NUMERICAL DETERMINATION OF RECTANGULAR BUSBARS TRANSIENT PARAMETERS

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Abstract – **The transient parameters of a system of two non-magnetic homogenous rectangular busbars are evaluated with finite element method and analytical relationships. Alternative ways of computation were adopted combining the two methods and the corresponding results were comparatively presented. The solid conductor characteristic has been pointed out compared to an ideal filiform conductor.**

Keywords: *transient parameters, finite element method*

1. INTRODUCTION

The characterization of a non-filiform circuit element can be made, in null initial field conditions and special boundary conditions, with the transient parameters, depending exclusively on the element intern structure and being the generalization of the constant parameters of the filiform circuits: the transient resistance $r(t)$, the transient conductance $g(t)$ and, for the elements having a finite stationary regime resistance and conductance, the transient inductance $l(t)$ and the transient capacitance $c(t)$ [1].

Theirs experimental determination supposes measurements in particular feeding regimes, which easily offer some of them. The others can be computed on theirs basis, using the analytical relationships established in [1].

Considering the numerical simulation like a substitute of the experiment, the transient parameters of a system of two non-magnetic homogenous rectangular busbars are evaluated with finite element method combined with analytical relationships.

2. NUMERICAL DETERMINATION

Using FLUX 2D program developed by CEDRAT Company and G2ELab, the transient parameters of the busbars system in figure 1 have been determined.

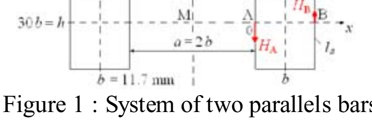

Figure 1 : System of two parallels bars.

The bars system was modeled with solid conductor components connected to the step and ramp signals via Circuit Coupling module. The numerical transient simulation begins at the moment corresponding to $\theta = 10^{-7}$ time constants τ ($\theta = t / \tau$) using the minimum time step allowed by the program (10^{-9} sec). At postprocessor level the following quantities were investigated: current $i(t)$, voltage $u(t)$, inductance $l(t)$.

The current $i(t)$ in the solid conductor is the total current passing the conductor cross section S. Its evaluation is computed as circulation of magnetic field strength H on the conductor perimeter l_S [5]:

$$i(t) = \oint_{l_S} H(t)\,\mathrm{d}l \tag{1}$$

The voltage $u(t)$ on the solid conductor results from :

$$u(t) = r_0 \cdot i(t) + \frac{\mathrm{d}\Psi(t)}{\mathrm{d}t}\ , \quad r_0 = \frac{\rho\, 2L}{S} \tag{2}$$

$$\Psi(t) = \frac{1}{i(t)} \int_0^{i(t)} \Phi_i(t)\,\mathrm{d}i(t) \tag{3}$$

$$\Psi(t) = \frac{4L}{i(t)} \int_0^{b} \int_0^{h/2} A_z(x, y, t)\ j(x, y, t)\,\mathrm{d}x\mathrm{d}y \tag{4}$$

where r_0 is the stationary resistance, evaluated with resistivity ρ (in copper) and conductor length L, $\Phi_i(t)$ is the interlinked with $\mathrm{d}i(t) = j(x, y, t)\cdot\mathrm{d}S$ magnetic flux and A_z is the normal on the cross section component of magnetic vector potential ($A_z = 0$ on $x = -a/2$).

From numerical values of the voltage and current were directly deduced, respectively, the transient resistance and the transient conductance and these were used by analytical relationships to compute the transient inductance and the transient capacitance. The numerical simulation offers also the transient inductance and it was use to mutually compute the transient resistance. Comparisons between directly numerical deduced transient parameters and those analytically computed were made. Finally, the solid conductor characteristic has been pointed out compared to an ideal filiform conductor.

