sigma_\beta^2}. \tag{21}$$

The over-temperature mean value is calculated with the relation:

$$\bar{\theta} = \int_0^\infty \theta g(\theta)d\theta \tag{22}$$

By replacing in the relation (22) the expression of the distribution function of the over-temperature given by the relation (21), one can obtain the expression of over-temperature mean value:

$$\bar{\theta} = A\int_0^\infty \theta\,\frac{\exp(-\alpha\theta)}{\sqrt{\theta}}\operatorname{ch}\left(\gamma\sqrt{\theta}\right)d\theta \tag{23}$$

The relation (23) can be calculated [10], and put in the form:

$$\bar{\theta} = \theta_n\left(\sigma_\beta^2 + \bar{\beta}^2\right) \tag{24}$$

The mean statistic value of the over-temperature depends on the over-temperature in the rate conditions, on the mean value of the load factor $\overline{\beta}$ and on the standard deviation σ_β of the load factor.

An application could be done, for the standard deviation value $\sigma_\beta = 0{,}1\overline{\beta}$ and the rate temperature θ_n=70 ºC, that corresponds to the power transformers with forced cooling [2]: with the relation (24), the dependency of the mean value of over-temperature on the mean values of load factor could be obtained (see Fig. 1).

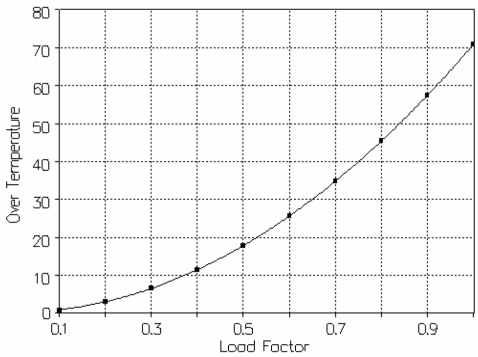

Fig. 1. Correlation between mean values of the over-temperature and load factor mean values (for $\sigma_\beta = 0{,}1\overline{\beta}$).

Some observations:

- In the case of lower values of the mean value of load factor, the relation (24) provides wrong values for the over-temperature because the iron losses become comparable to the copper losses and hypothesis is not respected;
- In the case when the transformer operates near the rate conditions, the relation (24) is validated. The curve from figure 1 is useful in determining the over-temperature according to statistical parameters of the load factor;
- The relationship (24) was also proposed by [9] but, the valuable conditions were not specified.
- In any case, to apply the relation (24) the hypothesis of a normal repartition for the load factor should be verified.

4. APLICATION – ESTABLISHING THE EQUIVALENT OVER-TEMPERATURE OF A TRANSFORMER USING THE LOAD CURVES FOR 60 DAYS

To establish the over-temperature in corelation with the load factor, the measured data of the load curves for a power transformer with aparent power S = 250 MVA and 400/110/20 kV voltages are used.

The load curves corespond to the daily active energy at the level of 110 kV. With the measurements for the active energy, made with a periodicity of 60 minutes, the values of the intensity of the electric current through the 110 V winding are obtained, with the relationship:

$$I_2 = \frac{\frac{W_i}{t}10^6}{\sqrt{3}U_2\cos\varphi}, \tag{25}$$

where W_i is active energy measured in MWh. It is considered the mean value of power factor $\cos\varphi$=0,92, as the transformer is equiped with the power factor control system.

The value of rate current intensity for the 110 kV winding, for t = 1 h, can be calculated:

$$I_{2n} = \frac{S}{\sqrt{3}U_2\cos\varphi} = \\ = \frac{250\cdot10^6}{\sqrt{3}110\cdot10^3\cdot0{,}92} = 1427{,}95 \text{ A}. \tag{26}$$

With the values of the current intensity, obtained with the relation (25) and the value of the rate curent intensity, the value of the load factor of the curent intensity β is obtained with the relation (2).

In this application, the periode of 60 days of load factor measurements is considered. After the identification and elimination of false data, the 2297 remained values of the load factor of the current intensity were clasified in 11 classes.

