RADIATIVE HEAT TRANSFER IN AXISYMMETRIC SYSTEMS OF NON-ISOTHERMAL SURFACES

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Abstract **– The paper discusses the radiative heat transfer between two cylindrical ideal surfaces mounted coaxially. This layout is frequently used in many thermal engineering applications.**

Keywords: *Radiative heat transfer, obstructed mutual visibility*

1. INTRODUCTION

The radiative heat transfer occurs in many engineering applications, such as power generation and high – temperature industrial processes. The advent of space age over the past thirty years brought the necessity to design tools to predict heat transfer in applications like rocket nozzles, reentry vehicles and space vehicles in vacuum. In an important number of cases the radiative heat transfer analysis needs to be performed for a radiatively non-participating medium and interaction with other heat transfer modes must be taken into account. Whatever the other heat transfer mode, the system requires non-isothermal boundary conditions, leading finally to complex integro-differential equations which can be solved almost exclusively by mean of numerical methods. The paper considers such an example of coupling radiation with convection for non-isothermal boundary conditions – that is fluid flow with heat transfer. Heat transfer between hot flue gas and cold water separated by a closed radiative enclosure that houses a radiatively non-participating medium, is analyzed. A mathematical model for the coupled radiative convective heat transfer is developed and a numerical algorithm for solving of heat transfer equations is proposed.

2. DESCRIPTION OF THE GEOMETRY CONSIDERED

The system analyzed in this paper serves as a starting point for generalization of the radiative heat transfer in enclosures consisting of surfaces with partial mutual visibility. It consists of two cylindrical radiative surfaces mounted coaxially. The inner surface is heated by a hot gas flow and the outer surface is cooled by an external water flow. The two radiative surfaces are separated by air. The system is designed so that non-isothermal conditions for the two surfaces are fulfilled. The high temperature of the flue gas ensures a large value for the difference of temperature between the two surfaces so that the radiative heat transfer has a much greater contribution to the overall heat transfer rate than natural convective heat transfer does. At the ends two flanges close the radiant enclosure.

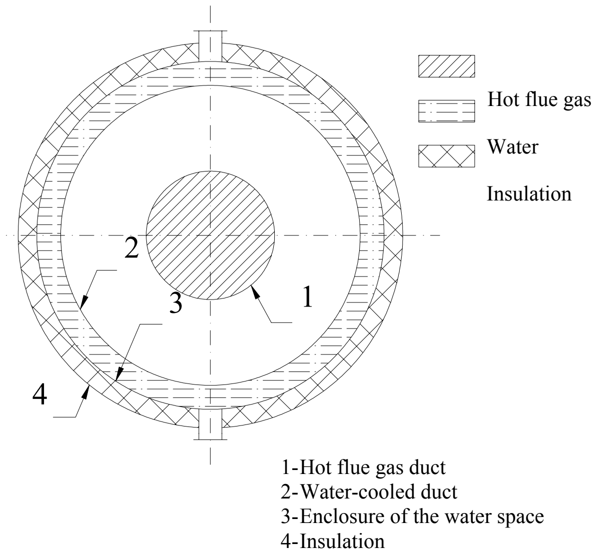

Figure 1: Cross section of the heat transfer system

The conjugated heat transfer system is depicted in cross section in figure 1. Axial flow of the two thermal agents ensures one-dimensional distributions of temperature:

$$T_{f1} = T_{f1}(z)\,,\ T_{f2} = T_{f2}(z)$$

$$T_1 = T_1(z)\,,\ T_2 = T_2(z)$$

3. THE MODEL

A series of simplifying assumptions are considered in order to reduce the complexity of the heat transfer problem. Though, the generality of the problem is not affected to an important extent by these assumptions:

- Temperature drop in the wall of the hot duct is neglected.
- The two radiant surfaces are considered black ($\varepsilon = 1$)
- Axial heat conduction for the two fluids is neglected

- Natural convection which occurs due to the presence of air between the two ducts is negligible in comparison with radiative heat transfer
- Axial heat conduction in the ducts is neglected.