2.1. Transient resistance and inductance determination at step current injection

At current step injection $i(t) = I_0 \cdot 1(t)$, for $t \geq 0$, the relationship between the transient resistance and the transient inductance is [1] ($\delta(t)$ = *the Dirac function*):

$$u(t) = I_0\left[\, l(0_+)\delta(t) + r(t)\right] = I_0\left[r_0 \cdot 1(t) + \frac{\mathrm{d}l(t)}{\mathrm{d}t}\right] \tag{5}$$

For $t > 0$, the first equality offers *the transient resistance at step current*, directly from the numerical value of voltage at step current:

$$r_{\text{step\_crt\_U}}(t) = \left.\frac{u_{\text{step\_crt\_NUM}}(t)}{I_0}\right|_{t>0} \quad (6)$$

Another way to obtain the transient resistance is using *the numerical transient inductance at step current* $l_{\text{step\_crt\_NUM}}(t)$ offered by FLUX (total inductance). The result was called *the transient resistance at step current from inductance*:

$$r_{\text{step\_crt\_L}}(t) = r_0 + \frac{\mathrm{d}l_{\text{step\_crt\_NUM}}(t)}{\mathrm{d}t} \quad (7)$$

Mutually, we can deduce the transient inductance from the transient resistance:

$$l(t) = L_\infty - \int_t^\infty \left(r(t) - r_0\right)\mathrm{d}t \quad (8)$$

An approximate expression is obtained using the numerical value of inductance in stationary regime L_{st} for its limit at infinity L_∞ and $r_{\text{step\_crt\_U}}(t)$ for transient resistance:

$$l(t) \approx L_{\text{st}} - \int_t^{10\cdot\tau} \left(r_{\text{step\_crt\_U}}(t) - r_0\right)\mathrm{d}t = l_{\text{step\_crt\_R}}(t) \quad (9)$$

where the integration was performed till the moment $t = 10\cdot\tau$, practically considered enough for transient process stabilization. This quantity was called *the transient inductance at step current from resistance.* The evolutions of these fours quantities are comparatively presented in figures 2, 3. It can be seen that the numerical results are not correct for $\theta < 10^{-6}$.

2.2. Transient resistance and inductance determination at ramp current injection

At current ramp injection $i(t) = t\cdot I_0\cdot 1(t)$, for $t \geq 0$, the relationship between the transient resistance and the transient inductance is [1]:

$$u(t) = I_0\left[l(0_+)\cdot 1(t) + \int_{0-\varepsilon}^{t} r(t)\mathrm{d}t\right] = I_0\left[r_0\cdot t + l(t)\right] \quad (10)$$

For $t > 0$, the first equality offers *the transient resistance at ramp current*, from the derivative of the numerical value of voltage at ramp current:

$$r_{\text{ramp\_crt\_U}}(t) = \left.\frac{1}{I_0}\frac{\mathrm{d}u_{\text{ramp\_crt\_NUM}}(t)}{\mathrm{d}t}\right|_{t>0} \quad (11)$$

Another way to obtain the transient resistance is using *the numerical transient inductance at ramp current* $l_{\text{ramp\_crt\_NUM}}(t)$ offered by FLUX. The result was called *the transient resistance at ramp current from inductance*:

$$r_{\text{ramp\_crt\_L}}(t) = r_0 + \frac{\mathrm{d}l_{\text{ramp\_crt\_NUM}}(t)}{\mathrm{d}t} \quad (12)$$

Mutually, we can deduce with (8) an approximate expression of the transient inductance from the transient resistance, using $r_{\text{ramp\_crt\_U}}(t)$ for the last:

$$l(t) \approx L_{\text{st}} - \int_t^{10\cdot\tau} \left(r_{\text{ramp\_crt\_U}}(t) - r_0\right)\mathrm{d}t = l_{\text{ramp\_crt\_R}}(t) \quad (13)$$

This quantity was called *the transient inductance at ramp current from resistance.* The evolutions of these fours quantities are comparatively presented in figures 4 and 5, where great errors can be remarked.

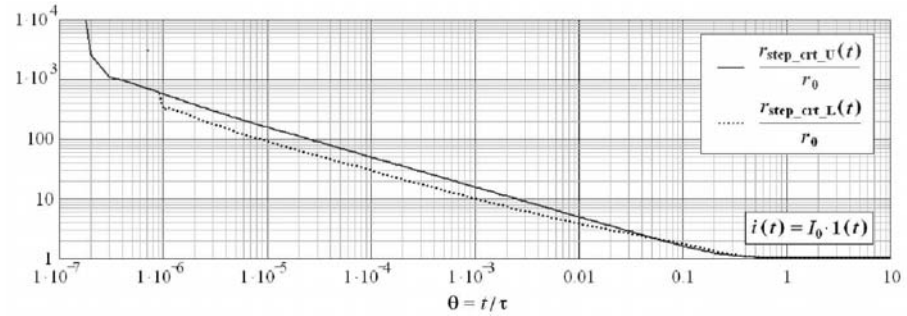

Figure 2 : Transient resistance at step current: solid line (6), dot line (7).

