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METODS OF MODELING FOR PHOTOVOLTAIC CELLS

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***Abstract* – This paper illustrates brass tacks of direct conversion solar energy in power energy, through photovoltaic cells, conversion based on photovoltaic effect. The theoretical foundations is presented and also the construction and working of the photovoltaic cells. Modeling method of photovoltaic cells is indicating in this paper too, and some experimental simulating was shown with Matlab application program.**

Keywords: *modeling, photovoltaic cells, solar radiation.*

1. INTRODUCTION

It is known that solar energy can be direct conversion in electrical energy by solid material semiconductor used *photovoltaic effect*. Photovoltaic generator, photovoltaic cells called, in opposition with electromechanical generator, productions d.c. electrical energy. Photovoltaic cells not environment pollution and can be used everywhere, even irradiate light [1].

Out of technological circuit and intermediately transformations, no motions, no vibrations and noise, modular building, lifetime over 25 years, we can tell that the future of energetic will belong to photovoltaic technology.

The discovery of photovoltaic effect belongs to franc physicist Edmond Becquerel, which in 1839, whit “wet battery ” experiment observed that the electrical power generate of battery increase if the plane of silver is sunny.

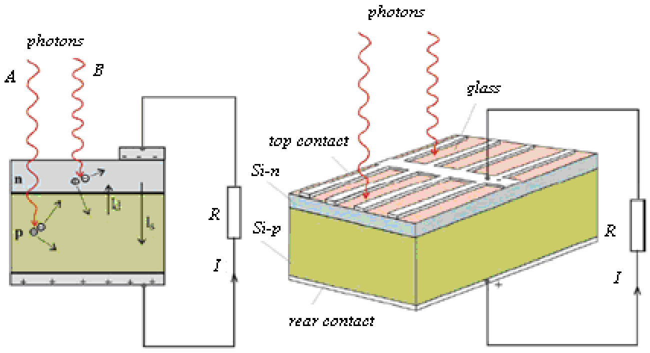
2. CONSTRUCTION AND WORKING OF PHOTOVOLTAIC CELL

Photovoltaic cell is an electronic device, working by reason of minority charge carriers. As initially material for fabrications is used semiconductor, usually crystalline silicon or polycrystalline, on it is surface by sundries technological method emergent overlay, how contain impurity for obtained junction pn. Figure 1 shows the constructive scheme adapted of PV cell, based on semiconductor material by type p [2].

Analyses of phenomenon, if the cell PV is incidence sunny (see figure 1) it is determined that this radiation can be equivalent whit a flux of photons energetically:

$$W = f \cdot \nu \cdot h \tag{1}$$

where, h is Planck’ constant, and ν is radiation frequency. If photon’ energy is enough large, then collision between photon and atom, the bonding electron bring into conduction electron, this electron get free, and generate a gol into crystal’ net. Therefore, action of photons generates couple electron-gol. This effect is called *inside photovoltaic effect*. In figure 1 on left side, photon A is low frequency and low energy too, photon B is high frequency and also a high energy (electromagnetic wave with small frequency get into material more deep).



a) b)

Figure 1. The construction scheme of photovoltaic cell

Electrical field of junction p-n have a direct action about new charge carrier, that field is having a potential barrier U_0 wich is dependence of semiconductor type used as long as 0,2 - 0,7 V. Spacial charge of junction will have a determinate role in division free charges – cuple electron-gol. The electrons will be directional to **n** area, the gols to **p** area of cell. This is the reason whence under light influence **p** area is pozitive loaded, while **n** area is negative loaded, that is appear electrical current through junction, determined by photovoltaic convertion of solar radiation (sunny). This circulation of current is from **n** area to **p** area into junction (figure 1 left side) that produce a drop voltage U on external charge R, conected to back connecting terminal and contact-grille on surface (figure 1 right side). Voltage U with junction is direct sense and will produce through junction current of diode I_d opposed by photovoltaic current I_s.