The heat transfer equations, on the assumptions made above, are the following:

Flue gas:

$$\rho_1 u_1 A_1 c_{p1} T_{f1} = \rho_1 u_1 A_1 c_{p1} T_{f1} + \frac{d}{dz}\left(\rho_1 u_1 A_1 c_{p1} T_{f1}\right)\cdot dz + + q_1 \cdot dS_1 \quad (1)$$

Flue gas duct:

$$\alpha_1 \left(T_{f1} - T_1\right)\cdot dS_1 = q_1 \cdot dS_1 \quad (2)$$

Cooling agent (water):

$$\rho_2 u_2 A_2 c_{p2} T_{f2} = \rho_2 u_2 A_2 c_{p2} T_{f2} + \frac{d}{dz}\left(\rho_2 u_2 A_2 c_{p2} T_{f2}\right)\cdot dz + + q_2 \cdot dS_2 \quad (3)$$

Cooling agent duct:

$$\alpha_2 \left(T_2 - T_{f2}\right)\cdot dS_2 = q_2 \cdot dS_2 \quad (4)$$

In equations (1) and (3) q_1 and q_2 are the radiative heat flux densities on the surfaces 1 and 2 respectively (figure 1).

For diffusely emmiting, black surfaces, the radiative heat flux density in a point described by the vector coordinate $\vec{r}$ is given by the following equation [1]:

$$q(\vec{r}) = \sigma_0 T^4(\vec{r}) - H(\vec{r}) \quad (5)$$

The irradiation of an elementary area dA belonging to an enclosure consisting of n is given by:

$$H(\vec{r}) = \int_{\bigcup_{j=1}^{n} S_j} \sigma_0 T^4(\vec{r}\,')dF_{dA-dA'} \quad (6)$$

in which $dF_{dA-dA'}$ is the infinitesimal view factor between dA and dA' respectively.

The irradiation of an elementary area dA can be easily determined if view factors between the elementary area considered and the rest of elementary areas dA' which compose the radiative enclosure are known.

The radiative heat flux density on the elementary area dA having the vector coordinate $\vec{r}$ takes the following form:

$$q(\vec{r}) = \sigma_0 T^4(\vec{r}) - H(\vec{r}) = \sigma_0 T^4(\vec{r}) - - \int_{\bigcup_{j=1,\, j\neq k}^{n} S_j} \sigma_0 T^4(\vec{r}\,')dF_{dA-dA'} \quad (7)$$

Index k designates the surfaces that are not visible from the $\vec{r}$ vector coordinate point. It can refer to the surface to which elementary area dA belongs if this surface is convex.

Due to the symmetry of the heat transfer system considered and the initial assumptions the temperatures of the two agents and of the heat transfer surfaces varies axially and the temperature of the end flanges varies radially.

Under these circumstances the domain of analysis is divided in axial direction into a n knots mesh having the size Δz. The end flanges are also divided into a m knots mesh having the size Δr.

The isothermal elements generated by the mesh are cylindrical for the flue gas and water ducts and annular for the end flanges. All elements have the same symmetry axis with the overall heat transfer system.

Although the surface elements defined in this manner are isothermal, there is no complete mutual visibility between all elementary areas that compose the surface elements. This situation leads to the necessity of redefining the view factor; the new concept is introduced in [2] and is called *obstructed view factor*.

The definition is similar to the standard view factor [1]: ratio between diffuse energy leaving the element dS_i directly toward visible zones of dS_j (and consequently intercepted by dS_j) and the total diffuse energy leaving the surface element dS_i.

This paper quotes an example [2] of such a obstructed view factor – that is from a surface element on the flue gas duct $dS_i = 2\pi R_1 \cdot dz$ having the axial coordinate z_i and a surface element from the water-cooled duct $dS_j = 2\pi R_2 \cdot dz$ having the axial coordinate z_j.

The surface element $dA_j = R_2 d\theta \cdot dz$ having the angular coordinate θ situated on surface S_2 is visible from surface S_1 from the elementary areas $dA_i = R_1 d\varphi \cdot dz$ situated on the arc $\Delta\varphi$ marked on the drawing (figure 2).

