APPLICATION OF PARITY MISCMATHCES IN DETECTION OF BAD DATA FOR POWER SYSTEM STATE ESTIMATION

Leonardo Geo MANESCU

LEG / INPG - UJF - UMR 5529, France; manescu@leg.ensieg.inpg.fr
GIE – IDEA France; manescu_adi@yahoo.fr

Abstract **– Traditionally system state estimation detects gross errors using measurement residuals. This paper proposes to employ parity mismatches for the same goal. With parity mismatch, the normalization allows a better detection of bad data in short lines. Unlike to the method of residuals, such approach separates the treatment of active and reactive flows and injections as well.**

Keywords: *state estimation, bad data detection, parity mismatch*

1. INTRODUCTION

A good state estimate helps power system control. Gross error identification leads to unbiased state estimation [1]. Traditionally this is done with measurement residuals (difference between given measurements and corresponding estimations) in different versions: ordered search, [2], grouped search, [3], or combinations [4 … 6].
A data validation method physically based is described in [7]. The relation between parameter errors and the measurement residuals is given in [8].
A parity mismatch (difference between the given and the calculated value of a constant/parameter) method was proposed for failure detection in complex plants, [9]. The parity mismatches are delivered by physical law. Being of different kinds, active/reactive power injection/line flow measurements are, decoupled in parity mismatches method.
Both parity mismatches and measurement residuals allow to identify gross errors present in a given set of measurements. The paper outlines relations between parity mismatches and the measurement residuals/ This add a new dimension and gives a physical significance to the bad data analysis.
For short lines, the residuals are of small magnitudes, [10…12]. Thus, the gross errors are difficult to identify. The proposed method normalizes line flow parity mismatches by using line impedances. This facilitates identification of gross errors in short lines.

2. MEASUREMENT RESIDUES

The given measurement vector z is modeled by:

$$z = h(x + e) \tag{1}$$

where x is the state vector, $h(x)$ is a nonlinear function of x and e (denoted the error vector of order m, whose i-th component is a normal noise with zero mean). Given z, the weighted least squares state estimate $\hat{x}$ is defined to be the value of x which minimize:

$$J(x) = (z - h(x))^T R^{-1} (z - h(x)) \tag{2}$$

where R is an (m x m) diagonal covariance matrix whose i-th diagonal element is σ_i^2 - the variance of i-th element, T denoted the transpose. The state estimation is then obtained from the sequence:

$$x^{k+1} = x^k + \left[H^T(x^k)R^{-1}H(x^k)\right]^{-1} H^T(x^k)R^{-1}\left(z - h(x^k)\right) \tag{3}$$

where $H(x^k)$ is the Jacobian matrix, [1]. The estimated measurement vector $\hat{z}$ is obtained from the state-estimated vector $\hat{x}$:

$$\hat{z} = H \cdot \hat{x} \tag{4}$$

The measurement residue vector is defined as:

$$r = z - \hat{z} \tag{5}$$

The vector r is related to the error vector e, through the residue sensitivity matrix W, [2], as:

$$r = W \cdot e \tag{6}$$

where $W = I - H\left(H^T \cdot R^{-1} \cdot H\right)^{-1} H^T \cdot R^{-1}$ (7)

The residuals (r) are modified to weighted residuals (r_w) and normalized residuals (r_n), using the covariance matrices, [2]. The residual covariance matrix is given by:

$$E\left(r \cdot r^T\right) = R - H \cdot \left[H^T \cdot R^{-1} \cdot H\right]^{-1} \cdot H^T = W \cdot R \tag{8}$$

Let $D = Diag.(W \cdot R)$ (9)

Then the weighted and normalized residuals are defined as:

$$r_w = \sqrt{R^{-1}} \cdot r \tag{10}$$

$$r_n = \sqrt{D^{-1}} \cdot r \quad (11)$$

The normalized residues have been the basis up until now for identification of gross errors.

3. PARITY MISMATCHES

The parity mismatches method, [13], is a model based formulation and uses physical laws of the system to estimate errors in measurement The equations are written such that the right hand side is a constant and the left hand side contains measurement. The difference between calculated and actual given value of a constant/parameter gives the parity mismatch. The method exploits the fact that the true values of measurements always satisfy the equations (for line flow and bus injections) and any gross error in the measurement will cause a large parity mismatch.