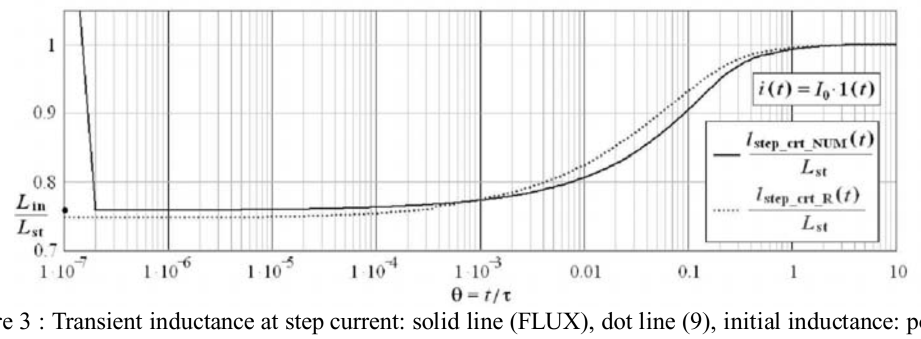

Figure 3 : Transient inductance at step current: solid line (FLUX), dot line (9), initial inductance: point (15).

With (10) we can also estimate the initial inductance:

$$l(0_+) = \frac{u(t)}{I_0} - \int_{0-\varepsilon}^{t} r(t)\mathrm{d}t = \text{ct.} \tag{14}$$

which is a constant value, not depending of the superior limit t up to which integration is performed. So, the average L_{in} of the n = 234 integral values corresponding to the analyzed moments can approximate $l(0_+)$: (15)

$$l(0_+) \approx L_{\text{in}} = \frac{1}{n}\sum_{k=1}^{n}\left[\frac{u_{\text{ramp\_crt\_NUM}}(t_k)}{I_0} - \int_0^{t_k} r_{\text{ramp\_crt\_U}}(t)\mathrm{d}t\right]$$

Comparatively, L_{in} is very close to $l_{\text{ramp\_crt\_R}}(10^{-7}\tau)$ and $l_{\text{step\_crt\_R}}(10^{-7}\tau)$. It is visible in the figures 4, 5. In the figure 6 is shown the solid conductor characteristic compared to an ideal filiform conductor, for which the (10) relation contains time-invariable parameters. The voltage at ramp current for filiform conductor is:

$$u_{\text{ramp\_crt\_FIL}}(t) = I_0[r_0 \cdot t + L_{\text{st}}]\Big|_{t>0} \tag{16}$$

Additionally, with (10) and using $l_{\text{ramp\_crt\_NUM}}(t)$ was deduced an approximate expression called the voltage at ramp current from inductance:

$$u_{\text{ramp\_crt\_L}}(t) = I_0[r_0 \cdot t + l_{\text{ramp\_crt\_NUM}}(t)] \tag{17}$$

In figure can be seen the asymptotically evolution of both the numerical value of voltage at ramp current $u_{\text{ramp\_crt\_NUM}}(t)$ and the voltage at ramp current from inductance $u_{\text{ramp\_crt\_L}}(t)$ towards the ideal values.

2.3. Transient conductance and capacitance determination at step voltage application

At voltage step application $u(t) = U_0 \cdot 1(t)$, for $t \geq 0$, the relationship between the transient conductance and the transient capacitance is [1]:

$$i(t) = U_0[c(0_+)\delta(t) + g(t)] = U_0\left[g_0 \cdot 1(t) + \frac{\mathrm{d}c(t)}{\mathrm{d}t}\right] \tag{18}$$

For t > 0, the first equality offers *the transient conductance at step voltage*, directly from the numerical value of current at step voltage:

$$g_{\text{step\_volt\_I}}(t) = \frac{i_{\text{step\_volt\_NUM}}(t)}{U_0}\Bigg|_{t>0} \tag{19}$$

