AN EXAMPLE OF SPICE MODELLING FOR ELECTROTHERMAL PHENOMENA

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Abstract **– Starting from the necessity of the functioning analysis of an electrical resistor through which is circulating a current, the paper evaluates the temperature, electrical resistance and power's way of variation. To exemplify it has been chosen a resistor with big range of variation of the temperature between the in off regime and nominal functioning regime. The analysis is based on the analogy of thermal and electrical phenomena and it uses SPICE modelling.**

Keywords: *electrical heating, resistance, SPICE modeling;*

1. INTRODUCTION

It is well known the fact that a metal conductor, through heating, modifies its electric resistance. Depending on the used material and the temperature that can be touch during its functioning, the electric resistance can become few times bigger than that from the ambient temperature.
The paper is proposing to analyze the functionality of an electric resistor through which is circulating a current and which suffers an auto-heating process.
It has been decided the study of a resistor's behavior which, during the functionality, depending on the working regime, it covers a larger range of temperature. For this, it has been decided to study the behavior of a resistor that forms the filament of an electric lightbulb. Its temperature varies between 300K (corresponding to the ambient temperature) when the lightbulb is turned off, and over 2200K in normal functioning regime.

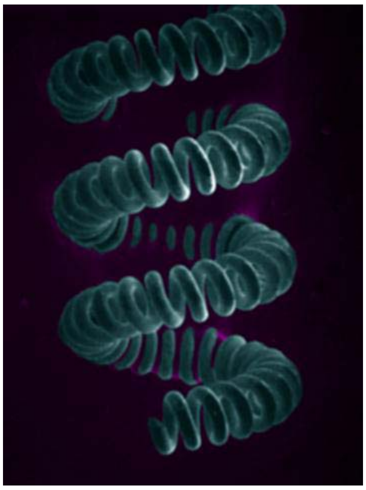

Figure 1: The filament

For study, we consider a lightbulb projected to develop a power of 100W when it is powered at the network voltage (220V / 50Hz). Its filament is made of a Wolfram wire being almost 2,5m long and having the diameter of 0.03mm. The filament of this kind of lightbulb is double spiral (like in fig.1).
For the considered lightbulb, in the beginning, the wolfram wire is reeled around a punch with the diameter of 0.12mm, and after that, the obtained coil is reeled around a punch with the diameter of 1mm. Ensuring for the two coils a reeling step almost equal with the double of the respective reeling wire width, it results a useful length of the filament of the electric lightbulb, obtained in this way, of about 38÷40mm.

2. MATHEMATICAL DESCRIPTION

The functionality of the lightbulb is characterized by the following set of equations:

$$i_{bulb} = \frac{u_{bulb}}{R_{bulb}(T)} \tag{1}$$

$$P_{bulb} = u_{bulb} \cdot i_{bulb} \tag{2}$$

$$P_{bulb} = P_{rad} + P_{stored} \tag{3}$$

where:

- P_{bulb} represents the power dissipation of a bulb
- P_{rad} represents the power radiation of a bulb under the form of light and temperature
- P_{stored} represents the power stored in the lightbulb to heat the filament

$$P_{rad} = C_1 \cdot \left(T^4 - T_{amb}^4\right) = \varepsilon_W \cdot c_n \cdot S \cdot \left(T^4 - T_{amb}^4\right) \tag{4}$$

$$P_{stored} = \frac{\mathbf{d}}{\mathbf{d}\,t}\left(W_{stored}\right) = C_T \cdot \frac{\mathbf{d}\,T}{\mathbf{d}\,t} \tag{5}$$

$$R_{bulb}(T) = \left(1 + \alpha_W \left(T - T_{amb}\right)\right) \cdot R_0 \tag{6}$$

where:

- $C_T = m \cdot c_{TW} = \dfrac{\pi \cdot d^2}{4} \cdot l \cdot \gamma_W \cdot c_{TW}$ represents the thermal capacitance of the filament

- $R_{bulb}(T)$ represents thermal resistance dependence.

