INITIAL MAGNETIC FIELD DISTRIBUTION AROUND RECTANGULAR BUSBARS

Grigore A. CIVIDJIAN

University of Craiova , gcividjian@elth.ucv.ro

Abstract – **The stationary magnetic field in and around the bus bars can be easy calculated using the Biot-Savart- Laplace law. The transient magnetic field depends essentially on the initial magnetic field on the bar surface. In the case of very thin high bars, this field can be determined analytically. In the paper, the initial magnetic field on the surface of arbitrary cross-section rectangular bar is studied. Using FEM and numerical conformal mapping the initial magnetic field around the bar is determined for a current step and simple formulas for ratio of fields on the two sides of the bar are proposed.**

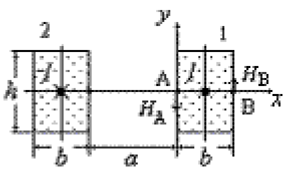
Keywords: *magnetic field, finite element method (FEM), numerical conformal mapping, Schwartz-Christoffel toolbox transient and stationary field.*

1. INTRODUCTION

In the case of uniform distributed direct current, the internal and external magnetic field can be evaluated using classical formulas and the ratio of the magnetic fields in middle of the two sides of the bar is given in [1]. In the case of current step, the initial magnetic field is only external and his vector potential satisfy the Laplace equation:

$$\Delta A = 0\,; \quad \left.\frac{\partial A}{\partial y}\right|_{x=-\frac{a}{2}} = \left.\frac{\partial A}{\partial y}\right|_{y=0} = 0 \qquad (1)$$

This means that the magnetic field can be considered static in the domain out of conductor.
The studied configuration is shown in fig. 1.



$$\eta = \left|\frac{H_B}{H_A}\right|_{t=0}$$

$$\eta_{av} = \left|\frac{H_{Bav}}{H_{Aav}}\right|_{t=0}$$

Fig. 1: Set of two rectangular bars

Also in [1] the transient magnetic field in conductor is theoretically solved in function of the ratio η of the magnetic fields on the two sides of the bar. In [2] this ratio is theoretically calculated, using the conformal transform of the domain from fig. 2 on the upper half plane, for the case of very thin high bars ($b/a \to 0$).

In this paper, for obtained in [2] exact formulas simpler approximations are proposed and the case $b \neq 0$ is analyzed, using the FEMM package for finite element analysis and Schwartz-Christoffel toolbox for numerical conformal mapping from MATLAB. The ratio η and η_{av} (of average magnetic fields) are evaluated. For the case of $b < h$ a new simple formula for η is proposed.

2. MAGNETIC FIELD PROBLEM

The magnetic field around the bars was solved using the FEMM finite element package, applied to the domain shown in fig. 2 and the results and problem data in fig. 3 and Tab. 1.

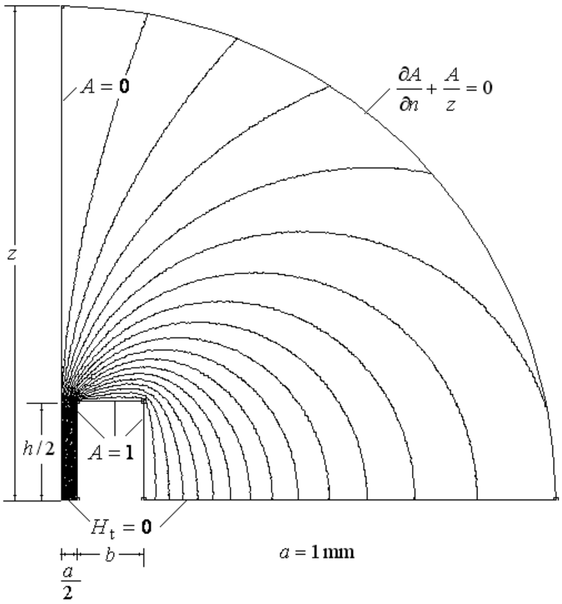


Fig. 2: Domain configuration and magnetic field pattern

b/a	h/a	z	Size	$\|H_A\|$	$\|H_B\|$	H_{Aav}	$\|H_{Bav}\|$
2	6	15	0.05	159	10.6	162	15.34
H_{bav}	H_{av}	I	η_{av}	η_{bav}	η	u_{0A}	u_{0B}
54.8	80.2	1.28	0.095	0.338	0.067	0.98	0.69

Tab. 1: Quantities for Fig. 3, $u_{0A} = \frac{H_A}{H_{Aav}}$; $u_{0B} = \frac{H_B}{H_{Bav}}$

The same field problem was solved using the Schwartz-Christoffel toolbox from MATLAB. The initial magnetic field pattern and the ratio of tangential magnetic field distribution along the bar perimeter are shown in fig. 4.