3.1. Line flow equations

Nodal voltages, $\underline{V}_i$, and currents, $\underline{V}_j$, are related by:

$$\underline{I}_{ij} = \underline{Y}_{ii} \cdot \underline{V}_i + \underline{Y}_{ij} \cdot \underline{V}_j \quad (12)$$

where:

$$\underline{Y}_{ii} = \frac{1}{R_{ij} + j \cdot X_{ij}} + j\frac{B'_{ij}}{2} = \underline{Y}_{jj} \quad \text{and}$$

$$\underline{Y}_{ij} = -\frac{1}{R_{ij} + j \cdot X_{ij}} = \underline{Y}_{ji}$$

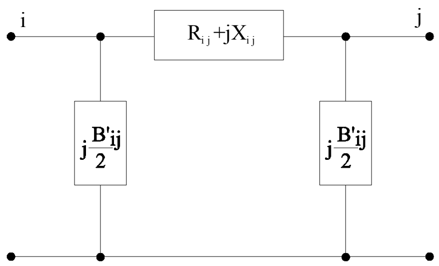

Figure 1: Transmission line as a two port network

If noting:

$$\underline{Y}_{ii} = G_{ii} + j \cdot B_{ii} \text{ and } \underline{Y}_{ij} = G_{ij} + j \cdot B_{ij} \quad (13)$$

the active power flow equation are given by:

$$P_{ij} = V_i^2 \cdot G_{ii} + V_i \cdot V_j \left[G_{ij} \cdot \cos\theta_{ij} + B_{ij} \cdot \sin\theta_{ij}\right] \quad (14)$$

where P_{ij} is the true active power measurement, V_i and V_j are the true voltage magnitudes, θ_{ij} is the difference between the branch voltages phase angles, G_{ij} and B_{ij} are the known constant conductance and susceptance of the line respectively. The equation (14) can be written with a constant in the right hand side as bellow:

$$\frac{P_{ij} - V_i \cdot V_j [G_{ij} \cdot \cos\hat{\theta}_{ij} + B_{ij} \cdot \sin\hat{\theta}_{ij}]}{V_i^2} = G_{ii} \quad (15)$$

The calculated value $\hat{G}_{ii}$ of G_{ii} can be obtained using (15), i.e.:

$$\frac{z_{ij} - \hat{V}_i \cdot \hat{V}_j [G_{ij} \cdot \cos\hat{\theta}_{ij} + B_{ij} \cdot \sin\hat{\theta}_{ij}]}{\hat{V}_i^2} = \hat{G}_{ii} \quad (16)$$

In (16) $\hat{V}_i$ and $\hat{V}_j$ are the estimated magnitudes of the state voltages, $\hat{\theta}_{ij}$ is the estimated difference between the terminal voltages and z_{ij} represents the measured active power line flow. Parity mismatches of the conductance parameter G_{ii} are given by:

$$PM_{ij}^1 = G_{ii} - \hat{G}_{ii} \quad (17)$$

The subscript ij corresponds to that of the measurement z_{ij}. Similarly it is possible to generate more parity mismatches PM^2, PM^3, from the same measurement, using different equation representation given below:

$$\frac{z_{ij} - \hat{V}_i^2 \cdot G_{ij} + \hat{V}_i \cdot \hat{V}_j \cdot B_{ij} \cdot \sin\hat{\theta}_{ij}}{\hat{V}_i \cdot \hat{V}_j \cdot \cos\hat{\theta}_{ij}} = \hat{G}_{ij} \quad (18)$$

$$PM_{ij}^2 = G_{ij} - \hat{G}_{ij} \quad (19)$$

$$\frac{z_{ij} - \hat{V}_i^2 \cdot G_{ij} + \hat{V}_i \cdot \hat{V}_j \cdot G_{ij} \cdot \cos\hat{\theta}_{ij}}{\hat{V}_i \cdot \hat{V}_j \cdot \sin\hat{\theta}_{ij}} = \hat{B}_{ij} \quad (20)$$

$$PM_{ij}^3 = B_{ij} - \hat{B}_{ij} \quad (21)$$

Parity mismatches PM_{ij}^4 to PM_{ij}^6 for the reactive power line flow are obtained from three similar representations as above.

