# JET ENGINE'S ACCELERATION'S CONTROLLER WITH RESPECT TO THE ROTATION SPEED

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Abstract – This paper deals with an automatic system for a jet engine's speed control during its acceleration/deceleration, independent of the engine's fuel pump, but connected to the engine's (single) spool. Starting from the functional scheme and the functional block diagram, one has established the system's nonlinear mathematical model, the linearised and the nondimensional mathematical model; based on this model, the block-diagram with transfer functions and the system's transfer function were issued. A stability study was performed, which has determined the system's stability domains.

*Keywords: jet engine, speed, fuel, control, actuator, throttle, acceleration, stability.* 

## **1. INTRODUCTION**

Modern aircraft gas-turbine engines, particularly the jet engines, have as most important controlled

parameter the speed *n*, both for various flight speeds and flight altitudes. The control parameter is, for most of the systems, the injection fuel flow rate  $Q_i$ . Especially during the dynamic regimes (for example the starting regime and the acceleration/deceleration regimes), a very important matter is also its variation

speed,  $\frac{dQ_i}{dt}$ , because of the possible side effects concerning the combustor gases' temperature, which can overheat the turbine's blades. So, the speed control systems, based on the injected fuel flow rate control, must be completed with additional fuel control systems based on another control parameter, such as the air flow rate, the fuel's injection pressure,

Figure 1 shows a constructive-functional scheme for an acceleration controller, based on the injected fuel flow rate's control, with respect to the engine's

the compressor's total (or static) pressure ratio, or the

engine's speed itself.



Figure 1: Controller's constructive-functional scheme

rotation speed. The engine's acceleration controller's main parts are, as follows: a) engine's speed centrifuge transducer; b) single active chamber actuator, with c) rigid feed-back; d) slide-valve command organ; e) fuel dosage rotation valve.

The controller operates opening/closing the dosage valve, according to the engine's speed's increasing/decreasing value.

## 2. SYSTEM'S MATHEMATICAL MODEL

## 2.1. System's motion non-linear equations

The non-linear mathematical model consists of the motion equations for each above-mentioned subsystem, as follows:

- speed centrifuge transducer's equation

$$F_{cp} - S_{c} p_{c} - m_{s} \frac{d^{2} x}{dt^{2}} - \xi \frac{dx}{dt} = 0, \qquad (1)$$

- flow rates' equations

$$Q_{1} = \mu_{1} L x \sqrt{2\rho^{-1}} \sqrt{p_{a} - p_{c}} , \qquad (2)$$

$$Q_{9} = \mu_{9} \frac{\pi d_{9}^{2}}{4} \sqrt{2\rho^{-1}} \sqrt{p_{a} - p_{A}} , \qquad (3)$$

$$Q_1 = \mu_r b z \sqrt{2\rho^{-1}} \sqrt{p_A} , \qquad (4)$$

$$Q_1 = Q_2 + Q_3,$$
 (

- C pressure chamber's equations

$$Q_2 = \frac{\mathrm{d}V_{\mathrm{c}}}{\mathrm{d}t} + \beta V_{\mathrm{c}} \frac{\mathrm{d}p_{\mathrm{c}}}{\mathrm{d}t}, \qquad (6)$$

$$V_{c} = V_{c0} - S_{c} x , \qquad (7)$$

- M pressure chamber's equations

$$Q_{3} = \frac{\mathrm{d}V_{\mathrm{M}}}{\mathrm{d}t} + \beta V_{\mathrm{M}} \frac{\mathrm{d}p_{\mathrm{C}}}{\mathrm{d}t}, \qquad (8)$$

$$V_{M} = V_{M0} + S_{1}z , \qquad (9)$$

- A pressure chamber's equations

$$Q_9 - Q_r = \frac{\mathrm{d}V_{\mathrm{A}}}{\mathrm{d}t} + \beta V_{\mathrm{A}} \frac{\mathrm{d}p_{\mathrm{A}}}{\mathrm{d}t}, \qquad (10)$$

$$V_{A} = V_{A0} - S_{A}x , \qquad (11)$$

- slide-valve's equation

$$S_{1}p_{c} + k_{6}(z+u) - k_{7}(v-z) - m_{1}\frac{d^{2}z}{dt^{2}} - \xi\frac{dz}{dt} = 0, (12)$$

