CONSTRICTION RESISTANCE OF BANDWIDTH REDUCTION

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Abstract **– In the paper, using conformal mappings, some new formulas are derived for supplementary steady-state resistance of flat homogeneous and isotropic conductors, caused by the current lines deformation near the bandwidth reduction and in the corners.**

Keywords*: *constriction resistance, corner resistance, conformal mapping, electric contact.

1. INTRODUCTION

It is well known that the current lines deformation in the vicinity of variable cross-section conductors lead to an increase of the steady state resistance, which can be taken into account by the so called "constriction resistance", used mainly in the electric contact theory, where simple formula is given for the case of axially-symmetric conductors [1]. In the paper, using conformal mappings similar formulas are obtained for the case of flat conductors. Moreover, a similar phenomenon occurs in the corner of flat conductors and for this case a "corner resistance" is introduced and evaluated.

2. CONSTRICTION RESISTANCE

In Fig. 1 the current lines in a half of symmetrical bandwidth reduction are shown and the values of corresponding constriction resistance are given for the case when the length of narrow and large parts tend to infinity.

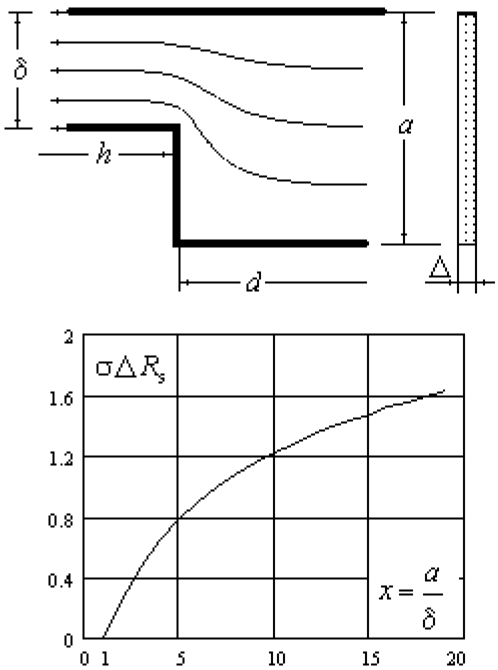

Fig. 1: Constriction resistance of bandwidth reduction

2.1. Constriction resistance for $h = d = \infty$

The analytical function

$$z(t) = \frac{a}{\pi}\left[\begin{array}{l}\mathrm{Arch}\,\dfrac{2t - x^2 - 1}{x^2 - 1} \\ -\dfrac{1}{x}\mathrm{Arch}\,\dfrac{(x^2+1)t - 2x^2}{(x^2-1)t}\end{array}\right]; \; x = \frac{a}{\delta} \qquad (1)$$

maps the upper half plane *t* (fig. 2 b) into the shaded domain from fig. 2 a [2].

The "constriction resistance" is defined as a difference between the real resistance of the constricted from *a* to *b* band and the sum of the resistances of the two segments, for the case $\overline{\mathrm{PN}} = d \to \infty, \overline{\mathrm{MQ}} = h \to \infty$:

$$R_s\left(\frac{a}{\delta}\right) = \frac{1}{\sigma\Delta}\lim_{\substack{r\to 0\\ R\to\infty}}\left[\frac{1}{\pi}\ln\frac{R}{r} - \frac{\overline{\mathrm{PN}}}{a} - \frac{\overline{\mathrm{MQ}}}{\delta}\right] \qquad (2)$$

where Δ is the (constant) thickness of the band and σ is the material conductivity.

After the computations, similar to given in annex of [3] and [4], the following formula was obtained for the constriction resistance of the Δ thickness band (Fig. 1):

$$R_s(x) = \frac{x}{\sigma\pi\Delta}\left[\left(1 + \frac{1}{x^2}\right)\ln\frac{x+1}{x-1} + \frac{2}{x}\ln\frac{x^2-1}{4x}\right]; \; [\Omega] \qquad (3)$$

The resistance of the shaded band from fig. 2 a) is equal to the sum of the resistances of two segments (with uniform distributed current) plus R_s.

