6TH INTERNATIONAL CONFERENCE ON ELECTROMECHANICAL AND POWER SYSTEMS
October 4-6, 2007 - Chişinău, Rep.Moldova

PHYSICO-MATHEMATICAL MODEL WITH FRICTION LOSSES FOR VUILLEUMIER MACHINES

Vlad Mario HOMUTESCU, Dan-Teodor BĂLĂNESCU, Adrian HOMUTESCU

Technical University "GHEORGHE ASACHI" of Iaşi, Technical University "GHEORGHE ASACHI" of Iaşi, Siemens VDO AG
mariohomutescu@gmail.com, balanescud@yahoo.com, spaimap@yahoo.com

Abstract **– The paper presents a new physico-mathematical model for Vuilleumier machines (thermal-acted heat pumps). The model takes into account the friction losses that appear in the heat exchangers of the machine. The pressure drop caused by friction inside a generic heat exchanger is determined. The coefficients of performance (COP's) calculated with the new model are smaller than the isothermal ones.**
The adjustment of the theoretical models in order to take into account the pressure losses inside the heat exchangers allow the emphasizing of some phenomena that appear when changing the rotation speed.

Keywords: *Vuilleumier machine, thermal-acted heat pump, physico-mathematical model, isothermal model, friction losses.*

1. INTRODUCTION

A Vuilleumier machine [1] (a thermal-acted heat pump) is a machine inside which a constant amount of gas evolves inside an almost constant total volume. The gas lies inside several heat exchangers and four variable volume chambers placed (most often) inside two cylinders, each cylinder being fit with its own displacer piston. There are three levels of temperature inside the machine.
The refrigerating effect is acquired by expanding the gas inside a low temperature chamber. Pressure variation inside the machine is acquired by heating the agent inside a high temperature chamber and by cooling the agent inside two intermediate temperature chambers.
According to the schematic diagram in fig. 1, a Vuilleumier machine is comprised of a cold cylinder 1 and a hot cylinder 15 inside which the cold displacer 3 and the hot displacer 13 work. The cold cylinder and displacer share a diameter inferior to the one shared by the hot cylinder and displacer. A drive comprised of crankshaft 18 and rods 17 and 19 provide movement for the displacers. The cold displacer splits the space inside its cylinder in two chambers: a low temperature one 4 and an intermediate temperature one 2. Inside the hot cylinder the hot displacer delimits a high temperature chamber 12 and an intermediate temperature chamber 14. Each cylinder is fit with its own heat exchanger set. The cold cylinder has a low temperature heater 5, a low temperature regenerator 6 and an intermediate temperature cooler 7. The hot cylinder is fit with an intermediate temperature cooler 9, a hot temperature regenerator 10 and a high temperature heater 11. The intermediate temperature cooling chambers are connected through pipe 8.

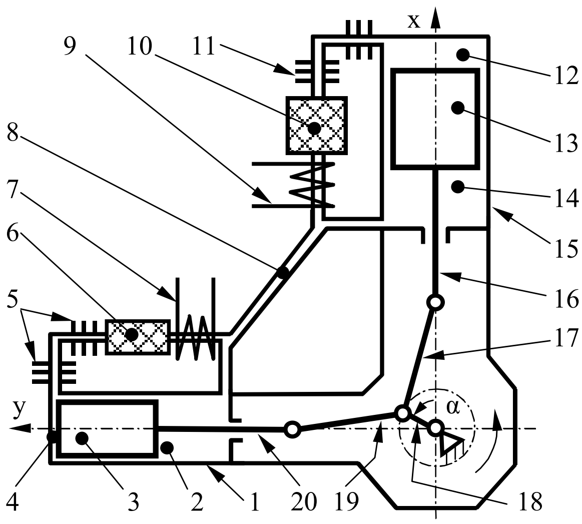

Figure 1: Vuilleumier thermal-acted heat pump:
1 and 15 - cylinder; 2 and 14 - intermediate temperature chamber; 3 and 13 - displacer; 4 - low temperature chamber; 5 - low temperature heater; 6 and 10 - regenerator; 7 and 9 - cooler; 8 - connection pipe; 11 - high temperature heater; 12 - high temperature chamber; 16 and 20 - stem; 17 and 19 - rod; 18 - crankshaft.