The heat flux radiated by the surface element $dS_i = 2\pi R_1 dz$ is:

$$dQ_{dS_i \rightarrow} = \sigma_0 T_i^4 \cdot dS_i = 2\pi R_2 dz \cdot \sigma_0 T_i^4 \quad (8)$$

The view factor between the elementary area belonging to the surface element from S_1, visible

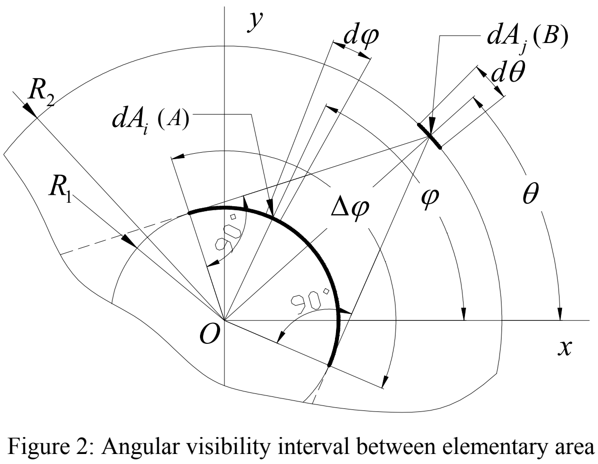

Figure 2: Angular visibility interval between elementary area dA_j on S_1 and a surface element $2\pi R_1 \cdot dz$

from B (figure 2), denoted $dA_i^v = R_1 \Delta\varphi \cdot dz$ and elementary area $dA_j = R_2 d\theta \cdot dz$ can be computed using the standard formula of the view factor [1]):

$$dF_{dA_i^v \to dA_j}^{S_1 - S_2} = \frac{1}{dA_i^v}\left[\int_{dA_i^v} \frac{\cos\theta_i \cos\theta_j}{\pi s^2} dA_i\right] dA_j \tag{9}$$

The heat flux radiated by the elementary area dA_i^v and intercepted by elementary area dA_j is given by:

$$dQ_{dA_i^v \to dA_j} = \left(\sigma_0 T_i^4 \cdot dA_i^v\right) \cdot dF_{dA_i^v \to dA_j}^{S_1 - S_2} = \\ = \sigma_0 T_i^4 \underbrace{\left(R_2 d\theta \cdot dz\right)}_{dA_j} \left[\int_{dA_i^v} \frac{\cos\theta_i \cos\theta_j}{\pi s^2} dA_i\right] \tag{10}$$

The heat flux intercepted by the surface element $dS_j = 2\pi R_2 \cdot dz$ is:

$$dQ_{\to dS_j} = \sigma_0 T_i^4 \cdot 2\pi R_2 \cdot dz \left[\int_{dA_i^v} \frac{\cos\theta_i \cos\theta_j}{\pi s^2} R_1 d\varphi \cdot dz\right] = \\ = 2\pi R_2 \sigma_0 T_i^4 (dz)^2 R_1 \int_0^{\Delta\varphi} \frac{\cos\theta_i \cos\theta_j}{\pi s^2} d\varphi \tag{11}$$

At this point it should be mentioned that the integration was carried out over the angular interval $[0, \Delta\varphi]$ instead of $[\theta - \Delta\varphi/2, \theta + \Delta\varphi/2]$, because the value of the integral does not depend on θ.

The obstructed view factor between dS_i and dS_j is:

$$dF_{dS_i \to dS_j}^{S_1 - S_2} = \frac{dQ_{\to dS_j}}{dQ_{dS_i \to}} = \frac{2\pi R_2 R_1 (dz)^2 \sigma_0 T_i^4 \int_{\Delta\varphi} \frac{\cos\theta_i \cos\theta_j}{\pi s^2} d\varphi}{\sigma_0 T_i^4 \cdot 2\pi R_1 dz} = \\ = R_2 dz \int_{\Delta\varphi} \frac{\cos\theta_i \cos\theta_j}{\pi s^2} d\varphi \tag{12}$$

Simple geometrical considerations (figure 2) give the value of $\Delta\varphi$, which is $\Delta\varphi = 2\arccos(R_1/R_2)$.