In the six equations that describe the phenomena that appear at the functioning of the lightbulb, are used the following physical constants:

- $\varepsilon_W \cong 0{,}3$ represents the degree of blackness of the filament around the functioning temperature
- $c_n = 5{,}67 \cdot 10^{-8}$ W/m^2•K^4 represents radiation constant of the black corp. (Ştefan - Boltzmann)
- $S = \pi \cdot d \cdot l = \pi \cdot 0{,}03 \cdot 10^{-3} \cdot 2{,}5 = 235{,}6 \cdot 10^{-6}$ m^2 represents the radiant surface of the wolfram wire from which is made the filament

Taking into account these data we can determine the value of the constant from the equation (4):

- $C_1 = \varepsilon_W \cdot c_n \cdot S \cong 4 \cdot 10^{-12}$ W/K^4
- $\gamma_W = 19{,}3 \cdot 10^3$ kg/m^3 - Wolfram' density
- $c_{TW} = 142$ J/kg•K - specific thermal capacitance of Wolfram

It results for the filament:

- $C_T = \frac{\pi \cdot d^2}{4} \cdot l \cdot \gamma_W \cdot c_{TW} = 4{,}84 \cdot 10^{-3}$ J/K
- $\alpha_W = 0{,}001$ K^{-1} - the coefficient of thermal variation of the Wolfram's resistance
- $T_{amb} = 300$ K - considered ambient temperature (the light bulb is turned off)
- $R_0 = \rho_W \cdot \frac{4 \cdot l}{\pi \cdot d^2} = 0{,}0528 \cdot \frac{4 \cdot 2{,}5}{\pi \cdot 0{,}03^2} \cong 187\ \Omega$ – filament's resistance at ambient temperature.

3. THE TRANSITORY REGIME MODELLING

The analysis of the six equations that describe the functionality of the bulb shows the fact that it can be realized a modelling using the block scheme from Fig.2.

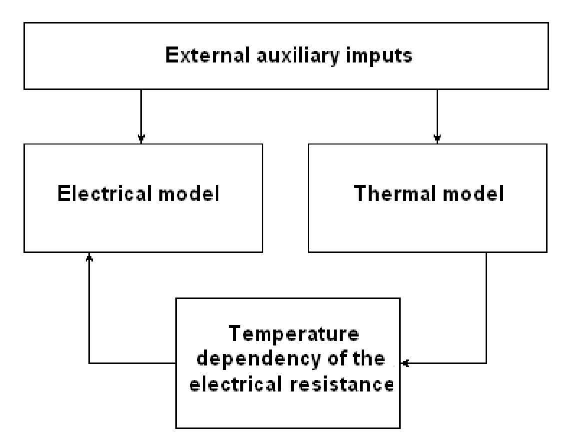

Figure 2: The block scheme

It is made an analysis based on SPICE modelling, based on the possibility of making formal analogies between different equations. The functional block that allows the modelling of auxiliary external inputs that intervene in the equations is presented in Fig.3.

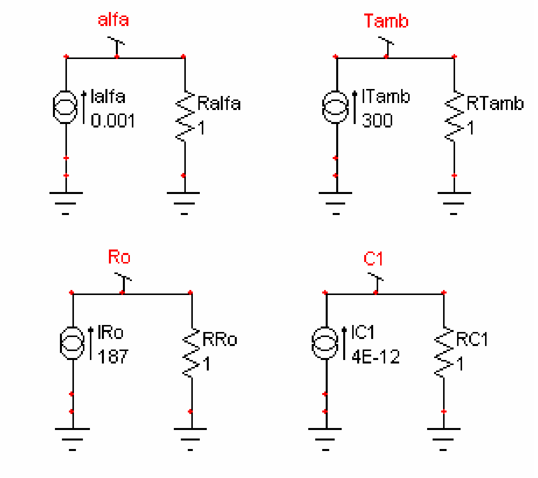

Figure 3: External inputs

External inputs are simulated through electrical circuits each of them being formed from a voltage source that discharge itself on a resistance with unitary value. In this way, in the corresponding node, it is obtained a voltage equal with the value of the current injected by the current source.

