DYNAMIC ELECTROMECHANIC CHARACTERISTIC OF DC ELECTROMAGNETS WITH NULL STARTING FLUX

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Abstract – **The operating characterization of the electromagnets' behavior usually takes into account their electromechanic characteristic into steady state conditions which is placed above the characteristic of resistance forces. The paper proposes an evaluation of steady state and dynamic electromechanic characteristics, during the moving of the DC electromagnet movable armature using an equivalent electromechanic energy conversion system with two freedom degrees. Finally, it draws the characteristics making and evaluation about their ratio and it gets some useful conclusions about dynamic behavior of these electromagnets.**

Keywords: *electromagnets, steady-state dynamic electromechanic characteristics.*

1. INTRODUCTION

To act the DC electromagnets means the values of the active forces higher than the resistance forces, hence the steady state electromechanic characteristic, F(δ), placed above the resistance forces characteristic, FR(δ), as shown in Fig.1.

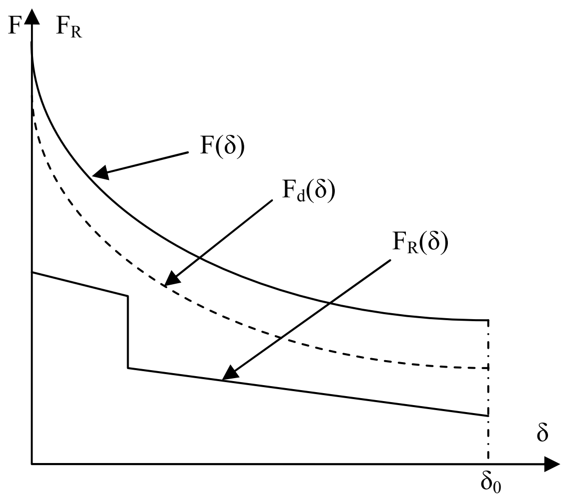


Figure 1: Comparison among characteristics.

The certainly action condition of these electromagnets is written usually at the level of the maximum gap, δ_0:

$$F(\delta_0) = k_{sig}\, F_R(\delta) \qquad (1)$$

and underlines the presence of the acting super-unit safety coefficient, k_{sig}.

Taking into account the fast increasing aspect of the steady state electromechanic characteristic, F(δ), the equation (1), usually implies the next condition:

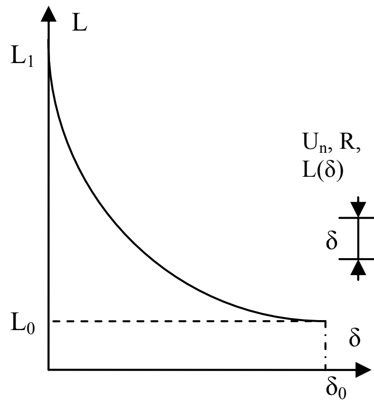
$$F(0) > F_R(0) \qquad (2)$$

The steady state electromechanic characteristic can be estimated by addition with neglecting the leakage magnetic fluxes, considering the variation of the inductance of the electromagnet type U-I vs. gap values, [1]:

$$L(\delta) = \frac{k^*}{\delta + a} = \frac{k^*}{\delta_0 - z + a}, \quad \delta = \delta_0 - z, \qquad L(\delta_0) = L_0, \quad L(0) = L_1 > L_0 \qquad (3)$$

where k and a, are constants respect to electromagnet construction, and z is the movable armature displacement respect to initial position (maximum gap, δ_0), Fig.2 and Fig.3, and it gets:

$$I_n = \frac{U_n}{R}, \quad \psi(\delta) = L(\delta) I_n, \quad F(\delta) = k_0 \psi^2(\delta), \qquad \psi(0) = L_0 I_n, \quad F(0) = k_0 \psi_0^2 \qquad (4)$$



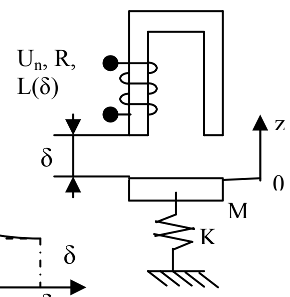


Figure 2: Inductance variation.

Figure 3: Electromagnet parameters.