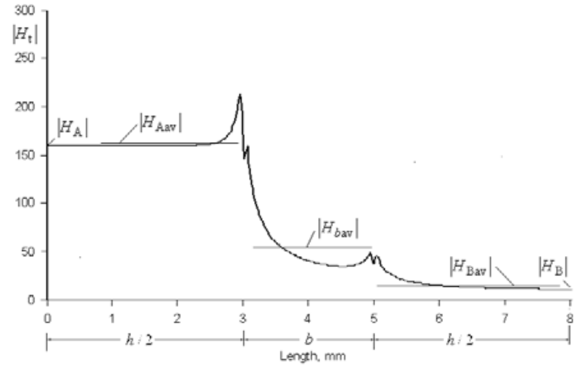


Fig. 3: Initial tangential magnetic field along bar perimeter

Such determined values of η for several ratio *b/a* are given in fig. 4 and 5.

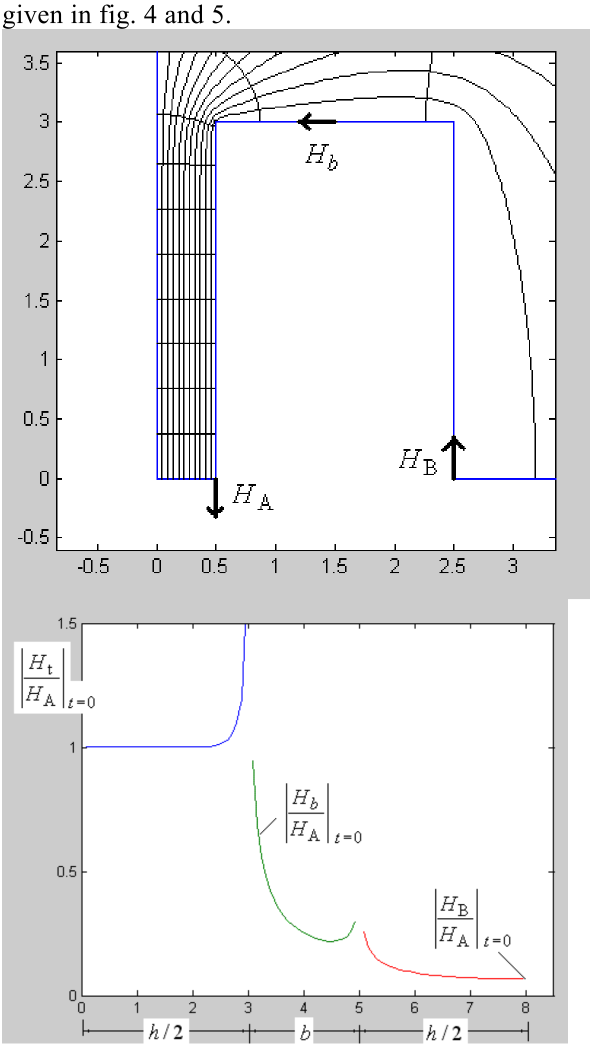


Fig. 4: Field lines and tangential field around bar (MATLAB)

3. RATIO OF INITIAL MAGNETIC FIELDS ON THE TWO SIDES OF THE BAR

For very thin bars, when $b/a \to 0$, by conformal mapping, an exact formula for the ratio η, of initial magnetic fields in the middle of the two opposite sides of the thin bar, was obtained [2]. It can be written as follows:

$$\eta(m) = \left|\frac{H_B}{H_A}\right|_{t=0} = \frac{E' - m^2 K'}{K' - E'}\,;\quad m' = \sqrt{1-m^2}$$

$$\frac{h}{a} = \frac{2}{\pi}\left[K' E\left(\frac{\sqrt{1-\frac{E'}{K'}}}{m'}, m'\right) - E' F\left(\frac{\sqrt{1-\frac{E'}{K'}}}{m'}, m'\right)\right] \tag{2}$$