The distance s between the centers of elementary areas dA_i and dA_j can be computed using the formula (figure 3):

$$s = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2} \tag{13}$$

The angles θ_i and θ_j can be computed using the formula: (figure 3):

$$\cos\theta_i = \frac{\overrightarrow{AB} \cdot \vec{n}_i}{\left\|\overrightarrow{AB}\right\| \cdot \left\|\vec{n}_i\right\|} \tag{14}$$

in which:

- $\overrightarrow{AB}$ is the vector with the s modulus between the centers of elementary areas dA_i and dA_j oriented from the center of dA_i towards the center of dA_j. In Cartesian coordinates, $\overrightarrow{AB}$ can be expressed as:

$$\overrightarrow{AB} = (x_B - x_A)\vec{i} + (y_B - y_A)\vec{j} + (z_B - z_A)\vec{k}$$

Hereinafter, the Cartesian coordinates will be converted to cylindrical coordinates.

The obstructed view factors between elements situated on other surfaces $dF_{dA_i \to dA_j}^{S_i - S_j}$ are computed in a similar manner. Their final forms are the following

Flue gas duct – flanges:

$$dF_{dA_i^v \to dA_j}^{S_1 - S_3(S_4)} = r \cdot dr \cdot \int_{\Delta\varphi} \frac{\cos\theta_i \cos\theta_j}{\pi s^2} d\varphi \tag{15}$$

Water –cooled duct – flue gas duct:

$$dF_{dS_j \to dS_i}^{S_2 - S_1} = R_d \cdot dz \int_{\Delta\theta} \frac{\cos\theta_i \cos\theta_j}{\pi s^2} d\theta \tag{16}$$

Elements from the water-cooled duct:

$$dF_{dS_i \to dS_j}^{S_2 - S_2} = R_2 \cdot dz \int_{\Delta\varphi} \frac{\cos\theta_i \cos\theta_j}{\pi s^2} d\varphi \tag{17}$$

Water-cooled duct - flanges:

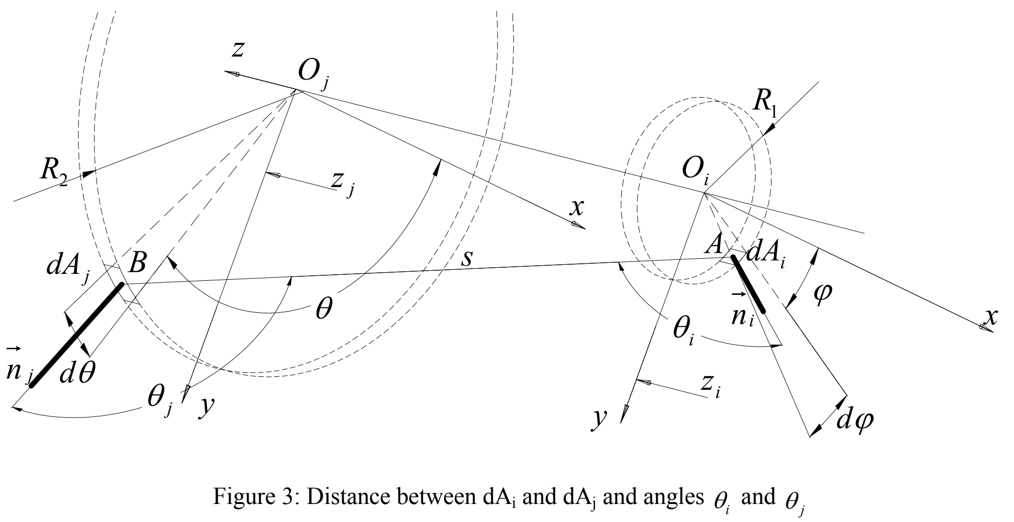

Figure 3: Distance between dA_i and dA_j and angles θ_i and θ_j

$$dF_{dS_i \to dS_j}^{S_2-S_3(S_4)} = r \cdot dz \int_{\Delta\varphi} \frac{\cos\theta_i \cos\theta_j}{\pi s^2} d\varphi \quad (18)$$