Further on, it prefers to consider the normalized values of the steady state attraction force which acts upon movable armature of the electromagnet, $F^*(\delta)$:

$$F^*(\delta) = \frac{F(\delta)}{F(0)} = \frac{L^2(\delta)}{L_0^2} = \left(\frac{\delta_0 + a}{\delta + a}\right)^2 = \\ = \left(\frac{\delta_0 + a}{\delta_0 + a - z}\right)^2 = \frac{1}{\left(1 - z^*\right)^2}, \quad (5)$$

where z^* means the normalized displacement of the movable armature:

$$z^* = \frac{z}{\delta_0 + a}, \quad z^*_{\max} = \frac{\delta_0}{\delta_0 + a}, \quad (6)$$

with a certainly sub-unit maximum value.

2. THE DC ELECTROMAGNET OPERATING WITH NULL STARTING FLUX INTO DYNAMIC CONDITIONS

To define the DC electromagnet operating with null starting flux (NSF) into dynamic conditions, when at initial position, δ_0 gap, and its movable armature is in mechanical equilibrium, the assembly is considered to be equivalent like an electromechanical system of energy conversion with two freedom degrees. The displacement of the movable armature, z, and the total dynamic flux, Ψ_d, can be described by a non-linear differential equations system, [1]:

$$\begin{cases} \ddot{z} + \Omega_0^2 z = \frac{1}{2} B_0 \psi_d^2 \\ \dot{\psi}_d + A_0 \psi_d = U_n + C_0 z \psi_d) \end{cases} \quad (7)$$

with:

$$\dot{z}(0) = z(0) = 0, \quad \psi_d(0) = 0 \quad (8)$$

where Ω_0^2, A_0, B_0, C_0 are constants respect to electromagnet and its coil construction, Fig. 3:

$$\Omega_0^2 = \frac{K}{M}, B_0 = \frac{1}{Mk^*}, A_0 = \frac{R}{L_0}, C_0 = \frac{A_0}{\delta_0 + a} \quad (9)$$

where K means the elastic constant of the spring from on the movable armature of the electromagnet with the mass M, k^* is a constant (see the equation 3), U_n, R, L_0, δ_0 with the significance mentioned before.
Considering the normalized values of the total dynamic magnetic flux, ψ_d^* and the displacement z^*,

$$\psi_d^* = \frac{\psi_d}{\psi_0}, \quad z^* = \frac{z}{\delta_0 + a}, \quad (10)$$

The equations system (7), becomes:

$$\begin{cases} \ddot{z}^* + \Omega_0^2 z^* = Q_0 \psi_d^{*2} \\ \dot{\psi}_d^* + A_0 \psi_d^* = A_0(\varepsilon + z^* \psi_d^*) \end{cases} \quad (11)$$

where:

$$\dot{z}^*(0) = z^*(0) = 0, \quad \psi_d^*(0) = 0, \\ \varepsilon = \frac{U}{U_n} \geq 1, \quad Q_0 = \frac{1}{2} B_0 \frac{\psi_0^2}{\delta_0 + a} \quad (12)$$

Using the finite difference method, the differential equations system (11), can be written as follow:

$$\begin{cases} \frac{z^*_{n+2} - 2z^*_{n+1} + z^*_n}{\tau^2} + \Omega_0^2 z^*_n = Q_0 \psi^*_{d,n}, z^*_0 = z^*_1 = 0 \\ \frac{\psi^*_{d,n+1} - \psi^*_{d,n}}{\tau} + A_0 \psi^*_{d,n} = A_0(\varepsilon + z^*_n \psi^*_{d,n}), \psi^*_{d,0} = 0 \end{cases} \quad (13)$$

and for an imposed integration step, τ, the solution is:

$$\begin{cases} z^*_{n+2} = \tau^2 (Q_0 \psi^{*\,2}_{d,n} - \Omega_0^2 z^*_n) + 2z^*_{n+1} - z^*_n, z^*_0 = z^*_1 = 0 \\ \psi^*_{d,n+1} = A_0 \tau(\varepsilon - \psi^*_{d,n} + z^*_n \psi^*_{d,n}) + \psi^*_{d,n} \\ \psi^*_{d,0} = 0, \quad n = 0, 1, 2... \end{cases} \quad (14)$$

which allow to evaluate the moving time of the movable armature, t_m, through the condition:

$$z^*_m = z^*_{\max} = \frac{\delta_0}{\delta_0 + a}, \quad t_m = n\tau \quad (15)$$