Here *K'* and *E'* are complementary complete and *F, E* are incomplete elliptic integrals of first and second kind and modulus *m* (co-modulus *m'*).
For $h > 2a$ the last equation can be replaced with simpler one:

$$m \approx \exp\left(-\frac{h/a + 0.25}{0.6}\right);\quad \frac{h}{a} > 2 \tag{3}$$

The eq. (2) is difficult to applied, even using the approximation (3). Therefore the approximation (4) is proposed, which how it can be seen in fig. 5, agree enough well with the exact values given by (2).
The values of η obtained with both methods are in good agreement and are plotted in fig. 5 and 6.

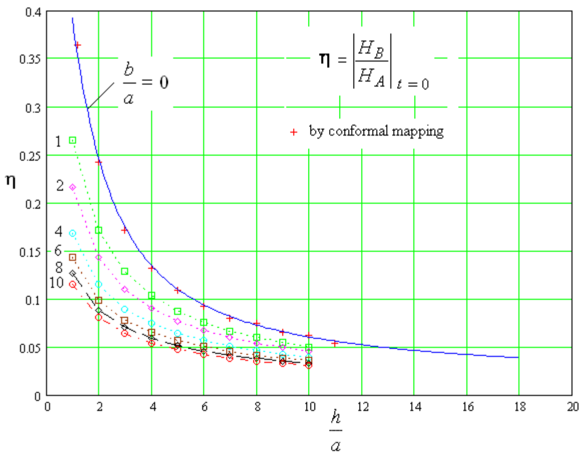


Fig. 5: η versus *h/a, b/a* - parameter

In fig. 6 it can be seen that for $b < h$ the ratio η of the magnetic fields in the middle height of the two sides of the bar depends only on the dimensionless parameter $x = \frac{h+b}{a}$. This is exactly the parameter used by Dwight for the determination of electro-dynamic forces between rectangular bars in direct current.

$$\eta(x) = \frac{\frac{\pi}{2} - \arctan(x)}{\frac{\pi}{2} + \arctan(x)}(0.7 + 0.02\,x) \tag{4}$$

$$x = \frac{h+b}{a}; \quad b < h$$

Thus, the proposed formula (4) has a much larger application, being valid also for $b \neq 0$, if $b < h$.

3.1. Ratio of initial average magnetic fields

The ratio of initial average magnetic field on the two sides of thin bar for $b/h \to 0$ is given by following exact formula [2]:

$$\eta_{\text{av}}(m) = \left| \frac{i\,K'(m)}{F(\lambda, m) - K(m)} \right| - 1; \quad \lambda = \frac{1}{m}\sqrt{\frac{E'}{K'}} \tag{5}$$

and can be enough well evaluated with the simple formula:

$$\eta_{\text{av}} \approx \frac{\frac{\pi}{2} - \arctan\frac{h}{2a}}{\frac{\pi}{2} + \arctan\frac{h}{2a}} \times \left[0.86 + 0.18\frac{h}{a} + 0.007\left(\frac{h}{a}\right)^2\right] \tag{6}$$

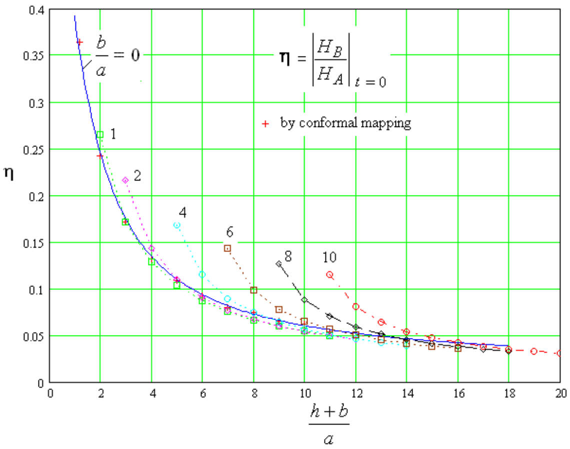


Fig. 6: η versus (*h*+*b*)/*a*, *b*/*a* - parameter

In fig. 7 can be seen that the agreement of these two formulas is enough good, at least for 1 < *h*/*a* <11. In same figure are plotted the values of η_{av} determined by numerical conformal mapping for 1< *b*/*a* <10. All of them are smaller than the corresponding values for $b/a \to 0$ and become much closer if plotted versus the Dwight parameter *x* (fig. 8).