2. PRESSURE DROP INSIDE THE HEAT EXCHANGERS

The theoretical models of the Vuilleumier heat pump assume that inside the heat exchangers only isothermal processes take place, and inside the chambers with variable volume the processes are isothermal (for isothermal physico-mathematical models [2]) or adiabatic (for adiabatic models [3], [4]). These models offer information about the maximum performances of the machine: heats exchanged with the heat sources, COP's, or about the temperature variation inside the adiabatic chambers. This information can be used for thermal and mechanical calculation of the machine.

In order to obtain numerical calculated results closer to the real functioning of the Vuilleumier machine physico-mathematical models that consider the various energy losses occuring in the real machine must be used. The energy losses can be taken into account with models with decoupled losses. These models assume that the energy losses that appear in the real functioning of the machine are produced by independent causes.

Real gas evolves inside the Vuilleumier machine. So, the gas flows through the heat exchangers (coolers, heaters and regenerators) with pressure losses caused by friction.

The pressure drop between two different sections varies during the cycle, because it depends on the speed and sense of the gas flow.

The instantaneous pressure differences between two sections of a generic heat exchanger can be calculated with the following relation [5]:

$$\Delta p = -2\,\mu_f\,\mathrm{Re}\frac{\mu\,w\,V_{sc}}{A_t\,d_h^2}\,, \qquad (1)$$

where:

μ_f = Fanning friction coefficient;

$\mathrm{Re} = \rho|w|d_h/\mu$ = Reynolds number;

ρ = gas density;

d_h = hydraulic diameter;

w = flow speed;

A_t = transversal flow area of heat exchanger;

V_{sc} = volume occupied by the gas inside the heat exchanger;

μ = dynamic viscosity.

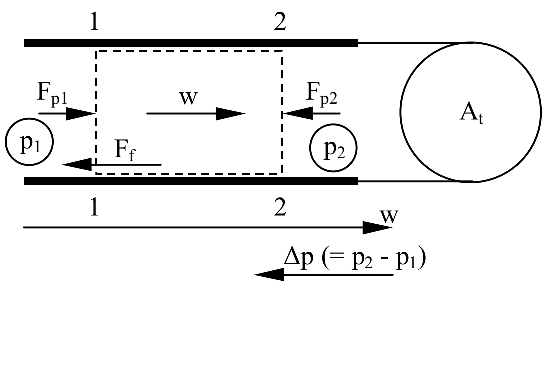

Figure 2: The model of the stationary flow used for calculating the pressure losses inside the heat exchangers.

The relation (1) results from the equality between friction and pressure forces, in accordance with the scheme on fig. 2 (scheme valid for a one-dimensional stationary flow). The existence of the friction shearing stress between layers of working gas implies that the flow must be at least two-dimensional. So, the flow must be simultaneously one and two-dimensional. This contradiction can be avoided with the hypothesis that the two- or three-dimensional flow can be transformed in an equivalent one-dimensional flow characterized by the medium instantaneous speed of the gas.

Eq. 1 is appliable to small time intervals, for which the flow can be considered stationary. The friction coefficient can be determined only experimentally, and is usually expressed as function of Reynolds number. In the literature information about various heat exchangers that can be used at Vuilleumier machines is available [5]. The sense of the gas flow inside the heat exchanger changes cyclically. So, a positive sense of the flow must be chosen. The gas speed oriented from the low temperature chamber toward the high temperature chamber is considered to be positive. Because the Reynolds number is always positive, it must be calculated using the gas speed in absolute value.

The gas speed can be calculated from the equation of continuity, using the one-dimensional flow hypothesis. The instantaneous mean speed can be considered, for example, the one from the transversal section in the middle of the heat exchanger.

The dynamic viscosity of the gas depends (practically) only on the temperature.

3. PHYSICO-MATHEMATICAL MODEL WITH FRICTION LOSSES. PERFORMANCES

The pressure inside the high temperature chamber, calculated with an isothermal or adiabatic theoretical model [2], [3], [4] was adopted as reference pressure. So, inside the high temperature chamber the pressure is the theoretical one. In all other variable volume chambers the pressure differs from the theoretical pressure, due to the pressure differences caused by the friction losses.

The isothermal model calculates the instantaneous pressure inside the Vuilleumier machine using the following expression [2]:

$$p(\alpha) = \frac{m\,R}{\sum_i \frac{V_i(\alpha)}{T_i}} \qquad (2)$$

where

$$\sum_i \frac{V_i(\alpha)}{T_i} = \frac{V_{lt}(\alpha)+V_{h1}}{T_{lt}} + \frac{V_{reg1}}{T_{reg1}} + \frac{V_{k1}+V_{it1}(\alpha)}{T_k} + \\ + \frac{V_{it2}(\alpha)+V_{k2}}{T_k} + \frac{V_{reg2}}{T_{reg2}} + \frac{V_{h2}+V_{ht}(\alpha)}{T_{ht}} . \qquad (3)$$

We used the following subscripts for dimensions inside machine chambers (volume V, temperature T, mass m etc.): h = heater; reg = regenerator; k = cooler; 1 = cold displacer; 2 - hot displacer; ht = high temperature; lt = low temperature; it = intermediate temperature.