The complex potential of the current density field *j* in *t*-plane will be:

$$W = -i\frac{I}{\pi d}\mathrm{Ln}t; \quad t = \rho e^{i\theta}$$
$$j = -i\overline{W'} = \frac{I}{\pi\rho\Delta}e^{i\theta} \tag{4}$$

where I [A] is the current and the complex conjugate of current density in z-plane is

$$\bar{j} = i\frac{\mathrm{d}W}{\mathrm{d}z} = i\frac{\mathrm{d}W}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}z} = \frac{I}{a\Delta}\sqrt{\frac{t-x^2}{t-1}} \tag{5}$$

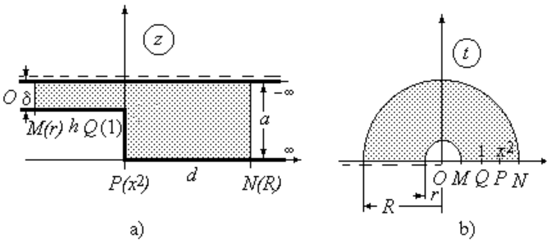

Fig. 2: Domains of the map for "constriction resistance"

2.2. Constriction resistance for $h = 0$, $d = \infty$

In case of frontal electric contact the length of the narrow part of the slab can be considered rather 0.

The analytical function:

$$t = \sin\left(\pi\frac{z}{a}\right) \tag{6}$$

maps the shaded domain from Fig. 3 into the upper half t-plane.
The complex potential in t-plane is:

$$\zeta = V + iU = U_0\,\mathrm{Arcos}\frac{t}{f} \tag{7}$$

The equipotential lines are confocal elliptical cylinders and the current lines are confocal hyperbolic cylinders with the focuses a_2 and a and with following half axes:

$$a_u = f\cosh\frac{U}{U_0}; \quad b_u = f\sinh\frac{U}{U_0}$$
$$a_v = f\cos\frac{V}{U_0}; \quad b_v = f\sin\frac{V}{U_0} \tag{8}$$

The total current injected in A_2A will be

$$I = \sigma V = \pi U_0\,\sigma \quad \text{[A/m]} \tag{9}$$

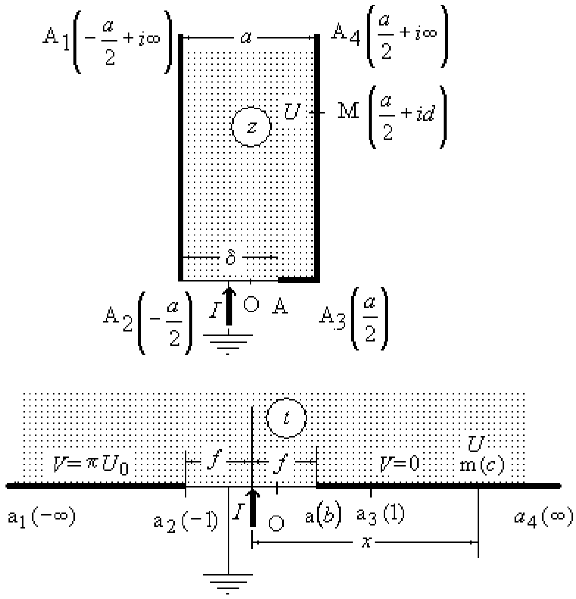

Fig. 3. Corresponding domains for constriction resistance at h=0 and d infinite

In Fig. 3 the following relations can be observed:

$$b = -\cos\left(\pi\frac{\delta}{a}\right); \quad c = \cosh\left(\pi\frac{d}{a}\right)$$
$$2f = b+1; \quad x = c + \frac{1-b}{2} \tag{10}$$

The resistance of the slab between grounded surface A_2A and the surface with the potential U, passing through the point M will be:

$$R = \frac{U}{I} = \frac{U_0\,\mathrm{Arch}\frac{x}{f}}{\pi\Delta\sigma U_0} = \frac{1}{\pi\Delta\sigma}\mathrm{Arch}\frac{x}{f} \tag{11}$$