3.2. Bus Injection Equations

The bus injection equation for active power at the i-th bus is given by:

$$P_{ii} = V_i^2 \cdot G_{ii} + V_i \sum_{j=2}^{n_b} V_j \cdot \left(G_{ij} \cdot \cos\theta_{ij} + B_{ij} \cdot \sin\theta_{ij}\right) (22)$$

where P_{ij} is the "true" active power injection at i-th bus and n_b is the number of buses in a given system.

For the purpose of obtaining parity mismatch, (22) can be rewritten as:

$$\frac{z_{ii} - \hat{V}_i \sum_{j=2}^{n_b} \hat{V}_j [G_{ij} \cdot \cos\hat{\theta}_{ij} + B_{ij} \cdot \sin\hat{\theta}_{ij}]}{\hat{V}_i^2} = \hat{G}_{ii} \quad (23)$$

The parity mismatch for the injection measurement can be then obtained as:

$$PM_{ij}^7 = G_{ii} - \hat{G}_{ii} \quad (24)$$

A parity mismatch PM_{ij}^8 can be generated similarly for the reactive power injection measurement. The other constants in (22) can also be used to generate parity mismatches.

4. PARITY MISMATCHES

Parity mismatches and measurement residuals arise because of gross errors are presented in measurements. Ideally both of them should have zero mean value. In practical cases it is possible to establish a relationship between the two. The power system equation (14) and (22) contain three types of variables: the state variables, the parameters/constants and the power flows. The equation can be recast in the form (15) or (23) with a constant/parameter on the right hand side. An error in any one of the variables of the LHS, will therefore almost always be reflected onto the RHS. A similar phenomenon occurs in measurement residues as well.

In the process of obtaining residuals, the information contained in large number of measurements first gets compressed into the minimal set of estimated state, through the least squares (or similar) technique. Obtaining the residuals encompasses decompression of information in the estimated state into the estimated measurements. In the parity mismatch method, the given measurements are transformed into calculated values of the system constants (and corresponding mismatches) by rewriting physical equations. Thus, both parity mismatches and measurement residuals are obtained using different reconstruction techniques. The residue r_{ij} is given by:

$$r_{ij} = z_{ij} - \hat{z}_{ij} \quad (25)$$

$$r_{ij} = z_{ij} - \hat{V}_i \cdot \hat{V}_j \begin{bmatrix} G_{ij} \cdot \cos\hat{\theta}_{ij} + \\ B_{ij} \cdot \sin\hat{\theta}_{ij} - G_{ii} \end{bmatrix} - G_{ii} \cdot \hat{V}_i^2 \quad (26)$$

Using (16) and (26) we get:

$$r_{ij} = -\hat{V}_i^2 \left(- G_{ii} + \hat{G}_{ii}\right) \quad (27)$$

It is to be noted that $\hat{G}_{ij}$ is a function of $\hat{V}_j$ and $\hat{\theta}_{ij}$

$$r_{ij} = \hat{V}_i^2 \cdot PM_{ij}^1 \quad (28)$$

Similar order parity mismatches can be related to other residues:

$$r_{ij} = -\hat{V}_i \cdot \hat{V}_j \cdot \cos\hat{\theta}_{ij} \cdot PM_{ij}^2 \quad (29)$$

$$r_{ij} = -\hat{V}_i \cdot \hat{V}_j \cdot \sin\hat{\theta}_{ij} \cdot PM_{ij}^3 \quad (30)$$

A similar set of relations can be developed for parity mismatches PM_{ij}^4 to PM_{ij}^6, obtained for the reactive power measurements, [14]. The relation between the measurement residue of the active power injection and its parity mismatch is:

$$r_{ii} = \hat{V}_i^2 \cdot PM_{ii}^7 \quad (31)$$

A similar relation can be developed for parity mismatch PM_{ii}^8 reactive power injection measurement, [14]. Parity mismatches PM_{ij}^1, PM_{ij}^4, PM_{ij}^7 and PM_{ij}^8 have simple relationship with measurement residuals.

The residue base methods give rise to constraints like observability and degeneration of measurement configurations ([3], [4], [5] and [6]). Decoupling line flow and injection measurements cannot be effected if limited local redundancy. However, this feature is intrinsic to the parity mismatch method. The procedure of generating the parity mismatches is much more direct and transparent than the measured residuals. Thw influence of other parameters (such as admittances) on the parity mismatches is also obvios. The parity mismatches have a strong physical significance because of their direct association with the parameters. The relations (28)...(31) enable one to assign this physical significance to residues, too. This helps in normalization based on sensitivities of state variables and parameters variables. Such normalization, using line impedance, follows.