Flanges – flue gas duct:

$$dF_{dS_j \to dS_i}^{S_3(S_4)-S_1} = R_1 \cdot dz \int_{\Delta\varphi} \frac{\cos\theta_i \cos\theta_j}{\pi s^2} d\theta \quad (18)$$

Flanges – water –cooled duct:

$$dF_{dS_j \to dS_i}^{S_3(S_4)-S_2} = R_2 \cdot dz \int_{\Delta\theta} \frac{\cos\theta_i \cos\theta_j}{\pi s^2} d\theta \quad (19)$$

Elements on flanges

$$dF_{dS_j \to dS_i}^{S_3-S_4} = r_i \cdot dr \int_0^{\Delta\theta} \frac{\cos\theta_i \cos\theta_j}{\pi s^2} d\theta \quad (20)$$

The basic equations that describe the heat transfer process become:

For the flue gas:

$$-\frac{\rho_1 u_1 A_1 c_{p1}}{P_1}\frac{dT_{f1}(z_i)}{dz} = \sigma_0 T_1^4(z_i) - \int_0^L \sigma_0 T_2^4(z_j) dF_{z_i-z_j}^{S_1-S_2} - $$
$$- \int_{R_d}^{R} \sigma_0 T_3^4(r_j) dF_{z_i-r_j}^{S_1-S_3} - \int_{R_d}^{R} \sigma_0 T_4^4(r_j) dF_{z_i-r_j}^{S_1-S_4} \quad (21)$$

Flue gas duct:

$$\alpha_1(T_{f1} - T_1) = \sigma_0 T_1^4(z_i) - \int_0^L \sigma_0 T_2^4(z_j) dF_{z_i-z_j}^{S_1-S_2} - $$
$$- \int_{R_d}^{R} \sigma_0 T_3^4(r_j) dF_{z_i-r_j}^{S_1-S_3} - \int_{R_d}^{R} \sigma_0 T_4^4(r_j) dF_{z_i-r_j}^{S_1-S_4} \quad (22)$$

Cooling agent (water):

$$\frac{\rho_2 u_2 A_2 c_{p2}}{P_2}\frac{dT_{f2}(z_i)}{dz} = \sigma_0 T_2^4(z_i) - \int_0^L \sigma_0 T_1^4(z_j) dF_{z_i-z_j}^{S_2-S_1} - $$
$$- \int_0^L \sigma_0 T_2^4(z_j) dF_{z_i-z_j}^{S_2-S_2} - \int_{R_d}^{R} \sigma_0 T_3^4(r_j) dF_{z_i-r_j}^{S_2-S_3} - $$
$$- \int_{R_d}^{R} \sigma_0 T_3^4(r_j) dF_{z_i-r_j}^{S_2-S_4} \quad (23)$$

In (21) and (23) P_1 and P_2 are the values of perimeter of the flow sections for flue gas and water respectively Water cooled duct:

$$\alpha_2(T_2 - T_{f2}) = \sigma_0 T_s^4(z_i) - \int_0^L \sigma_0 T_1^4(z_j) dF_{z_i-z_j}^{S_2-S_1} - $$
$$- \int_0^L \sigma_0 T_2^4(z_j) dF_{z_i-z_j}^{S_2-S_2} - \int_{R_d}^{R} \sigma_0 T_3^4(r_j) dF_{z_i-r_j}^{S_2-S_3} - \int_{R_d}^{R} \sigma_0 T_4^4(r_j) dF_{z_i-r_j}^{S_2-S_4}$$

(24)

4. CONCLUSIONS

A four-equation analytical model for the conjugated heat transfer convection – radiation was developed. The system consisting of four equations containing non-linear and integral terms can be solved applying numeric techniques. The approach used in this paper was focused on the study of radiative heat transfer in systems of surfaces that obstruct partially each other's visibility.

References

[1] Modest, M., F., Radiative Heat Transfer, McGraw Hill Book Company, New York 1993

[2] Diaconu, B., M., Contribuții la intensificarea transferului de căldură prin contact and prin radiație, PhD thesis, UTCB, 2004

[3] Leca, A., Cerna-Mladin, E., Stan, M., Transfer de căldură and masă. O abordare inginerească, Editura Tehnică, Bucureşti 1998