For the solutions $z^*_m(n\tau)$ and $\psi^*_{d,n}(n\tau)$, for n ≤ m, and for an imposed value τ, it can define in any time of the movable armature displacement, the gap values, δ:

$$\delta_n(n\tau) = (\delta_0 + a) z^*_n(n\tau), \quad 0 \leq n \leq m \quad (16)$$

and also for the dynamic attraction force, $F_{d,n}(n\tau)$, considering the values of the dynamic magnetic flux $\psi^*_{d,n}(n\tau)$:

$$\psi_{dn}(n\tau) = \psi_0 \psi^*_{dn}(n\tau), \quad F_{dn}(n\tau) = k_0 \psi_{dn}(n\tau) \quad (17)$$

Taking into account the relations (43), (16) and (17), it gets the normalized dynamic electromechanic characteristic, $F_d^*(\delta)$:

$$F_d^*(\delta) = \frac{F_d(\delta)}{F(0)} = \psi_d^{*2}(\delta), \quad (18)$$

and making a comparison with the normalized steady state electromechanic characteristic, $F^*(\delta)$, it can define a safety dynamic coefficient of electromagnet acting, K_d:

$$K_d(\delta) = \frac{F_d^*(\delta)}{F^*(\delta)} = \frac{F_d(\delta)}{F(\delta)} \quad (19)$$

Useful information about DC electromagnets' operating behavior into dynamic conditions with NSF, it can obtain from time evolution of normalized current, i^*:

$$i^* = \frac{i}{I_n}, \quad I_n = \frac{U}{R}, \quad i^* = \psi_d^* (1 - z^*)\,, \qquad (20)$$

or making an evaluation about the influence of the changing values of the supply voltage upon acting time, through changing of the ε coefficient from the relations (14).

nτ [s]	z*	Ψ	Ψ_d	F*	F_d*	δ [mm]	i*
0.002	0	1	0.2944	1	0.086671	3	0.2944
0.003	0.00256	1.002567	0.407296	1.00514	0.16589	2.98912	0.406253
0.004	0.013787	1.01398	0.502295	1.028155	0.252301	2.941405	0.49537
0.005	0.040835	1.042574	0.583036	1.08696	0.339931	2.82645	0.559228
0.006	0.088977	1.097668	0.65356	1.204874	0.42714	2.621846	0.595408
0.007	0.158862	1.188866	0.718295	1.413401	0.515947	2.324837	0.604185
0.008	0.244767	1.324095	0.781625	1.753229	0.610938	1.959739	0.590309
0.009	0.334609	1.502876	0.847176	2.258635	0.717706	1.577912	0.563703
0.01	0.412114	1.70101	0.916983	2.893436	0.840858	1.248515	0.539081
0.011	0.461007	1.855312	0.99073	3.442183	0.981546	1.04072	0.533996
0.012	0.470352	1.888046	1.065291	3.564716	1.134844	1.001005	0.564229
0.013	0.439549	1.784277	1.135014	3.183644	1.288257	1.131917	0.63612
0.014	0.381125	1.615835	1.193235	2.610922	1.423809	1.380219	0.738463
0.015	0.319662	1.469857	1.235081	2.16048	1.525424	1.641437	0.840272
0.016	0.286242	1.401036	1.260637	1.962901	1.589206	1.78347	0.899789
0.017	0.309467	1.448156	1.276671	2.097156	1.629888	1.684767	0.881584
0.018	0.405739	1.682762	1.295617	2.831687	1.678624	1.27561	0.769935
0.019	0.57216	2.337322	1.332428	5.463072	1.775364	0.568321	0.570066

Table 1: Parameter values @ coefficient ε = 1.

Nτ [s]	z*	Ψ	Ψ_d	F*	F_d*	δ [mm]	i*
0.002	0	1	0.35328	1	0.124807	3	0.35328
0.003	0.003686	1.0037	0.488755	1.007414	0.238882	2.984333	0.490564
0.004	0.019853	1.020256	0.602843	1.040922	0.363419	2.915623	0.615054
0.005	0.058803	1.062477	0.700303	1.128857	0.490424	2.750088	0.744055
0.006	0.128138	1.146971	0.786843	1.315541	0.619122	2.455414	0.902486
0.007	0.228875	1.296806	0.86908	1.681706	0.7553	2.027282	1.127029
0.008	0.353082	1.545792	0.953853	2.389472	0.909836	1.4994	1.474458
0.009	0.484158	1.938576	1.047123	3.758078	1.096466	0.942331	2.029927
0.01	0.600292	2.501823	1.152699	6.25912	1.328714	0.448761	2.883848
0.011	0.680825	3.133076	1.27098	9.816167	1.615389	0.106494	3.982076