Grossly erroneous power flow measurements on short lines are known to lead to small residues, [10…12]. The reason is that compression and reconstruction of the information is much stronger in short lines. During the process of compression, the gross error biases the estimated state. The biased estimate state has a large amplification for short lines and proportionately smaller amplification in the long line measurements connected to the short line. Since the short line measurement is erroneous and the amplified estimated measurement is very close to the erroneous measurement, it shows a small residue.

However, the measurements in nearby long lines even through true, amplify the bias in the estimate, and generally show residues larger than those for the short line. A normalized based on the impedance magnitude is found to overcome this difficulty with short lines ([14]). By this normalization, the parity mismatches corresponding to long lines are attenuate while the parity mismatches of the short lines get amplified resulting in better visibility of short lines. The normalization is carried out by defining the normalized parity mismatch:

$$PM_{ij}^{N} = PM_{ij} \Big/ IPM_{ij} \tag{32}$$

where IPM_{ij} is the impedance of the line ij. The normalized parity mismatches are then subject to statistical tests.

5. STATISTICAL TESTING

Grossly erroneous measurements distort mean μ and variance σ^2 of the set of mismatches $\{PM_{ij}^N\}$. If the grossly erroneous measurements are removed iteratively, the mean μ and variance σ^2 tend toward their true values. A standard error D_{ij} for a measurement z_{ij} is needed for the purpose of identification:

$$D_{ij} = \left(PM_{ij}^{N} - \mu\right) \Big/ \sigma \tag{33}$$

The statistical test for the active power flow measurement in described by the algorithm:

(a) Obtain the system state estimation, $\hat{x}$, from the given measurement set.

(b) Write parity equation (15) for measurements of the active power line flow.

(c) Obtain parity mismatches for the active power line flows.

(d) Calculate mean μ and variance σ^2 for these parity mismatches.

(e) Select a statistical confidence threshold λ.

(f) Compute standard error, D_{ij}, using (33).

(g) If $D_{ij} > \lambda$, then z_{ij} is grossly erroneous.

(h) Eliminate these grossly erroneous measurements from $\{PM_{ij}^N\}$.

(i) Repeat step (d) to (h), until (g) indicates bad data.

(j) The measurements corresponding to the eliminated mismatches are claimed to be grossly erroneous.

Similarly are tested the parity mismatches corresponding to the reactive power flow, active/reactive power injections.

6. PARITY MISMATCHES

The parity mismatch method was tested on IEEE 57 bus system, [15]. The measurement configuration consists of all line flows measured at one end and all the bus injection measurements. In the illustrative example gross errors are incorporated in the short and medium lines on IEEE 57-bus system. The given measurements, the true values and the associated errors are given in Table 1. The measurement of active power flow corresponding on line 37-38, the active power injection to bus 16 and the reactive power injection of bus 11 include gross errors of small magnitudes.

Sr. No.	Measure ment	True value	Given value	Error
1	P 1-2	1.0207	1.2207	0.2088
2	P 37-38	-0.2106	-0.31065	-0.0992
3	Q 49-50	0.04494	-0.255	-0.3016
4	Q 11	0	-0.1	-0.1008
5	P 16	-0.43	-0.33	0.0982
6	Q 40	0	-0.4	-0.3016
7	P 57	-0.06	0.173	0.2409

Table 1: Given data for IEEE 57 bus system

The parity mismatches for the set of data is then obtained. The line flow parity mismatches are normalized using the criterion from (32). Table 2 indicates the iteration number, the type of mismatch, the measurement, the actual mismatch magnitude and the standard error on the larger of the mismatches which got caught in the threshold test. The parity mismatches corresponding to the active power line flow, the reactive power line flow the active power injection and the active and reactive injection measurements are then separated for statistical testing.

This can be done in the case of parity mismatches because they are directly derived from physical laws. The nature of the error propagation is specific to each (variables relationship) law The results of statistical test are indicated in Table 2. The last two rows outline that if the (normalized) parity are too small, then they escape the threshold net and defy identification.