The dependence of the resistance varying with the temperature (equation (6)) will be modelled with the help of an unlinear current source (called „*Resistance*"), defined through the equation:

$$I = (1 + V(alfa) \cdot (i(Vtemp) - V(Tamb))) \cdot V(R_0) \quad (7)$$

Where $i(Vtemp)$ represents the modelling of the temperature through the mediation of the current that crosses the continuous voltage source *Vtemp* with the value „0V".(Fig.4)

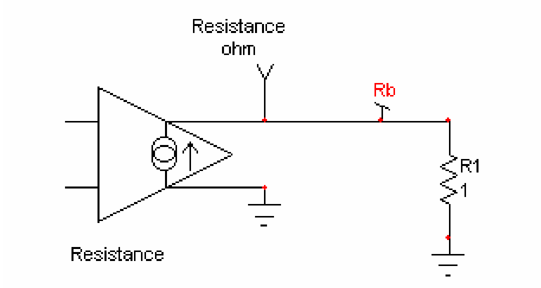

Figure 4: "Resistance"

The electric model portion (Fig.5) can be realized using a null voltage source, „*Vcurrent* =0V", that measures the current through the lightbulb's filament i_{bulb}. The unlinear current source („*Ohm*") models Ohm's law (described in the equation (1)) and it is defined by the equation:

$$I = (V(plus) - V(minus)) / V(Rb) \quad (8)$$

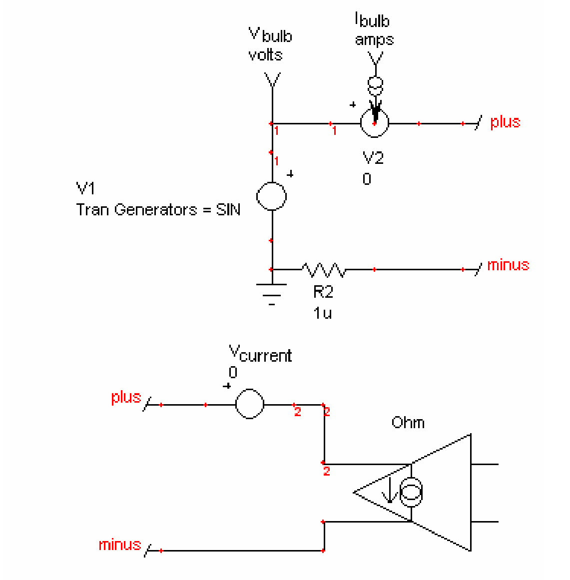

Figure 5: The electric model

The subcircuit from Fig.6 realizes the thermal model of the electric lightbulb. The electrical power transformed into heat, „P_{bulb}", defined through the equation (2), is simulated with the help of an unlinear voltage source, called *„Pconsumed"*, and it is defined by the equation:

$$V = (V(plus) - V(minus)) \cdot i(Vcurrent) \qquad (9)$$

The power radiation of the bulb, „P_{rad}", is expressed through the equation (4) and it is modelled with the unlinear voltage source *„Pradiation"*, with the equation that defines it:

$$V = V(C1) \cdot (pwrs(i(Vtemp),4) - pwrs(V(Tamb),4)) \quad (10)$$

The heat storage is modelled with a coil, called „L_{CT}", with the inductance equal with the caloric capacitance of the filament of the bulb. The modelling with the help of the coil is based on the fact that the equation with powers resulted from the combination of the equations (3)and (5):

$$P_{bulb} - P_{rad} = C_T \cdot \frac{\mathrm{d}T}{\mathrm{d}t} \qquad (11)$$

can be simulated with an equation with voltages, in the form of:

$$Pconsumed - Pradiation = L_{CT} \cdot \frac{\mathrm{d}i_{L_{CT}}}{\mathrm{d}t} \qquad (12)$$

The current through the coil is measured with the null voltage source *Vtemp* and is equal in value (taking into account the analogy presented before) with the absolute temperature at which the filament is at a certain moment.