3. PHOTOVOLTAIC CELL MODEL METHODS

A detailed approach to PV cell module or array modeling based on a mathematical description of the equivalent electrical circuit of a PV cell is given in [3] and [7]. Three models are used to describe the equivalent electrical circuit of a PV cell module or array: the one-diode, the two-diode, and the empirical model. The most commonly used configuration is the one-diode model that represents the electrical behavior of the *pn*-junction. The two-diode model allows for a more detailed description of the recombination process of charge carriers both on the surface and in the bulk material. The empirical model is a good fit for the measured I-U curve and has a less number of parameters than in the other two models. The parameters of this model (P_{max}, I_{sc}, U_{oc}, etc) are usually given in the manufacturer's data sheet, which allows modeling with an acceptable accuracy.

3.1. One-Diode Model

The photovoltaic cell is represented as an equivalent circuit containing a current generator (modeling the conversion of solar radiation to electric energy), a diode (accounting for the physical properties of the semiconductor cells) and two resistances, shunt and series resistances. The characteristic equation of the PV cell model $I_s=f(U_s, I_s)$ is obtained by applying Kirchoff's current law to the equivalent circuit Figure 2, where I_s [A] and U_s [V] are the terminal current and voltage of the model respectively:

$$I_s = I_{ph} - I_d \; I_{sh} \tag{1}$$

Where:

I_s –terminal current of cell
U_s - voltage of output
I_{ph} – photocurrent of cell
I_d – diode loss current due to charge carrier recombination.
I_{sh} - diode loss current due to shunt resistance.

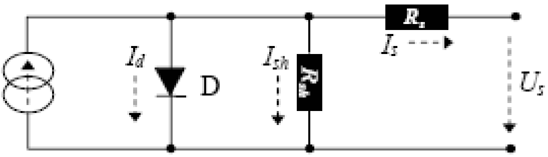


Figure 2. Equivalent circuit of the PV cell one-diode model.

The photocurrent I_{ph} [A] is directly dependent on the solar radiation E_s and the ambient temperature T_a and is modeled by:

$$I_{ph} = P_1 E_s \left[1 + P_2 (E_s - E_0) + P_3 (T_j - T_0)\right] \tag{2}$$

Where E_0 =1000W/m² and T_0 =298.15K correspond to a reference solar radiation and a reference ambient temperature, respectively. P_1 [Am²/W], P_2 [m²/W] and P_3 [1/K] are constant parameters
The I-U characteristic of a PV cell is also influenced by the temperature of the cell. The cell temperature is a simple linear function of the cell junction temperature T_j [K] and the global solar radiation E_s [W/m²]. Equation 3 below describes the junction temperature, where the ambient temperature T_a [°C] determines the crossing point of the function on the vertical axis:

$$T_j = (T_a + 273.15) + \frac{E_s}{800W/m^2}(T_{op} - 20) \tag{3}$$

Where, *NOCT* is a parameter and called "*Normal Operating Cell Temperature*". It is given by the PV cell module manufacturer, mostly between 45 and 49°C.
The diode loss current I_d [A] due to charge carrier recombination is given by:

$$I_d = I_{sat}\left[exp\left(\frac{e_0}{\alpha f N_s k} \cdot \frac{U_s + R_s I_s}{T_J}\right) - 1\right] \tag{4}$$

Where

$I_{sat} = P_4 T_j^3 \, exp\left(-\frac{E_g}{kT_j}\right)$ -saturation current, [A]

e_0 - electron charge, [C]
a_f- *ideality* factor of the photovoltaic array, [-]
N_s- number of cells in series, [-]
K- Boltzmann's constant, [J/K]
R_s- series resistance, [Ω]
E_g - gap energy, [eV] and
P_4- correction parameter, [A/K³]
Finally, the shunt current I_{sh} [A] is calculated from:

$$I_{sh} = \frac{U_s + R_s I_s}{R_{sh}} \tag{5}$$

Where R_{sh} [Ω] is shunt resistance.

The four variables generating this model are the two input variables, solar radiation E_s [W/m²] and ambient temperature T_a [°C], as well as the two output terminal variables, PV cell current I_s [A] and voltage U_s [V], as shown in figure 3.

Figure 3. Block diagram of the PV cell with input/output variables.