- actuator's equation

$$S_{A}p_{A} - k_{8}y - m_{2}\frac{d^{2}y}{dt^{2}} - \xi\frac{dy}{dt} = 0, \qquad (13)$$

$$v = k_4 y , \qquad (14)$$

where  $F_{cp}$  is the axial force due to the centrifuge masses rotation, which expression is, according to [7] and [8],  $F_{cp} = F_{cp}(n, x) = (K_1 + K_2 x)n^2$ ;  $p_A, p_C, p_M$ pressures;  $S_c$ ,  $S_A$ ,  $S_1$  – pistons' surfaces;  $p_a$ -supplying pressure (assumed as constant);  $V_A, V_C, V_M$  – pressure chambers' volumes;  $\beta$  – fluid's compressibility coefficient;  $\beta$ -fluid's density;  $Q_1, Q_2, Q_3, Q_9, Q_r$ -fluid flow rates;  $\mu_1, \mu_9, \mu_r$  – flow rate's co-efficient; L, b – slide valve's slot width;  $d_9$  – drossel's diameter;  $\xi$  viscous friction co-efficient,  $m_2$  – actuator's mobile ensemble's mass,  $m_1$  – pressure sensor's mobile slide valve's mass;  $k_6, k_7, k_8$ -spring elastic constants; xsensor's slide valve's displacement; z-transducer's slide valve's displacement; u-pressure sensor's spring preset; y-actuator's rod's displacement, v-lug's displacement;  $k_4$  - rigid feed-back's co-efficient, based on the cam's profile.

#### 2.2 Linear mathematical model

The above-described 14 non-linear motion equations are building the system's non-linear mathematical model, very difficult to be used for studies.

Assuming the small-disturbances hypothesis, one can obtain a linear form of the model; so, assuming that
each X parameter can be expressed as

$$X = X_0 + \frac{\Delta X}{1} + \frac{(\Delta X)^2}{2} + \dots + \frac{(\Delta X)^n}{n}, \quad (13)$$

(where  $X_0$  is the steady state regime's X-value and  $\Delta X$  -deviation or static error) and, neglecting the terms which contains  $(\Delta X)^r, r \ge 2$ , one obtains a new form of the equation system (1)...(14), particularly in the neighborhood of a steady state operating regime, as follows:

$$k_{cpn}\Delta n - k_{cpx}\Delta x - S_{c}\Delta p_{c} - m_{s}\frac{d^{2}}{dt^{2}}\Delta x - \xi\frac{d}{dt}\Delta x = 0, (15)$$

$$\Delta Q_1 = k_1 \Delta x - k_{1C} \Delta p_C, \qquad (16)$$

$$\Delta Q_2 = -S_C \frac{\mathrm{d}}{\mathrm{d}t} \Delta x + \beta V_{C_0} \frac{\mathrm{d}}{\mathrm{d}t} \Delta p_C , \qquad (17)$$

$$\Delta Q_3 = S_1 \frac{\mathrm{d}}{\mathrm{d}t} \Delta z + \beta V_{M0} \frac{\mathrm{d}}{\mathrm{d}t} \Delta p_C , \qquad (18)$$

$$\Delta Q_1 = \Delta Q_2 + \Delta Q_3, \qquad (19)$$

$$S_1 \Delta p_c + k_6 (\Delta z + \Delta u) - k_7 (\Delta z + \Delta v) -$$

$$-m_1 \frac{\mathrm{d}^2}{\mathrm{d}t^2} \Delta z - \xi \frac{\mathrm{d}}{\mathrm{d}t} \Delta z = 0 , \qquad (20)$$