The program doesn't offer the value of capacitance $c_{\text{step\_volt\_NUM}}(t)$, so, a similar to (7) relationship can't be established. Mutually, we can deduce the transient capacitance from the transient conductance:

$$c(t) = C_\infty - \int_t^\infty (g(t) - g_0)\mathrm{d}t \tag{20}$$

An approximate expression can be obtained using the numerical value of capacitance in stationary regime C_{st} for its limit at infinity C_∞ and $g_{\text{step\_volt\_I}}(t)$ for transient conductance, by analogy with (9):

$$c(t) \approx C_{\text{st}} - \int_t^{10\cdot\tau} (g_{\text{step\_volt\_I}}(t) - g_0)\mathrm{d}t = c_{\text{step\_volt\_G}}(t) \tag{21}$$

This quantity was called *the transient capacitance at step voltage from conductance.* It must be noted that C_{st} is established in 2.4 section of the paper, for another feeding regime. The evolutions of the two quantities are presented in figures 7 and 8.

2.4. Transient conductance and capacitance determination at ramp voltage application

At voltage ramp voltage $u(t) = t \cdot U_0 \cdot 1(t)$, for $t \geq 0$, the relationship between the transient conductance and the transient capacitance is [1]:

$$i(t) = U_0\left[c(0_+)\cdot 1(t) + \int_{0-\varepsilon}^{t} g(t)\mathrm{d}t\right] = U_0[g_0 \cdot t + c(t)] \tag{22}$$

For t > 0, the first equality offers *the transient conductance at ramp voltage*, from the derivative of the numerical value of current at ramp voltage:

$$g_{\text{ramp\_volt\_I}}(t) = \frac{1}{U_0}\frac{\mathrm{d}i_{\text{ramp\_volt\_NUM}}(t)}{\mathrm{d}t}\Bigg|_{t>0} \tag{23}$$

With (22) we can estimate the initial capacitance:

$$c(0_+) = \frac{i(t)}{U_0} - \int_{0-\varepsilon}^{t} g(t)\mathrm{d}t = \text{ct.} \tag{24}$$

which is a constant value, not depending of the superior limit t up to which integration is performed. So, the average C_{in} of the n = 234 integral values corresponding to the analyzed moments can approximate $c(0_+)$: (25)

$$c(0_+) \approx C_{\text{in}} = \frac{1}{n}\sum_{k=1}^{n}\left[\frac{i_{\text{ramp\_volt\_NUM}}(t_k)}{U_0} - \int_0^{t_k} g_{\text{ramp\_volt\_I}}(t)\mathrm{d}t\right]$$

Mutually, we can deduce an approximate expression of the transient capacitance from the transient conductance, using $g_{\text{ramp\_volt\_I}}(t)$ for the last:

$$c(t) \approx C_{\text{in}} + \int_0^t (g_{\text{ramp\_volt\_I}}(t) - g_0)\mathrm{d}t = c_{\text{ramp\_volt\_G}}(t) \tag{26}$$

This quantity was called *the transient capacitance at ramp voltage from conductance.* The evolutions of the two transient parameters are presented in figures 9 and 10. In figures 10 and 8, the initial capacitance C_{in} is also visible, close to both $c_{\text{ramp\_volt\_G}}(10^{-7}\tau)$ and $c_{\text{step\_volt\_G}}(10^{-7}\tau)$.

The expression (26) allows the approximation of stationary regime capacitance C_{st}, required by (21).

$$C_{\text{st}} \approx c_{\text{ramp\_volt\_G}}(10\,\tau) \tag{27}$$

In fig. 11 is shown the solid conductor characteristic compared to an ideal filiform conductor, for which the (22) relation contains time-invariable parameters. The current at ramp voltage for filiform conductor is:

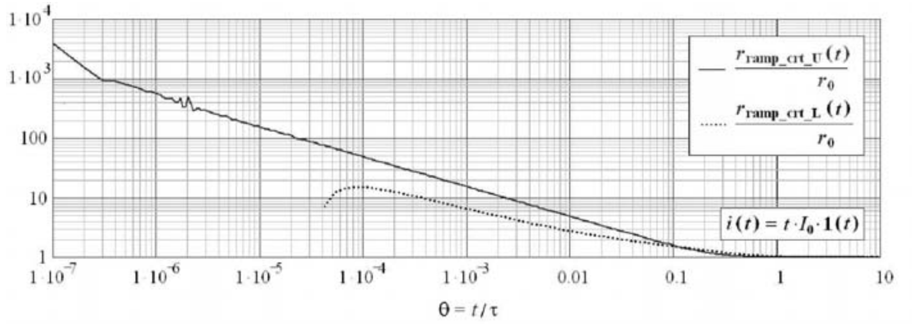

Figure 4 : Transient resistance at ramp current: solid line (11), dot line (12).

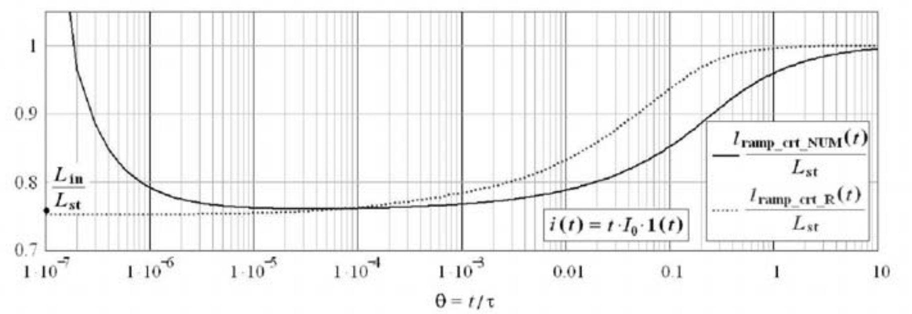

Figure 5 : Transient inductance at ramp current: solid line (FLUX), dot line (13), initial inductance: point (15).

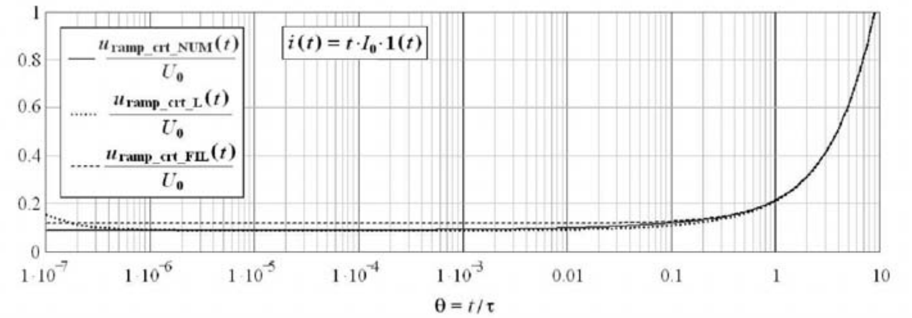

Figure 6 : Voltage evolution at ramp current: solid line (FLUX), dot line (17), dash line (16).

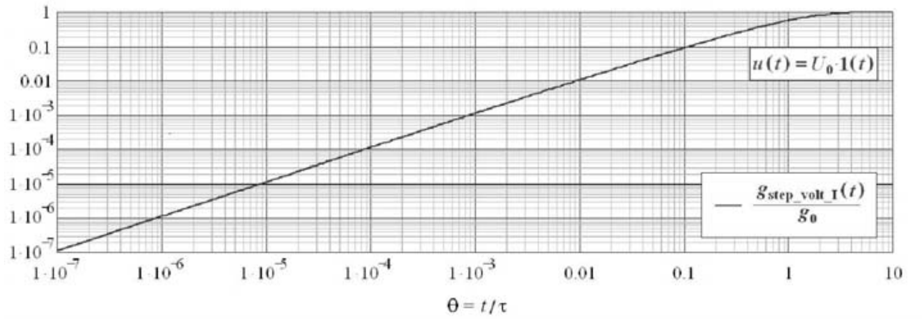

Figure 7 : Transient conductance at step voltage: solid line (19).