The relations between input and output variables are:

$$I_s = P_1E_s\left[1+P_2\left(E_s-E_0\right)+P_3\left(T_j-T_0\right)\right]- \\ -P_4T_j^3\,exp\left(-\frac{E_g}{kT_j}\right)\left[exp\left(\frac{eo}{\alpha_f N_s k}\cdot\frac{U_s+R_sI_s}{T_j}\right)-1\right]-\frac{U_s+R_sI_s}{T_j} \quad (6)$$

The set of parameters (P_1, P_2, P_3, P_4, R_s and R_{sh}) can be obtained from the modules manufacturer's data sheet. Table 1 gives these parameters for the *photo watt BPX 47-451A 45Wp Si* PV module [7].

Parameter	P_1	P_2	P_3	P_4	Rs	Rsh
Value	2.96	-8.6E-4	0.0037	1272.3	1.29	154.1

Table 1. Parameters of the one-diode model of the *photo watt BPX 47-451A* PV module

3.2 Two-Diode Model

The two-diode model, is derived from the same equivalent circuit of the one-diode model, with the main difference that the recombination current *Id* is replaced by two currents I_{d1} and I_{d2}. Recombination of minority carriers, both on the surface and in the bulk material, is the major determinant of the open-circuit voltage, occurring readily at trapping levels of the depletion zone. When modeling the recombination phenomena, the first diode is associated with neutral (base and emitter) regions, whereas the second diode simulates the space-charge recombination effect by incorporating a separate current component I_{d2} with its own exponential voltage dependence [3]. The characteristic equation of this model is obtained by the same manner as for the one-diode model. Figure 4 shows the equivalent circuit of the two-diode model. The terminal current I_s [A] is given by:

$$I_s = I_{ph} - \left(I_{d1} + I_{d2}\right) - I_{sh} \quad (7)$$

Where

$$I_{d1} + I_{d2} = I_{sat1}\left[\exp\left(\frac{e_0}{\alpha\cdot\alpha f N_s k}\cdot\frac{U_s+R_sI_s}{T_J}\right)-1\right]+ \\ I_{sat2}\left[\exp\left(\frac{e_0}{\beta\cdot\alpha f N_s k}\cdot\frac{U_s+R_sI_s}{T_J}\right)-1\right] \quad (8)$$

α and β are fit parameters that are set to 1 and 2 respectively in the two-diode model.

The dependence of the saturation currents on temperature is given by:

$$I_{sat1} = P_{01}T_j^3\,exp\left(-\frac{E_g}{kT_j}\right) \quad (9)$$

$$I_{sat2} = P_{02}T_j^{\frac{5}{2}}\,exp\left(-\frac{E_g}{2kT_j}\right) \quad (10)$$

Where P_{01} [A/K³] and P_{02} [A/K$^{5/2}$] are constant parameters.

Furthermore, the photocurrent I_{ph} [A] is proportional to the solar radiation E_s and is assumed to be linearly dependent on the cell temperature T_j:

$$I_{ph} = (P_1 + P_2T_j) \times E_s \quad (11)$$

Where P_1 [Am²/W] and P_2 [Am²/WK] are constant parameters.

The shunt-current I_{sh} is expressed in the same way as for the one-diode model.

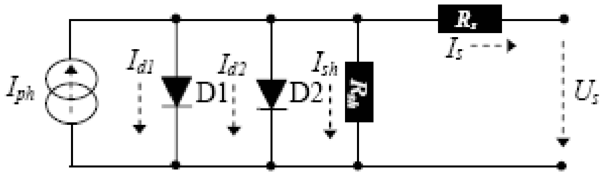


Figure 4. Equivalent circuit of the PV cell two-diode model

The relationship between the voltage U_s [V] of a PV cell
and the current I_s [A] is given by the two-diode model as:

$$I_s = P_1E_s\,[1+P_2\,(E_s-E_0)+P_3\,(T_j-T_0)]\,P_{01}T_j^3\,exp\left(-\frac{E_g}{kT_j}\right) \\ \left[exp\left(\frac{e_0}{\alpha\cdot\alpha f N_s k}\cdot\frac{U_s+R_sI_s}{T_J}\right)-1\right]+ \\ P_{02}T_j^{\frac{5}{2}}\,exp\left(-\frac{E_g}{2kT_j}\right)\left[exp\left(\frac{e_0}{\beta\cdot\alpha f N_s k}\cdot\frac{U_s+R_sI_s}{T_J}\right)-1\right]-\frac{U_s+R_sI_s}{R_{sh}} \quad (12)$$