The adiabatic models [3], [4] calculate the instantaneous pressure by integrating a system of differential equations.

In the connection pipes between the chambers and heat exchangers the pressure losses were neglected. Due to this hypothesis, the pressure inside the two intermediate temperature chambers is the same.

The pressure for a certain section x-x of the machine is calculated with the following relation:

$$p_x(\alpha) = p(\alpha) - \Delta p_{x-ht} , \qquad (4)$$

where $\Delta p_{x\text{-}ht}$ is the sum of the pressure differences between section x-x and high temperature chamber.

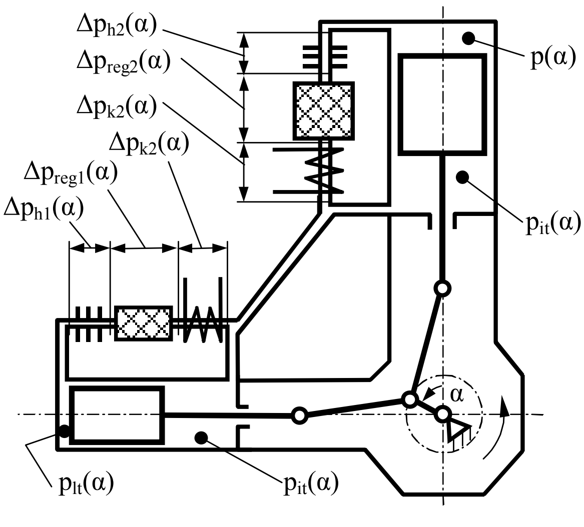

Figure 3: Pressure differences between the extreme sections of the heat exchangers

The instantaneous pressures inside the intermediate temperature chambers and inside the low temperature chamber are:

$$p_{it}(\alpha) = p(\alpha) - (\Delta p_{h2} + \Delta p_{reg2} + \Delta p_{k2}) ; \qquad (5)$$

$$p_{lt}(\alpha) = p(\alpha) - (\Delta p_{h2} + \Delta p_{reg2} + \Delta p_{k2}) - \\ - (\Delta p_{k1} + \Delta p_{reg1} + \Delta p_{h1}) . \qquad (6)$$

The heats cyclically exchanged inside each chamber are calculated with the relations:

$$Q_{ltf} = \int_0^{2\pi} p_{lt}(\alpha)\left[\frac{dV_{lt}(\alpha)}{d\alpha}\right] d\alpha ; \qquad (7)$$

$$Q_{it1f} = \int_0^{2\pi} p_{it}(\alpha)\left[\frac{dV_{it1}(\alpha)}{d\alpha}\right] d\alpha ; \qquad (8)$$

$$Q_{it2f} = \int_0^{2\pi} p_{it}(\alpha)\left[\frac{dV_{it2}(\alpha)}{d\alpha}\right] d\alpha ; \qquad (9)$$

$$Q_{ht} = \int_0^{2\pi} p(\alpha)\left[\frac{dV_{ht}(\alpha)}{d\alpha}\right] d\alpha . \qquad (10)$$

When friction is taken into account, inside the high temperature chamber - taken as reference - the heat exchanged remains unchanged, because the pressure is the same as in the theoretical model.

Inside the isothermal model of the Vuilleumier machine [2] an assumption is made that in all of functional chambers - including here the heat exchangers - the temperature is constant. So, for a whole thermodynamic cycle the heats exchanged with the exterior in coolers, regenerators and heaters are zero. The energy balance for a cycle takes the following expression:

$$\sum_{i=1}^{10} Q_i = Q_{ht} + Q_{it1} + Q_{it2} + Q_{lt} = \\ = L_{ext} = L_f + L_{sd} \qquad (11)$$

where

L_{ext} = work cyclically exchanged with the exterior by the machine taken as a whole;

L_f = work against friction;

L_{sd} = work exchanged by the entire machine as a result of the pressure differences on the transversal area of the stems.