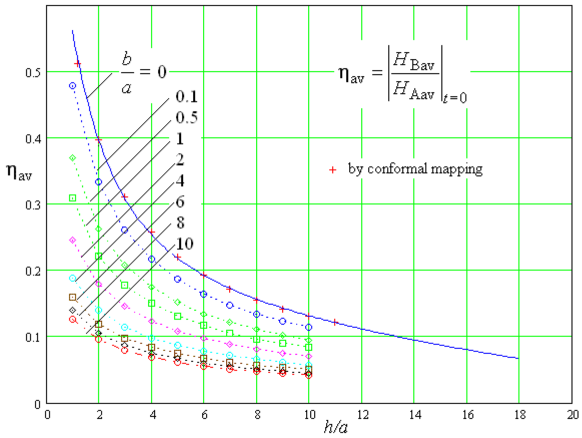


Fig. 7: η_{av} versus *h*/*a*, *b*/*a* – parameter (*t* = 0)

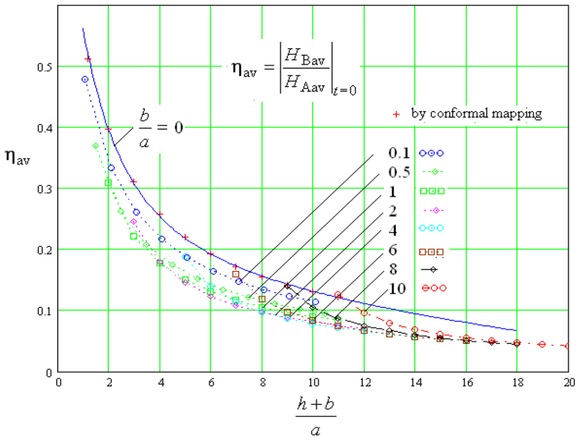


Fig. 8: η_{av} versus (*h*+*b*)/*a*, *b*/*a* - parameter

3.2. Initial magnetic field in the middle of topside

We will define the ratio of the magnetic field in the middle of *b*-length topside of the bar to the magnetic field in point A as:

$$\eta_b = \left| \frac{H_b}{H_\text{A}} \right|_{t=0} \tag{7}$$

The obtained using rectmap function from sc-toolbox values are independent on *h*/*a*. They are given in Tab. 3 and plotted in fig. 9.

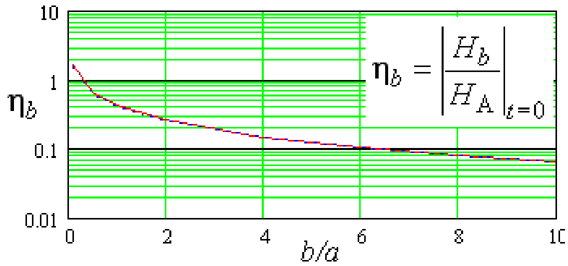


Fig. 9: η_b versus *b*/*a*

3.3. Average magnetic field on *b*-length topside

The ratio η_{bav} is almost independent on *h/a* and can be approximated with following formula:

$$\eta_{bav} \approx \frac{\pi - 4\arctan\dfrac{b}{2a+b}}{\pi + 4\arctan\dfrac{b}{2a+3b}} \times \left[1 + \frac{h}{100a} + \frac{b}{a}\left(0.03 + \frac{h}{2000a}\right)\right] \quad (8)$$

In fig. 10 the values of η_{bav} are plotted for h/a = 1 and 10 and it can be observed they are enough close to each other (see also Tab. 5).

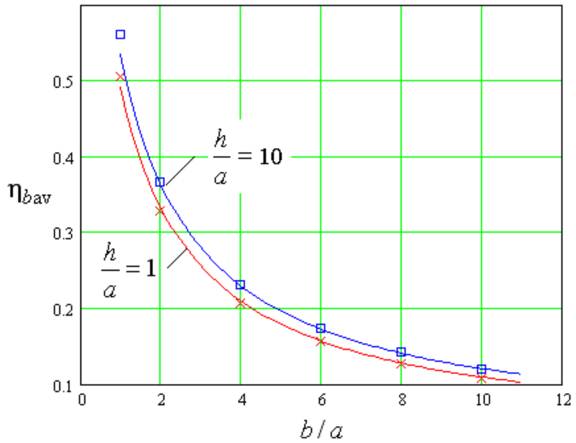


Fig. 10: η_{bav} versus *b/a, h/a* – parameter (8)

Using these ratios, the average initial magnetic field between the bars can be evaluated as follows:

$$H_{\text{Aav}} = \frac{I}{h(\eta_{\text{av}} + 1) + 2b\,\eta_{bav}} \quad (9)$$

I is the magnitude of the current step in the bar.