Table 2: Parameter values @ coefficient ε = 1.2.

nτ [s]	z*	Ψ	Ψ_d	F*	F_d*	δ [mm]	i*
0.002	0	1	0.4416	1	0.195011	3	0.4416
0.003	0.00576	1.005793	0.610944	1.01162	0.373253	2.97552	0.618043
0.004	0.031021	1.032014	0.753756	1.065053	0.568148	2.868161	0.80279
0.005	0.091879	1.101175	0.876896	1.212587	0.768947	2.609513	1.063313
0.006	0.200246	1.250385	0.989484	1.563462	0.979078	2.148954	1.54702
0.007	0.357944	1.557496	1.102869	2.425795	1.21632	1.478739	2.675333
0.008	0.553476	2.239519	1.229572	5.015445	1.511848	0.647729	6.166852

Table 3: Parameter values @ coefficient ε = 1.5.

3. CONSIDERATIONS ABOUT ACTING BEHAVIOUR OF THE DC ELECTROMAGNET WITH NSF

It considers a real DC electromagnet with NSF with a dynamic behavior described by the non-linear differential equations system (11), with the solution (14), with the following constants:

$$\Omega_0^2 = 3 \cdot 10^5 [rad/s],\ Q_0 = 10^5 [1/s],\ A_0 = 160 [1/s],$$
$$\tau = 10^{-3} [s],\ \delta_0 = 3 [mm],\ \delta_0 + a = 4.25 [mm] \tag{21}$$

The changing of the voltage supply values means the changing of the ε coefficient. The computation results that describe the dynamic behavior of this electromagnet for three values of the ε coefficient (1, 1.2 and 1.5) are reported in the tables 1, 2 and 3.
The evolution gap δ, and current i^*, are shown in Fig.4 and Fig.5. The curve aspects are influenced by the increasing of the voltage supply.

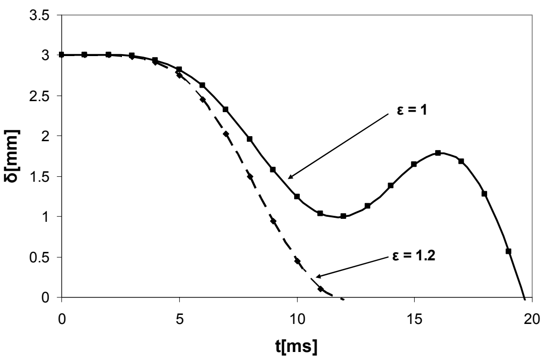


Figure 4: Gap evolution vs. time. Comparison between characteristics @ coeffcient ε = 1 (solid line) and coeffcient ε = 1.2 (dash line).

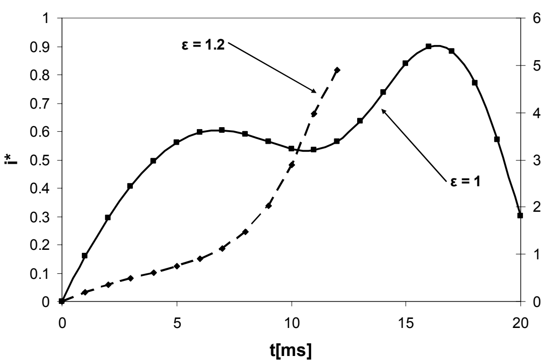


Figure 5: Normalized current vs. time. Comparison between characteristics @ coefficient ε = 1 (solid line) and coefficient ε = 1.2 (dash line).

In Fig.6 is presented the acting time t_{act}, against supply voltage, through ε coefficient, see the relation (12_3), and it's shown a limit value of this coefficient, ε_{lim}, after that the acting time doesn't decreases significantly. Hence, the real voltage supply for the DC electromagnet coil with NSF, must go beyond this limit value. The data is reported in table 4.

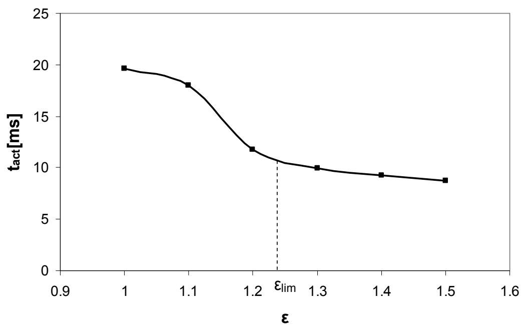


Figure 6: Acting time vs. coefficient ε.

t_{act} [s]	ε
0.0196296	1
0.0180041	1.1
0.0117553	1.2
0.0099846	1.3
0.0092612	1.4
0.008727	1.5

Table 4: Acting time for different ε coefficients.