Where

P_1, P_2, P_{01}, and P_{02} constant parameters, [A.m²/W, A.m²/W.K, A/K³ and A/K$^{5/2}$]

α and β fit diode parameters, equal 1 and 2 respectively.

The other parameters in equation 12 are the same as for the one-diode model. The set of parameters (P_1,

P_2, P_{01}, P_{02}, R_s and R_{sh}) can be obtained from the modules manufacturer's data sheet. Table 2 gives these parameters for the PV module SM50 [7].

Parameter	P_1	P_2	P_{01}	P_{02}	R_s	R_{sh}
Value	0.306	0.179E-4	1.708E-4	1.880	1.381E-4	0.13

Table 2. Parameters of the two-diode model of the photovoltaic module SM50

3.3 Empirical Model

Many parameters are used within the one-diode and two-diode models. Some of them have known values and others are physical constants

This model describes the behavior of a PV cell via the equivalent electrical circuit shown in figure 5, which consists of a current source I_{ph}, a parallel-connected diode D and a series resistor R_s.

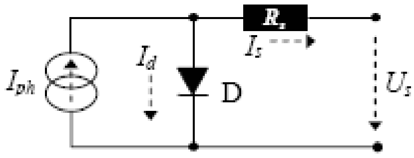


Figure 5. Equivalent circuit of the PV cell empirical model.

The main advantage of this model is the limited number of parameters, which can be found in manufacturer's data sheet. The equation describing the I-U curve of the PV cell is:

$$I_s = I_{ph} - I_d \tag{13}$$

The photocurrent and diode loss current cannot be measured by a simple manner. Therefore, a few number of parameters which can be measured easily such as open-circuit voltage (U_{oc}); short-circuit current (I_{sc}); and maximum power (P_{max}) are used to represent this model.

In Equation 13, the following simplification is used: $I_{ph} \approx I_{sc}$ and substituting $\delta = e_0/a_{fk}T_j$ (see I_d in equation 4), the I-U curve can be expressed as:

$$I_d = I_{sc}\left[1 - \left(\frac{I_{sat}}{I_{sc}}\right)\exp\delta\left(U_s + R_s I_s\right)\right] \tag{14}$$

Since δ and R are unknown, two conditions are required to find them:

1. At $I_s = 0$, then $U_s = U_{oc}$
2. At the maximum power point $U_s(I_{s,max}) = P_{max}/I_{s,max}$

From condition 1:

$$U_{oc} = \left\langle U_s \big|_{I_s=0} \right\rangle = \frac{1}{\delta} ln\left(\frac{I_{sc}}{I_{sat}}\right) \text{ or } \delta = \frac{1}{U_{oc}} ln\left(\frac{I_{sc}}{I_{sat}}\right) \tag{15}$$

It was found that a typical value of the ratio I_{sat}/I_{sc} for a silicon cell at standard test conditions (T_0=25°C, E_0=1000 W/m²) ranges approximately from 10^{-8} to 10^{-10}. The accuracy of calculations of the fit is affected only slightly when this ratio varies within that range. Thus, in order to reduce the number of measurements, it is assumed that $I_{sat}/I_{sc} = 10^{-9}$. Substituting this value into Equations 14 and 15 then:

$$U_s = U_{oc}\left[1 + \frac{1}{20.7}\ln\frac{I_{sc} - I_s}{I_{sc}}\right] - R_s I_s \tag{16}$$