Without friction, L_{ext} and L_{sd} are equal. For a theoretical machine (for which stem diameters are null and the working agent is the ideal gas) $L_{ext} = 0$. With friction taken into account, $L_{ext} = L_f$ only if stem diameters are null. Usually $L_{sd} > 0$, because the pressure inside the machine is far greater than the pressure on the other side of the stems. Work L_{ext} can be produced or consumed, depending on the working conditions. Normally $L_{ext} < 0$, and the machine consumes external mechanical energy, because $|L_f| > L_{sd}$. A critical rotation speed, for which $L_{ext} = 0$ (meaning $L_{sd} = |L_f|$), exists.

The efficiencies for the Vuilleumier machine working as refrigerator or as heat pump are determined with the following relations:

$$\varepsilon_r = \frac{Q_{lt}}{Q_{ht} - L_{ext}}, \quad (12)$$

$$\varepsilon_{hp} = \frac{|Q_{it1} + Q_{it2}|}{Q_{ht} - L_{ext}}. \quad (13)$$

For rotation speeds under the critical one (for which the work exchanged with the exterior L_{ext} is produced) the COP definition relations above show that the denominator of the fractions decreases. It can be demonstrated that $\varepsilon_{hp} = \varepsilon_r + 1$.

4. NUMERICAL EXAMPLE

A Vuilleumier machine featuring the following dimensions is assumed: D_1= 0.1 m; d_1= d_2 = 0.02 m; D_2 = 0.12 m; $r_1 = r_2$ = 0.05 m; $l_1 = l_2$ = 0.2 m; $f_{TDP1} = f_{BDP1} = f_{TDP2} = f_{BDP2}$ = 0.001 m; $V_{h1} = V_{h2} = V_{k1} = V_{k2}$ = 0.05 V_{SD2}; $V_{reg1} = V_{reg2}$ = 1.2 V_{SD2}, where V_{SD2} = volume swept by the high temperature displacer, D = cylinder diameter, d = stem diameter, r = crankshaft radius, l = rod length, f = dead space length.
The machine works at 1000 rpm with a total mass of hydrogen m = 0.0207 kg (corresponding to a pressure of 50 bar in the machine, at an ambient temperature of 15 ºC; R_{H2} = 4121 J/(kg K)) between temperatures $T_{ht} = T_{h2}$ = 923 K; $T_{k1} = T_{k2} = T_{it1} = T_{it2}$ = 343 K and $T_{lt} = T_{h1}$ = 278 K.

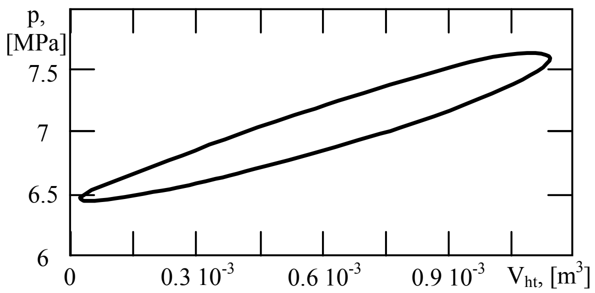

Figure 4: Pressure variation inside the high temperature chamber.

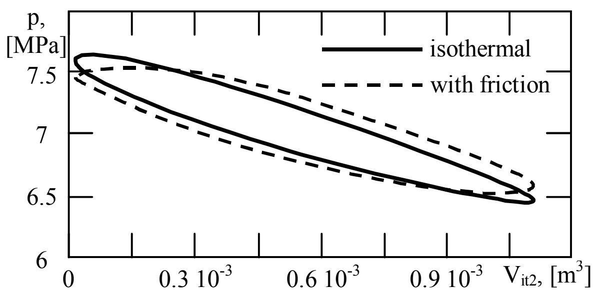

Figure 5: Pressure variation inside the intermediate temperature chamber it2.

The numerical solution of the described isothermal model with friction of the Vuilleumier machine leads to the results displayed in fig. 4 to fig. 9, as well as inside table 1.

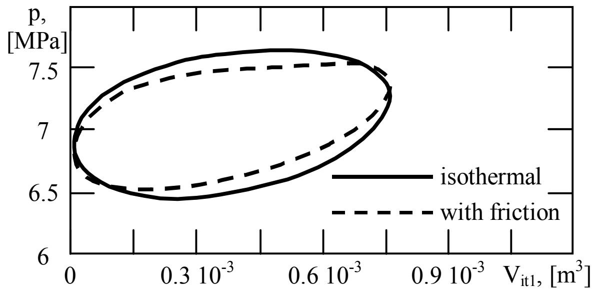

Figure 6: Pressure variation inside the intermediate temperature chamber it1.