In Fig.7 are shown the normalized steady state and dynamic electromechanic characteristic, $F^*(\delta)$ and $F_d^*(\delta)$, for ε = 1.2, which underline the dynamic attraction force values always less then steady state attraction force.

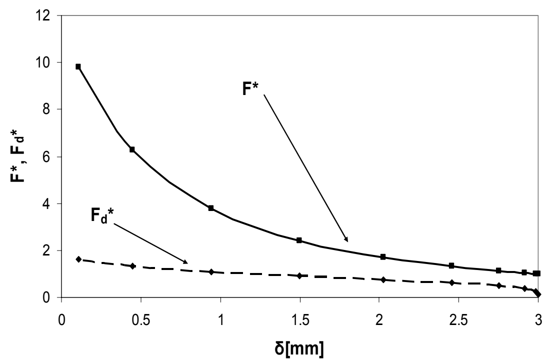


Figure 7: Normalized electromechanic characteristics vs. gap. Comparison between steady-state characteristic, F^* (solid line) and dynamic characteristic, F_d^* (dash line).

The curves $F^*(t)$, $F_d^*(t)$ and $K_d(t)$ for $\varepsilon = 1$ and $\varepsilon = 1.2$, are presented in Fig.8. It notices that normalized dynamic forces, F_d^* values are in every moment less then normalized steady state forces, F^* values, more less like acting time is smaller, and moving velocity of the movable armature is bigger. Actually, at designing of these electromagnets, the safety acting coefficient, K_{sig}, must exceed the minimum values of the dynamic safety acting coefficient, K_d.

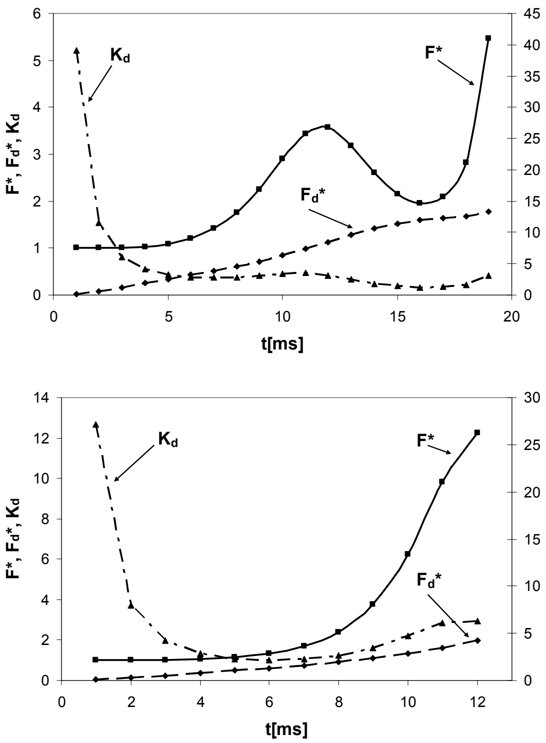


Figure 8: Normalized electromechanic characteristics and the dynamic safety acting coefficient vs. time. Comparison between steady-state characteristic, F^* (solid line), dynamic characteristic, F_d^* (dash line) and the dynamic safety acting coefficient, K_d (dash dot line), @ coefficient $\varepsilon = 1$ (upper diagram) and coefficient $\varepsilon = 1.2$ (lower diagram).

4. CONCLUSIONS

It considers the DC electromagnets with NSF like electromechanical systems of energy conversion with two freedom degrees and with the assumption of neglecting the leakage magnetic flux, their dynamic behaviour can be described by non-linear differential equations type (7). Using the method of finite differences it defines the time evolution of the displacement of movable armature and the total magnetic flux at different values of voltage supply, the ε coefficient, from the relation (12_3).

Hence, for each considered ε value, it estimates the time evolution of the gap δ, the normalized current through the coil, i^*, the attraction normalized steady state force, F^* and the attraction normalized dynamic force, F_d^*.

Considering the ratio of normalized steady state force, F^*, to normalized dynamic force, F_d^*, it defines the dynamic safety coefficient for the DC electromagnets with NSF, with minimum values to be used for a real designing of these electromagnets. It gets interesting information about the influence of the voltage supply upon dynamic behavior of DC electromagnets with NSF.

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