Iter .No .	Type of mismatch	Measu-rement	Mismatch magnitude	Std. error
1	PM^{1N}	P_{1-2}	4.01	8.323
1	PM^{4N}	P_{36-40}	3.02	7.115
2	"	Q_{49-50}	1.37	5.505
1	PM^{7}	P_{57}	0.11	5.455
1	PM^{8}	Q_{40}	0.23	6.515
-	PM^{1N}	P_{37-38}	0.38	Small
-	PM^{8}	Q_{11}	0.025	Small

Table 2: Measured, mismatch and error for IEEE 57

Remark: Flows measurements on lines L_{1-2} and L_{37-38} (short lines) and L_{49-50} (medium length) are grossly erroneous. This becomes more visible if their impedances are normalized.

6. CONCLUSION

In this paper, network parameters are employed to generate parity mismatches for the purpose of identification of gross errors. Relations between measurement residuals and parity mismatches have been derived. The physical request of parity mismatches delivers one facility of normalization that improves the delectability of gross errors in short lines.
The statistical test is unique in the sense that the elimination of grossly erroneous measurement is a part of the statistical testing procedure and does not have the observability or loss of information constraints. It has also been observed that the statistical test is suited for large systems with large global redundancy. Incorporation of parallel computing could enhance the power of this method.

References

[1] F.C. Schweppe, J.Wildes, "Power System Static State Estimation", IEEE Trans. on Power Apparatus and Systems, Vol.PAS-89, No.1, Jan. 1970, pp.120-135.

[2] E.Handschin, F.C.Schweppe, J.Kohlas, A.Fiechter, "Bad Data Analysis for Power System State Estimation", IEEE Trans. on Power Apparatus and Systems, Vol. PAS-94, No.2, March/April 1975, pp.329-337.

[3] L. Mili, Th.Van Cutsem, M.Ribbens Pavella, "Hypothesis Testing and Identification. A New Approach for Bad Data Analysis in Power System State Estimation", IEEE Trans. on Power Apparaus and Systems, Vol.Pas-103, No.11, Nov.1984, pp.3239-3252.

[4] I.W.Slutsker, "Bad Data Identification in Power System State Estimation Based on Measurement Compensation and Linear Residual Calculation"", IEEE Trans. on Power Systems, Vol.4, No.1, Feb.1989, pp. 53-60.

[5] F.Zhuang, R.Balasubrahmanian, "Bad Data Processing in Power System State Estimation by Direct Data Deletion and Hypothesis Tests", IEEE Trans. on Power Systems, Vol.PWRS-2, No.2, May 1987, pp.321-330.

[6] B.M.Zhang, S.Y.Wang, N.D.Xiang, "A Linear Recursive Bad Data Identification method with Real Time Application to Power System State Estimation", IEEE Trans on Power Systems, Vol.7, No.3, Aug.1992, pp.1378-1385

[7] P.Bonanomi, G.Gramberg, "Power System Data Validation and State Calculation by Network Search Techniques", IEEE Trans. on Power Apparatus and Systems, Vol.PAS-102, No.1, Jan. 1983, pp.238-249.

[8] W.I.Liu, F.F.Wu, S.Liu, "Estimation of parameters Errors from Measurement Residuals in State Estimation", IEEE Trans. on Power Systems, Vol. 7, No.1, Feb.1992, pp.81-89.

[9] H.A.Mangalvedekar, S.D.Varwandkar, "Masking in Power System State Estimation", International Conference on Stability and Load Management, Nagpur, India, January 1996.

[10] L.Mili, V.Phaniraj, P.J.Rousseeuw, "Least Median Squares Estimation in Power Systems", IEEE Trans. on Power Systems, Vol.6, No.2, May 1991, pp.511-523.

[11] M.K.Celik, A.Abur, "Use of Scaling in WLAW Estimation of Power Systems States", IEEE Trans. on Power Systems, Vol.7, No.2, May 1992, pp.684-692.

[12] M.K.Celik, W.H.Edwin Liu, "An Incremental measured Placement algorithm for State Estimation", IEEE Trans. on Power Systems, Vol.10, No.3, Aug. 1995, pp.1698-1703.

[13] J.J.Gertler, "Survey of Model Base Failure Detection and Isolation in Complex Plants", IEEE Systems Magazine, Vol.8, No.6, December 1988, pp.3-11.

[14] J.D.Parakkuth, "Power System State Estimation and Gross Error Identification for Large Systems", M.Tech dissertation, ITT -Pawai, Bombay, India.

[15] M.A.Pai, "Computer Methods in Power System Analysis", Tata McGraw Hill Publishing, 1978.