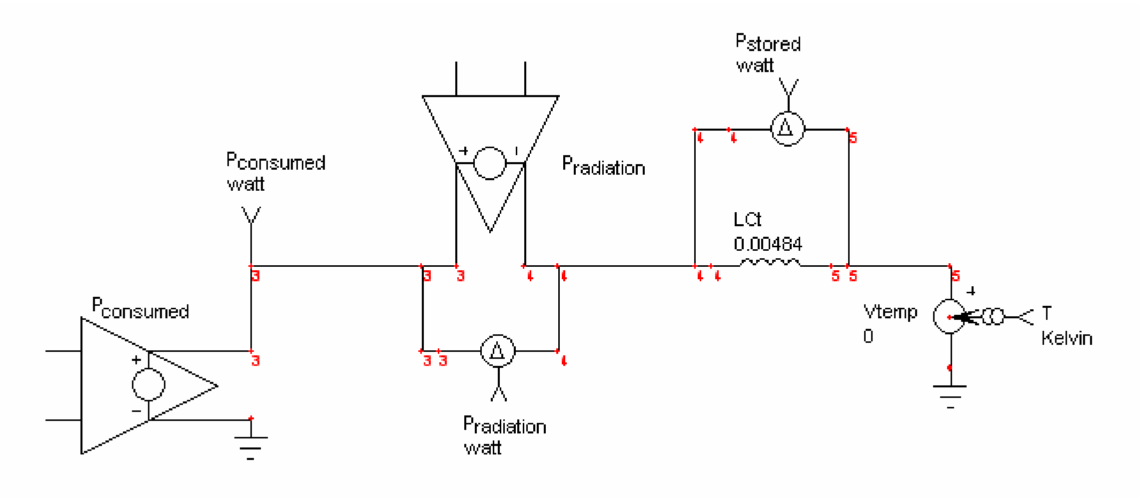

Figure 6: The thermal model

4. CONCLUSIONS

The SPICE model resulted after the union of the previously presented submodels had permitted the simulation of the electrical lightbulb's functionality. We considered it to be supplied from an unsinusoidal source with the effective value of the voltage of 220V and the frequency of 50 Hz. The synthetic results of the simulations are presented in Fig. 7, 8 and 9.

The analysis of the graphic representations from Fig. 7÷9 allows us to present the following conclusions:

- While the filament's temperature grows from 300K to a value almost of 2230K, the electric resistance is almost tripling, touching the medium value of 543Ω;
- The consumed powers, power radiation and power stored of the bulb, in normal functioning

regime, presents sinusoidal evolutions with double frequency comparing with that of the power supply. As a result of this fact, the light radiation emitted by the electrical lightbulb presents intensity oscillations with the frequency of 100Hz;

- The power radiation has small oscillations around the value of 95W, while the stored power oscillates around the zero value, to maintain the functional temperature of the lightbulb.

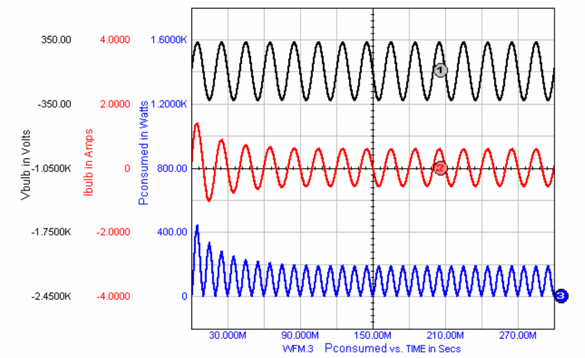

Figure 7: V_{bulb}, I_{bulb}, $P_{consumed}$

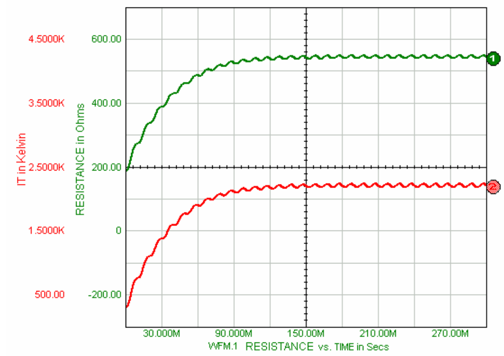

Figure 8: *Temperature, Resistance*

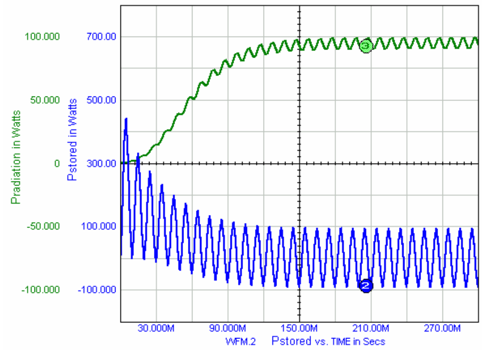

Figure 9: $P_{radiation}$, P_{stored}

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