4. CONCLUSIONS

For $h > b$ the ratio η of magnetic fields in the middle of the two sides of the bar can be evaluated with simple formula (4). Using (6), (8), (9), fig. 7 or 8 H_{Aav} can be determined. Considering $H_{\text{A}} \approx H_{\text{Aav}}$ and using fig. 5, 6, 9 and (4) the initial magnetic fields H_{B} and H_b can be determined.

References

[1] G. A. Cividjian, *Current distribution in rectangular busbars,* Revue Roumaine des Sciences Techniques, Série Electrotechnique et Energetique, t. 48, no. 2-3, pp. 313-320, Bucarest, 2003.

[2] G. A. Cividjian, *Initial magnetic field distribution around high rectangular busbars,* Simp. Național de Electrotehnică Teoretică SNET '07, Bucharest, 2007 (unpublished).

Annex: Numerically determined field ratios

*h**b*	1	2	4	6	8	10
1	0.265	0.216	0.169	0.143	0.127	0.115
2	0.171	0.144	0.115	0.099	0.088	0.081
3	0.128	0.11	0.09	0.078	0.07	0.064
4	0.103	0.09	0.074	0.065	0.059	0.054
5	0.087	0.077	0.064	0.057	0.051	0.047
6	0.075	0.067	0.057	0.05	0.046	0.042
7	0.067	0.06	0.051	0.045	0.041	0.038
8	0.06	0.054	0.046	0.041	0.038	0.035
9	0.054	0.049	0.042	0.038	0.035	0.033
10	0.05	0.045	0.039	0.035	0.033	0.03

Tab. 2: Values of η

*h**b*	1	2	4	6	8	10
1	0.422	0.256	0.147	0.104	0.08	0.065
2	0.423	0.258	0.149	0.105	0.081	0.066
3	0.419	0.256	0.147	0.104	0.081	0.066
4	0.416	0.254	0.147	0.104	0.08	0.066
5	0.414	0.253	0.146	0.103	0.08	0.066
6	0.413	0.252	0.145	0.103	0.08	0.065
7	0.411	0.251	0.145	0.103	0.08	0.065
8	0.411	0.251	0.145	0.102	0.079	0.065
9	0.41	0.25	0.144	0.102	0.079	0.065
10	0.409	0.25	0.144	0.102	0.079	0.065

Tab. 3: Values of η_b

*h**b*	1	2	4	6	8	10
1	0.309	0.246	0.189	0.159	0.14	0.127
2	0.222	0.18	0.14	0.119	0.105	0.096
3	0.178	0.145	0.114	0.098	0.087	0.079
4	0.15	0.124	0.098	0.084	0.075	0.068
5	0.131	0.108	0.086	0.074	0.067	0.061
6	0.117	0.097	0.078	0.067	0.06	0.055
7	0.106	0.088	0.071	0.062	0.055	0.051
8	0.097	0.081	0.066	0.057	0.051	0.047
9	0.089	0.075	0.061	0.053	0.048	0.044
10	0.083	0.07	0.057	0.05	0.045	0.041

Tab. 4: Values of η_{av}

*h**b*	1	2	4	6	8	10
1	0.505	0.329	0.207	0.156	0.126	0.107
2	0.539	0.352	0.222	0.167	0.136	0.115
3	0.55	0.359	0.227	0.171	0.139	0.118
4	0.554	0.362	0.229	0.173	0.14	0.119
5	0.557	0.363	0.23	0.173	0.141	0.12
6	0.558	0.364	0.231	0.174	0.141	0.12
7	0.559	0.365	0.231	0.174	0.142	0.12
8	0.56	0.365	0.231	0.174	0.142	0.12
9	0.56	0.366	0.231	0.174	0.142	0.121
10	0.561	0.366	0.231	0.174	0.142	0.121

Tab. 5: Values of η_{bav}