The geometrical resistance of considered domain will be:

$$R_g(x) = \frac{1}{\pi}\mathrm{Arch}\frac{x}{f} = \frac{1}{\pi}\mathrm{Arch}\frac{1-b+2c}{1+b} \tag{12}$$

The corresponding constriction resistance will be:

$$R_s\big|_{h=0} = \lim_{d\to\infty}\left[\frac{1}{\pi}\mathrm{Arch}(\alpha c+\beta) - \frac{d}{a}\right];$$
$$\alpha = \frac{2}{1+b}; \quad \beta = \frac{1-b}{1+b} \tag{13}$$

Replacing the Arch with Ln

$$\mathrm{Arch}(z) = \mathrm{Ln}\left(z + \sqrt{z^2-1}\right) \tag{14}$$

$$R_s\big|_{h=0} = \lim_{c\to\infty}\left[\frac{1}{\pi}\ln\left(\alpha\,c+\beta+\sqrt{(\alpha\,c+\beta)^2-1}\right)-\frac{d}{a}\right] = \frac{\ln(\alpha)}{\pi}$$

Taking into account the eq. (10) and replacing the cosh with exponential function, it results for the constriction resistance of the rectangular cross-section slab for $h = 0$ and $d = \infty$:

$$R_s\big|_{h=0} = \frac{1}{\pi\,\Delta\,\sigma}\ln\frac{2}{1-\cos\pi\dfrac{\delta}{a}} \quad [\Omega] \tag{15}$$

Comparing the eq. (15) and (3), we can observe in Fig. 4 that the values of the constriction resistance are almost coincident in both cases, that means that, at least for large d, the length of narrow part h is not important and can be considered equal to zero.

$$\lim_{\delta/a\to 0}\left(R_s\big|_{h=\infty} - R_s\big|_{h=0}\right)\Delta\,\sigma = \frac{2}{\pi}\left[1+\ln\frac{\pi}{8}\right] \approx 0.042 \tag{16}$$

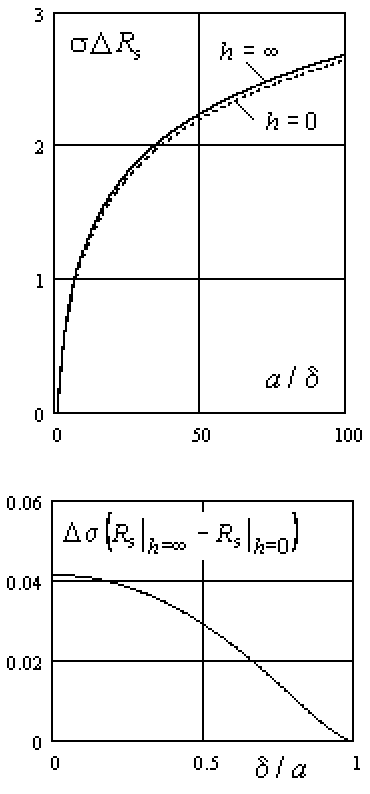

Fig. 4: Constriction resistance for h=∞ and h=0 at d=∞

2.3. Constriction resistance for *h* = 0 and *d* finite

In the case of finite length of wide part of the plate the following analytical function can be used:

$$w = \frac{d}{K(m)}F(z,m);\;\; z = \mathrm{sn}\left(\frac{K(m)}{d}w, m\right) \tag{17}$$

where F and K are the incomplete, respectively complete elliptic integrals of first order with module m and sn the elliptic sinus. This function maps the shaded domain from Fig. 5 into the upper half z-plane [2].

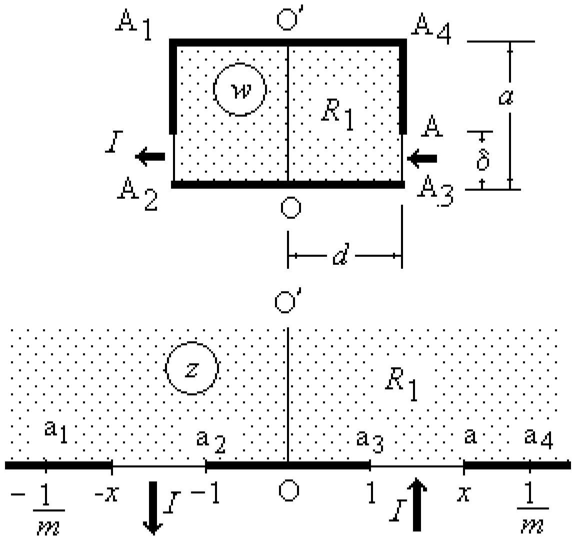

Fig. 5: Corresponding domains for constriction resistance at h=0 and d finite

When the point A coincides with A_4 the geometric resistance R_1 of the half rectangle, between A_3A_4 and symmetry axis OO'will be:

$$R_1(m) = \frac{d}{a} \tag{18}$$

If as argument of elliptic integral F is taken the angle φ ($z = \sin\varphi$) there is the functional equation [5]:

$$F\left(\frac{\pi}{2}+i\chi, m\right) = K(m) + i\,F\left(\arcsin\frac{\mathrm{th}\,\chi}{m'}, m'\right) \tag{19}$$

If as argument of F is considered x, this equation become:

$$F(x,m) = K(m) + i\,F\left(\frac{\sqrt{x^2-1}}{x\,m'}, m'\right) \tag{20}$$

In particular,

$$F\left(\frac{1}{m}, m\right) = K(m) + i\,K'(m) \tag{21}$$

For the point A_4 it results the following equation, which determine the module *m*:

$$d + ia = \frac{d}{K(m)} F\left(\frac{1}{m}, m\right) = d\left[1 + i\frac{K'(m)}{K(m)}\right]$$
$$\Rightarrow \quad R_1(m) = \frac{d}{a} = \frac{K(m)}{K'(m)} \tag{22}$$

where R_1 is also two times the geometric resistance between the strip a_3a_4 and the symmetry axis OO' (Fig. 5).
For small and large *m* can be used the following approximations:

$$R_1(m) \approx \begin{cases} \dfrac{\pi}{2\ln\dfrac{4}{m} - \dfrac{m^2}{4}}; & m < 0.6 \\ \dfrac{2}{\pi}\left[\ln\dfrac{4}{\sqrt{1-m^2}} - \dfrac{1-m^2}{5-m^2}\right]; & m > 0.85 \end{cases} \tag{23}$$

For $d < 0.6\,a$ the module *m* can be evaluated with the formula, derived from first eq. (23):

$$m \approx 4\exp\left(-\frac{\pi}{2}\frac{a}{d}\right) < 0.3 \tag{24}$$

The equation (22) and the approximations (23) are shown in Fig. 6.

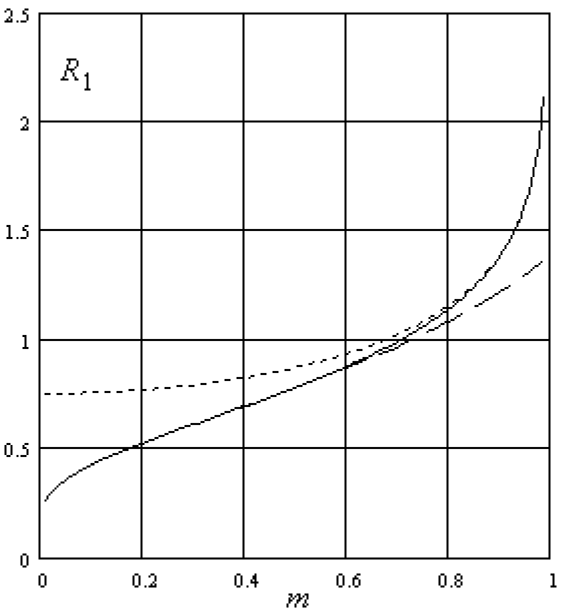

Fig. 6: The two approximations (23)

Applying the eq. (20) to the point a, we can write:

$$d + i\,\delta = d\left[1 + i\frac{F\left(\dfrac{\sqrt{x^2-1}}{x\,m'}, m'\right)}{K(m)}\right] \tag{25}$$