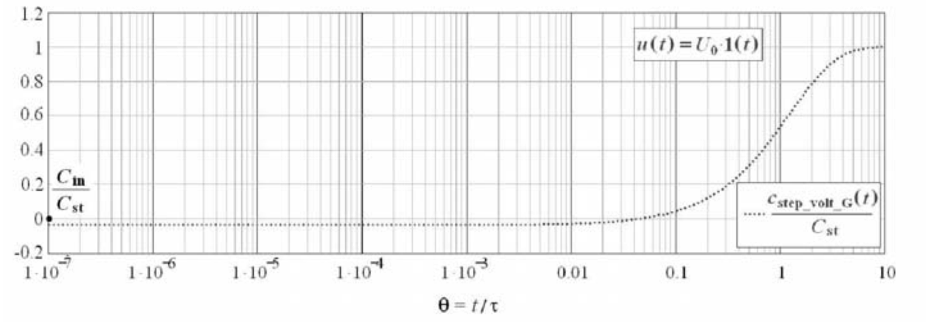

Figure 8 : Transient capacitance at step voltage: dot line (21), initial capacitance: point (25).

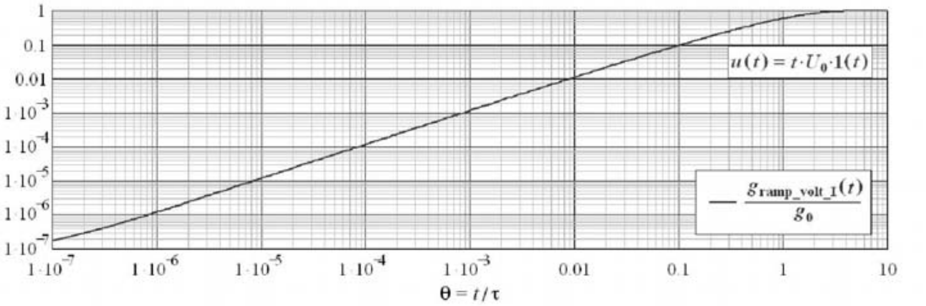

Figure 9 : Transient conductance at ramp voltage: solid line (23).

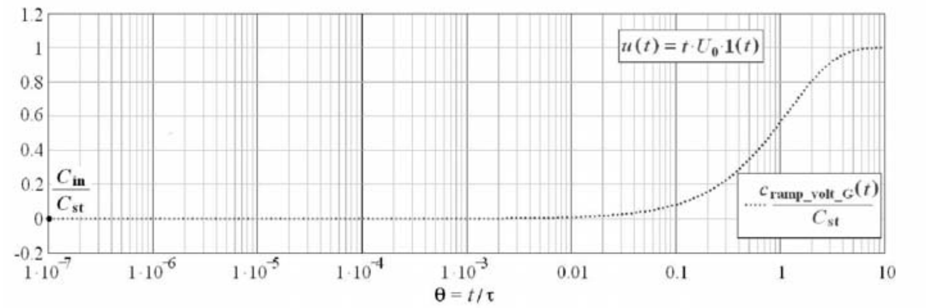

Figure 10 : Transient capacitance at ramp voltage: dot line (26), initial capacitance: point (25).

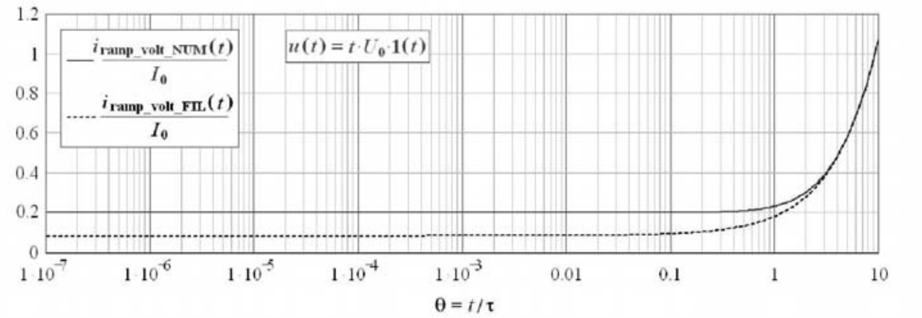

Figure 11 : Current evolution at ramp voltage: solid line (FLUX), dash line (28).