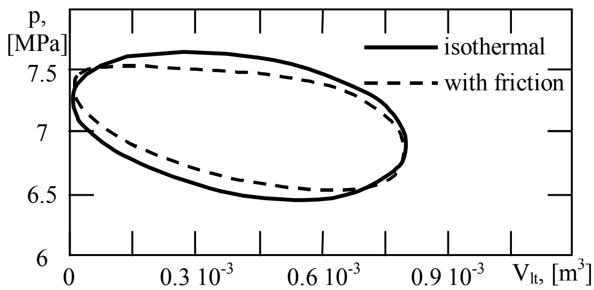

Figure 7: Pressure variation inside the low temperature chamber.

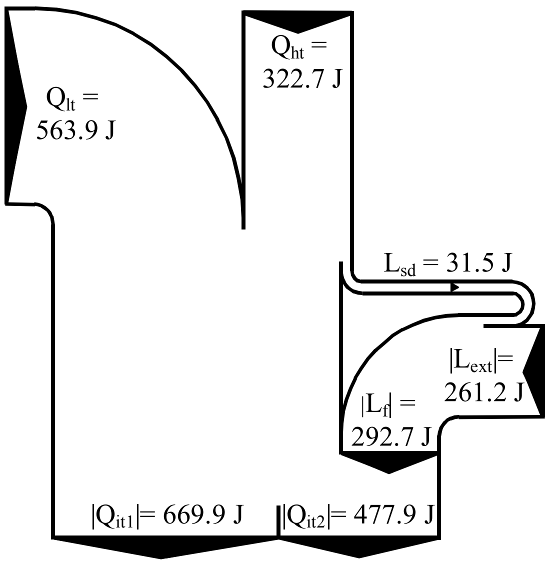

Figure 8: Cyclical balance of energy for the Vuilleumier heat pump.

The pressure variations inside the machine chambers are shown in fig. 4, fig. 5, fig. 6 and fig. 7. In the isothermal Vuilleumier machine with friction the interval for which the gas pressure inside chambers it1, it2 and lt varies is smaller than the corresponding interval for the isothermal machine without friction. The reducing of the pressure differences is a cause for the reduction of the heat pump COP.

In fig. 8 the energy balance of the analyzed thermal-acted Vuilleumier heat pump is presented, for a rotation speed of 1000 rpm (for which $|L_f| > L_{sd}$). The picture emphasizes the heats and works implied in the functioning of the machine.

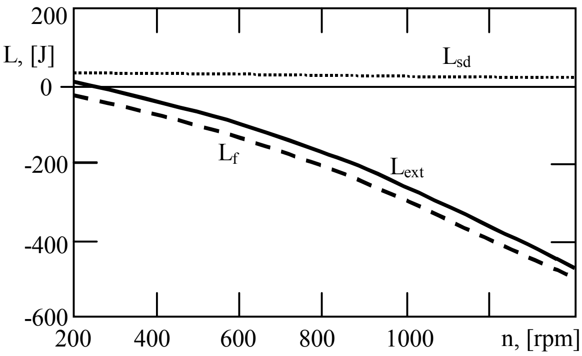

Figure 9: Works L_{ext}, L_f and L_{sd} in function of the rotation speed.

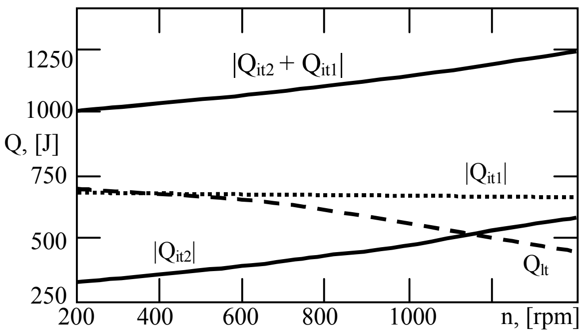

Figure 10: Heats Q_{lt}, Q_{it1} and $|Q_{it2}|$ in function of the rotation speed.

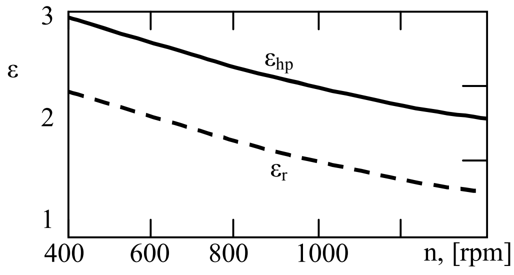

Figure 11: COP's in function of the rotation speed.