From condition 2:

$$U_{oc} = \left\langle U_s \middle| I_s = I_{s\max} \right\rangle = \left(\frac{P_{\max}}{I_{s\max}}\right) \tag{17}$$

$$\left.\frac{\partial U_s}{\partial I_s}\right|_{I_s = I_{s\max}} = \left.\frac{\partial}{\partial I_s}\left(\frac{P_{\max}}{I_s}\right)\right|_{I_s = I_{s\max}} = \frac{-P_{\max}}{I^2_{s\max}} \tag{18}$$

The current at the maximum power point $I_{s,max}$ is unknown. Therefore, substituting equation 18 into equation 17:

$$\frac{P_{\max}}{I_{s\max}} = U_{oc}\left[1 + \frac{1}{20.7}\ln\frac{I_{sc} - I_{s\max}}{I_{sc}}\right] - R_s I_{s\max} \tag{19}$$

Differentiating equation 17 according to Equation 19 we get.

$$\frac{P_{\max}}{I^2{}_{s\max}} = \frac{U_{oc}}{20{,}7}\left[\frac{1}{I_{sc} - I_{s\max}}\right] - R_s \tag{20}$$

Combining equations 19 and 20:

$$\mathrm{I_{s\max}}\left[1 + \frac{1}{20.7}\left(\frac{\mathrm{I_{s\max}}}{\mathrm{I_{sc}} - \mathrm{I_{s\max}}} + \ln\frac{\mathrm{I_{sc}} - \mathrm{I_{s\max}}}{\mathrm{I_{sc}}}\right)\right] - \frac{2\mathrm{P_{\max}}}{\mathrm{U_{oc}}} = 0 \tag{21}$$

This equation has to be solved numerically in order to determine the value of $I_{s,max}$. Then R_s is calculated using equation 20, and this value is substituted into equation 16 to find the I-U curve of the PV cell.

4. EFFECT OF SOLAR RADIATION E_S AND JUNCTION TEMPERATURE T_J

The I-U curves of the PV generator vary with solar radiation E_s [W/m²] and junction cell temperature T_j [°C]. Therefore, the values of U_{oc}, I_{sc}, and P_{max} at any combination of E_s and T_j are needed. The parameters of the model at standard test conditions are known from manufacturer's data sheet. Now, the parameters at any other E_s, T_j combinations must be calculated. T_j is the junction cell temperature, which is related to the ambient temperature Ta by the linear relation.

$$T_j = T_a + (A + B \times E_s) \tag{22}$$

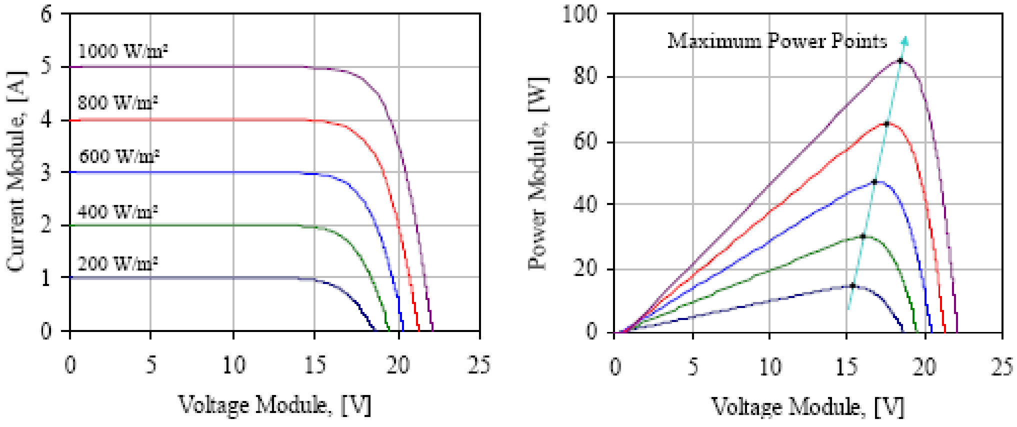
Where A [°C] and B [°C.m²/W] are constants.
Defining $I_{sc,STC}$, $U_{oc,STC}$, $P_{max,STC}$ as the short-circuit current, open-circuit voltage and maximum power at standard test conditions respectively, these parameters (I_{sc}, U_{oc}, and P_{max}) can be computed, to a good degree of accuracy, at any ambient temperature and solar radiation by the following equations:

$$I_{sc=} \frac{I_{scSTC} E_s}{E_0} \left(1 + \frac{i_{coef}\left(T_j - T_0\right)}{U_{oSTC}} \right) \tag{23}$$