$$\Delta v = k_4 \Delta y \,, \tag{21}$$

$$\Delta Q_r = k_{rz} \Delta z + k_{rA} \Delta p_A, \qquad (22)$$

$$\Delta Q_9 = -k_{_{9A}} \Delta p_A \,, \tag{23}$$

$$\Delta Q_{9} - \Delta Q_{r} = S_{A} \frac{\mathrm{d}}{\mathrm{d}t} \Delta y + \beta V_{A0} \frac{\mathrm{d}}{\mathrm{d}t} \Delta p_{A}, \quad (24)$$

$$S_{A}\Delta p_{A} - k_{8}\Delta y - m_{2}\frac{d^{2}}{dt^{2}}\Delta y - \xi\frac{d}{dt}\Delta y = 0.$$
 (25)

The above system's used annotations are:

$$k_{cpn} = \left(\frac{\partial F_{cp}}{\partial n}\right)_{0}; k_{cpx} = \left(\frac{\partial F_{cp}}{\partial x}\right)_{0};$$
$$k_{1x} = \left(\frac{\partial Q_{1}}{\partial x}\right)_{0} = \mu_{1}L\sqrt{2\rho^{-1}(p_{a} - p_{C0})};$$

$$k_{1c} = -\left(\frac{\partial Q_1}{\partial p_c}\right)_0 = \frac{\mu_1 L x_0 \sqrt{2\rho^{-1}}}{2\sqrt{(p_a - p_{c0})}}; k_{cA} = \left(\frac{\partial Q_r}{\partial p_A}\right)_0 = \frac{\mu_r b z_0 \sqrt{2\rho^{-1}}}{2\sqrt{p_{A0}}};$$

$$k_{9A} = -\left(\frac{\partial Q_{9}}{\partial p_{A}}\right)_{0} = \frac{\mu_{90}\pi d^{2}\sqrt{2\rho^{-1}}}{8\sqrt{(p_{a} - p_{A0})}}.$$
 (26)

Substituting the expressions for  $Q_1, Q_2, Q_3, Q_9, Q_r$  in the equations (15), (19), (20), (22)...(25) and applying the Laplace transformer (s being the image of the derivation operator), one obtains the linear form of the mathematical model, as follows:

$$k_{cpn}\Delta n - S_c\Delta p_c - (m_s s^2 + \xi s + k_{cpx})\Delta x = 0, \quad (27)$$

$$(S_{C}s + k_{1x})\Delta x = S_{1}s\Delta z + [\beta(V_{C0} + V_{M0})s + k_{1C}]\Delta p_{C}, \quad (28)$$

$$S_1 \Delta p_c + k_6 \Delta u - k_7 \Delta v = \left[ m_1 \mathrm{s}^2 + \xi \mathrm{s} + \left( k_7 + k_6 \right) \right] \Delta z , \quad (29)$$

$$\Delta v = k_{A} \Delta y \,, \tag{30}$$

$$-k_{r}\Delta z = S_{A}s\Delta y + \left[\beta V_{A0}s + \left(k_{9A} + k_{rA}\right)\right]\Delta p_{A}, \quad (31)$$

$$S_{A}\Delta p_{A} - \left(m_{2}\mathrm{s}^{2} + \xi \mathrm{s} + k_{8}\right)\Delta y = 0.$$
 (32)