It results for δ:

$$\delta(x,m) = d\,\frac{F\left(\sqrt{\dfrac{1-1/x^2}{1-m^2}}, \sqrt{1-m^2}\right)}{K(m)} \tag{26}$$

The geometric resistance between the stripe A_3A and OO' will be $R_1(1/x)$ and the constriction resistance:

$$R_s\big|_{h=0}(x,m) = \frac{1}{\sigma\,\Delta}\left[R_1\left(\frac{1}{x}\right) - R_1(m)\right] \tag{27}$$

The corresponding dimension ratio results from (22) and (26):

$$\frac{\delta}{a} = \frac{F\left(\sqrt{\dfrac{1-1/x^2}{1-m^2}}, \sqrt{1-m^2}\right)}{K'(m)} \tag{28}$$

Taking values for $m \in (0, 1)$ and $x \in (1, 1/m)$, the constriction resistances can be calculated for $h = 0$ and finite values of the length *d* of the wide part of the slab, for varies dimensions ratios (Table 1).
In Fig. 6 are given the values of constriction resistance, calculated with (27) (solid lines) and with eq. (15) (dots). It can be observed that for $d > a / 2$ the simple formula (15), obtained for $d / a = \infty$ can be used for constriction resistance evaluation.
For accurate evaluation of the equations (27) and (28), the following approximations of elliptic integral *K* and *K*' were used:

$$K(m) = \begin{cases} \dfrac{\pi}{2}\left[1 + \dfrac{m^2}{4} + \dfrac{9m^4}{64}\right]; & m < 0.1 \\ \left(\dfrac{5-m^2}{4}\right)\ln\dfrac{4}{\sqrt{1-m^2}} - \dfrac{1-m^2}{4}; & m > 0.97 \end{cases}$$
$$K'(m) \approx \left(1 - \frac{m^2}{4}\right)\ln\frac{4}{m} - \frac{m^2}{4}; \quad m < 10^{-3} \tag{29}$$

For $m > 1\text{-}10^{-7}$ the elliptic integral *F* can be approximated as:

$$F(x,m) \approx \mathrm{arth}(x), \quad m > 1 - 10^{-7} \qquad (30)$$

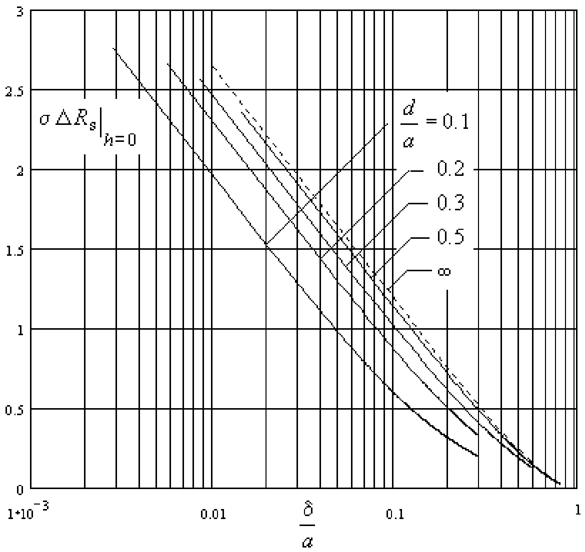

Fig. 7: Constriction resistance for h = 0, *d/a* - parameter (27)

d/a	0.01	0.02	0.03	0.04	0.05
m	$2.4 \cdot 10^{-68}$	$3.11 \cdot 10^{-34}$	$7.28 \cdot 10^{-23}$	$3.5 \cdot 10^{-17}$	$9.08 \cdot 10^{-14}$
d/a	0.1	0.2	0.3	0.4	0.5
m	$6.07 \cdot 10^{-7}$	$1.55 \cdot 10^{-3}$	$2.14 \cdot 10^{-2}$	$7.85 \cdot 10^{-2}$	0.172

Table 1: Corresponding values of R_1 and m (22).

3. CONCLUSIONS

For *d/a* >0.5, the length *h* of narrow part of the band can be considered zero and the new formula (15) can be used for constriction resistance evaluation of bandwidth reduction. For smaller *d/a*, the new algorithm given by eq. (22), (27), (28) and Fig. 7 have to be used.

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