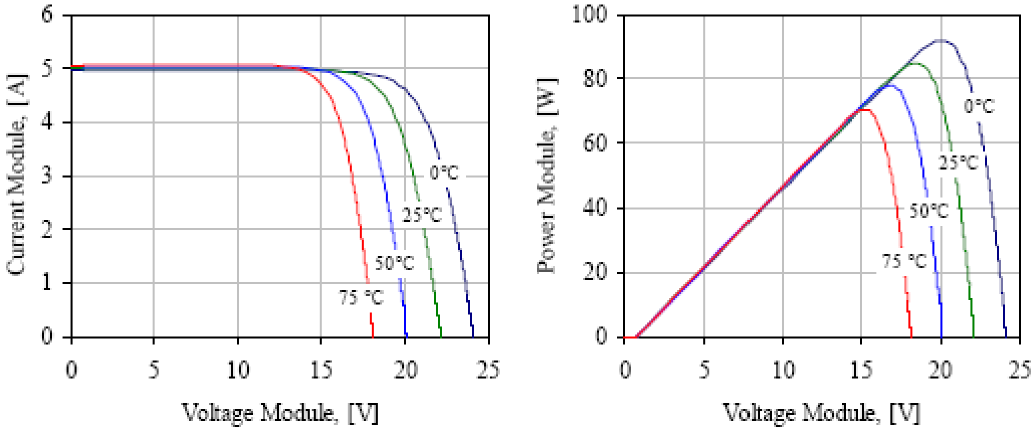
$$U_{oc=} U_{ocSTC} \left(1 + \frac{i_{coef}\left(T_j - T_0\right)}{U_{oSTC}} \right) \ln\left(2{,}72 + \theta \left(E_s - E_0\right)\right) \tag{24}$$

$$P_{\max=} P_{\max STC} \left(1 + \frac{i_{coef}\left(T_j - T_0\right)}{U_{oSTC}} \right) \tag{25}$$

Where i_{coef} temperature coefficient of short-circuit current, [A/°C] u_{coef} temperature coefficient of open-circuit voltage, [V/°C] and θ constant equal to 0.0005, [m²/W]. The serial resistor R_s is now calculated by substituting these parameters into equations 19 and 20.
By substituting the resulting value of R_s into equation 16, the I-U curve of the PV generator is determined.



(a) Influence of solar radiation (cell temperature Tj= 25°C)



(b) Influence of cell temperature (solar radiation E_0=1kW/m²)

Figure 6. I-U characteristics of BP 585 High-Efficiency Monocrystalline PV Module

As demonstrated in Figure 6, an increase in solar radiation causes the output current to increase and the horizontal part of the curve moves upward. An increase in cell temperature causes the voltage to move leftward, while decreasing temperature produces the opposite effect. Thus, the I-U curves display how a photovoltaic module responds to all possible loads under different solar radiation and cell temperature conditions.

An operating point of a photovoltaic module will move by varying solar radiation, cell temperature, and load values. For a given solar radiation and operating temperature, the output power depends on the value of the load. As the load increases, the operating point moves along the curve towards the right. So, only one load value produces a PV maximum power. The maximum power points line, which is positioned at the knees of the I-U curves, has a nearly constant output voltage at varying solar radiation conditions. When the temperature varies, the maximum power points are generated in such a manner that the output current stays approximately constant.

5 CONCLUSIONS

Three PV generator models have been presented, whereas the empirical model is simpler than other one or two diode models. While for the one or two diode models six parameters must be determined, and these parameters are difficult to measure precisely to obtain an acceptable accuracy of the models, the empirical model uses five parameters which can be found in the manufacturer's data sheet of the PV module.

Modeling of photovoltaic modules are not difficulty of realize then when is know the model of photovoltaic cell. Also have been demonstrated that the temperature and the solar radiation influenced suggestive the system performances.

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