In fig. 9, fig. 10 and fig. 11 the behavior of the isothermal Vuilleumier machine with friction when the rotation speed changes is presented. It can be observed that when the rotation speed increases the terms $|L_f|$, $|L_{ext}|$ and $|Q_{it2}|$ increase too. The heat taken from the cold source Q_{lt} diminishes and L_{sd} and $|Q_{it1}|$ lessen slowly. The coefficients of performance of the heat pump and of the refrigerator also become smaller when the rotation speed increases.

n = 1000 rpm		Model	
		isothermal	with friction
Q_{ht}	[J/cycle]	322.7	322.7
Q_{it2}	[J/cycle]	-313.7	-477.9
Q_{it1}	[J/cycle]	-679.5	-669.9
Q_{lt}	[J/cycle]	707.8	563.9
L_{sd}	[J/cycle]	37.3	31.5
L_{ext}	[J/cycle]	37.3	-261.2
L_f	[J/cycle]	0.0	-292.7
ε_{hp}	-	3.078	1.966
ε_r	-	2.193	0.966

Table 1: Calculated results for n = 1000 rpm.

5. CONCLUSIONS

The adjustment of the theoretical models in order to take into account the pressure losses inside the heat exchangers - losses caused by friction - allow the emphasizing of some phenomena that appear when changing the rotation speed. From this point of view the model with decoupled losses is more advanced than the theoretical models (the rotation speed does not affect the functioning and the performances of a machine simulated with isothermal or adiabatic models).

A numerical example illustrates the functioning of the Vuilleumier machine accordingly to the proposed physico-mathematical model. The numerical simulation of the isothermal Vuilleumier machine with friction functioning led to the following conclusions:

- the pressure inside chambers it1, it2 and lt varies inside a smaller interval than in the case of the isothermal machine without friction; finally, this leads to a reduction of the COP's;
- $|L_f|$, $|Q_{it2}|$ and the term $|Q_{it1} + Q_{it2}|$ (the productivity of the heat pump) rise when the rotation speed rises;

- when the rotation speed rises the heat taken from the low temperature heat source Q_{lt} becomes smaller, and L_{sd} and $|Q_{it1}|$ lessen slowly;
- the work cyclically exchanged with the exterior L_{ext} becomes smaller when the rotation speed rises; a critical value of the rotation speed exists, for which this work is null;
- for rotation speeds below the critical one the machine produces work; for values above the critical rotation speed the machine spends work;
- the COP becomes smaller when the rotation speed rises.

In conclusion, the paper presents a new physico-mathematical model with decoupled energy losses for the Vuilleumier machine. The model appreciates the performance losses of the machine due to the friction losses.

References

[1] C.A. Homutescu, Gh. Savitescu, E. Jugureanu, V.M. Homutescu, *Introduction in Stirling Machines*, CERMI Printhouse, Iaşi, 2003 (in Romanian).

[2] V.M. Homutescu, E. Jugureanu, C.A. Homutescu, A. Homutescu, *Isothermal Model for Vuilleumier Machines*, First International Conference "Advanced Concepts in Mechanical Engineering", Iaşi, 2004, in Bul. I.P.I., Tom L (LIV), Fasc. 6C, p. 111 ... 116, ISSN 1011-2855.

[3] V.M. Homutescu, A. Homutescu, D.-T. Bălănescu, *Semi-Adiabatic Physico-Mathematical Model of the Vuilleumier Heat Pump*, Proceedings of the 5th International Conference on Electromechanical and Power Systems SIELMEN 2005, October 2005, Chişinău, Republic of Moldova, Vol. I, p. 510-513, ISBN GEN 973-716-208-0.

[4] V.M. Homutescu, E. Jugureanu, A. Homutescu, *Adiabatic Physico-Mathematical Model of the Vuilleumier Thermal-Acted Heat Pump*, The 2nd International Conference "Advanced Concepts in Mechanical Engineering", June 2006, in Bul. I.P.I., Tom LII (LVI), Fasc. 6C, p. 151 ... 156, ISSN 1011-2855.

[5] I. Urieli, *Stirling Cycle Machine Analysis*, http://www.ent.ohiou. edu/~urieli/index.html and http://www.sesusa.org/DrIz/index.html, excerpts of "Urieli I., Berchowitz D.M., Stirling Cycle Machine Analysis. Athens, Ohio, 1984".