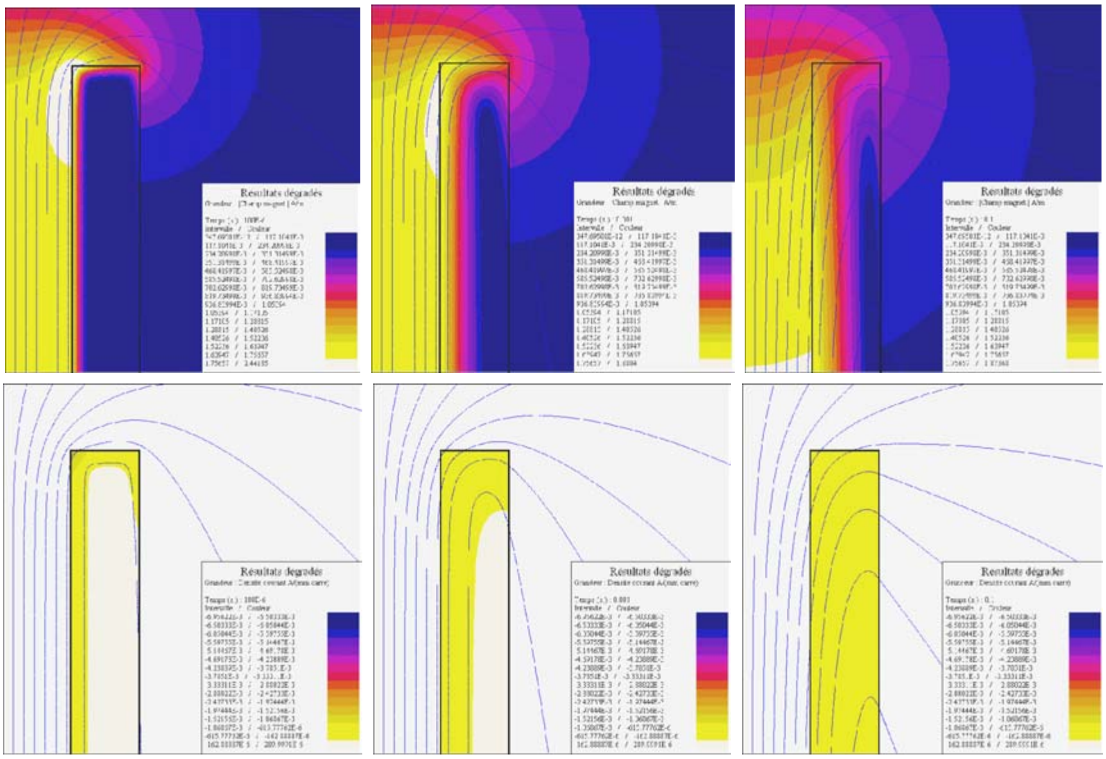

Figure 12 : Magnetic field pattern (up), current density distribution (down) at θ = 10^{-2}, 10^{-1}, 10, for step current.

$$i_{\text{ramp\_volt\_FIL}}(t) = U_0\left[g_0 \cdot t + C_{\text{st}}\right]\Big|_{t>0} \qquad (28)$$

compared to the numerical value of current at ramp voltage $i_{\text{ramp\_volt\_NUM}}(t)$. His asymptotically evolution towards the ideal values can be remarked.

Using visualization facilities offered by FLUX postprocessor, in the figure 12 are presented the magnetic field pattern and the current density distribution in a little part of the studied domain at three moments of transient process corresponding to θ = 10^{-2}, θ = 10^{-1}, θ = 10, for step current injection.

4. CONCLUSIONS

The transient parameters – resistance, conductance, inductance and capacitance – of a system of two non-magnetic homogenous rectangular busbars were evaluated with finite element method and analytical relationships. Numerical simulations in particular feeding regimes had easily offered some of them and the others could be computed using analytical relationships. Comparisons between directly numerical deduced transient parameters and those analytically computed show acceptable results at step current injection excepting the beginning of the transient process where the numerical simulation was strongly affected by the unavoidable coarse time mesh. Great errors were founded at ramp current when operators like integrals or derivatives were applied to the interpolated numerical signals. The solid conductor characteristic has been pointed out compared to an ideal filiform conductor.

Acknowledgments

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