In table 1 the classes and number of values of β for each class are presented.

No. crt.	1	2	3
Class	(0,14-0,17]	(0,17-0,20]	(0,20-0,23]
β_i	14	57	123
No. crt.	4	5	6
Clasa	(0,23-0,26]	(0,26-0,29]	(0,29-0,32]
β_i	222	258	304
No. crt.	7	8	9
Class	(0,32-0,35]	(0,35-0,38]	(0,38-0,41]
β_i	208	189	29
No. crt.	10	11	
Class	(0,41-0,44]	(0,45-0,47]	
β_i	9	2	

Table 1. Classes and number of values of the load factor.

It should be mentioned that a complex analysis of the load curves ask an appropriate establishing of the observation period, and the form of the time-load variation, because:

- Daily load curve, with the observation period of 24 hours, usually has two load characteristics: one for summer day (normally, in the interval 18 – 25 June) and the one for a winter day (normally, 18 – 25 December),
- Annual load curve, with the observation period of 8760 hours (12 months or 365 days), usually has specific time-variation.

In figure 2, the dependence between the statistic frequency and load factor values is presented.

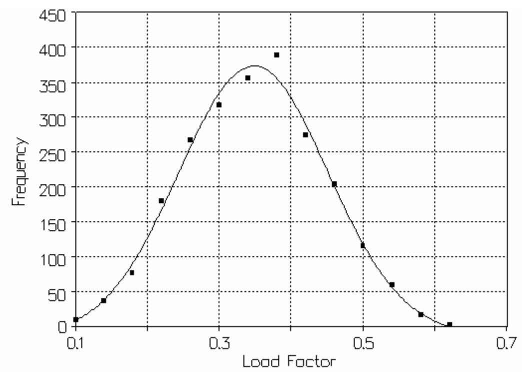

Fig. 2. Dependency of the statistic frequency and load factor established for a power transformer of 250 MVA and 400/110/20 kV.

Figure 2 suggests that load factor for this case respects a normal distribution of the probability. The concordace of empirical and normal repartition function was verified with Kolmogorov – Smirnov test.

Using the previous method, the over-temperature for observed transformer was established.

The mean value of load factor of the current intensity $\overline{\beta}$ and standard deviation σ_β of the load factor were calculated with the relations:

$$\overline{\beta} = \frac{1}{n}\sum_{i=1}^{n}\beta_i \, , \tag{27}$$

$$\sigma_\beta = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(\beta_i - \overline{\beta}\right)^2} \tag{28}$$

The results of data proceding conducts to the values:

$$\overline{\beta} = 0{,}29318 \quad ; \quad \sigma_\beta = 0{,}055834 \tag{29}$$

With the relation (24) and (29), the mean value of the over-temperature is obtained:

$$\overline{\theta} = 6{,}23514 \tag{30}$$

The obtained value is relative small because the transformer was no-loaded. Also, in the application it was not separated the workly days and holiday days of loading.

If consideration environment temperature θ_a=20ºC result for average transformer temperature value:

$$\overline{\theta}_{tr} = 6{,}23514 \tag{31}$$

5. CONCLUSIONS

For establishing the equivalent over-temperature of power transformers a statistical thermal model is proposed. The thermal model assimilates the transformer with a homogeneous body having as heat sources the copper losses. On the basis of the statistical density distribution of the load factor, the density distribution of the over-temperature is established.

In hypothesis of a normal distribution of the load factor, power transformer over-temperature depends on the sum of the square of mean value of the load factor of the current intensity and its standard deviation.

The normality of statistical distribution of a power transformer load factor is verified.

For a power transformer in operation, with data of the active energy, measured from 60 to 60 minutes for a period of 60 days, considering the normal distribution of the load factor, the mean value of over-temperature is obtained.

The statistic correlation between the insulation winding over-temperature and load factor could be used for estimating the lifetime curves of the transformer insulation in various functioning conditions.

This case study shows that the use of this transformer is non-economically: the load factor is small, and transformer should be replaced.

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