Using, also, the generic annotation  $\overline{X} = \frac{\Delta X}{X_0}$ , the above mathematical model can be transformed in a non-dimensional one:

$$k_n \overline{n} - k_c \overline{p_c} - (T_s^2 \mathbf{s}^2 + 2T_s \omega_0 \mathbf{s} + 1) \overline{x} = 0 , \quad (33)$$

$$k_{x1}(T_x \mathbf{s} + 1)\overline{x} = \tau_z \mathbf{s}\overline{z} + (\tau_c \mathbf{s} + 1)\overline{p_c}, \qquad (34)$$

$$k_{cz} \overline{p_c} + k_u \overline{u} - k_v \overline{v} = (T_1^2 \mathbf{s}^2 + 2T_1 \omega_0 \mathbf{s} + 1) \overline{z} , \quad (35)$$

$$\overline{v} = k_{vy} \overline{y} , \qquad (36)$$

$$-k_{AZ}\overline{z} = \tau_{y}s\overline{y} + (T_{A}s + 1)\overline{p_{A}}, \qquad (37)$$

$$k_{Ay} \overline{p_A} = (T_2^2 \mathbf{s}^2 + 2T_2 \omega_0 \mathbf{s} + 1) \overline{y}$$
. (38)

Based on this mathematical model, one has built the block diagram in fig. 2, where the annotations are

$$k_{n} = \frac{k_{cpn}n_{0}}{k_{cpx}x_{0}}, k_{c} = \frac{S_{c}P_{c0}}{k_{cpx}x_{0}}, T_{s} = \sqrt{\frac{m_{s}}{k_{cpx}}}, 2T_{s}\omega_{0} = \frac{\xi}{k_{cpx}},$$

$$k_{x1} = \frac{k_{1x}x_{0}}{k_{1c}P_{c0}}, T_{x} = \frac{S_{c}}{k_{1x}}, T_{x} = \frac{S_{1}}{k_{1c}}, T_{c} = \frac{\beta(V_{c0} + V_{M0})}{(k_{7} + k_{6})z_{0}},$$

$$k_{cz} = \frac{S_{1}P_{c0}}{(k_{7} + k_{6})z_{0}}, k_{u} = \frac{k_{6}u_{0}}{(k_{7} + k_{6})z_{0}}, k_{u} = \frac{k_{7}v_{0}}{(k_{7} + k_{6})z_{0}},$$

$$k_{vy} = k_{4}\frac{y_{0}}{v_{0}}, T_{1} = \sqrt{\frac{m_{1}}{k_{7} + k_{6}}}, 2T_{1}\omega_{0} = \frac{\xi}{k_{7} + k_{6}},$$



Figure 2: Controller's block diagram with transfer functions



Figure 3: Simplified block diagram with transfer functions

$$T_{2} = \sqrt{\frac{m_{2}}{k_{s}}}, 2T_{2}\omega_{0} = \frac{\xi}{k_{s}}, k_{Az} = \frac{k_{rz}z_{0}}{(k_{9A} + k_{rA})p_{A0}},$$
$$\tau_{y} = \frac{S_{A}y_{0}}{(k_{9A} + k_{rA})p_{A0}}, T_{A} = \frac{\beta V_{A0}}{(k_{9A} + k_{rA})}, k_{Ay} = \frac{S_{A}p_{A0}}{k_{8}y_{0}}.$$
(39)

#### 2.3 Simplified mathematical model

Based on some practical observation, one can make some supplementary hypothesis that could be further involved in the mathematical model simplifying.

Thus, the hydraulic fluid is a non-compressible one, so  $\beta = 0$ ; the inertial effects are very small, as well as the viscous friction, so the terms containing *m* and  $\xi$ are becoming null. The fluid flow rate  $Q_3$  is very small, comparative to the actuator's fluid flow rate  $Q_2$ , so  $Q_1 \approx Q_2$ .

So, the new, simplified, equations system is:

$$k_{n}\overline{n} - k_{c}\overline{p_{c}} = \overline{x}, \qquad (40)$$

$$k_{x1}(T_x \mathbf{s} + 1)\overline{x} = \tau_z \mathbf{s}\overline{z} + \overline{p_c} , \qquad (41)$$

$$k_{cz} \overline{p_c} + k_{\mu} \overline{u} - k_{\nu} \overline{v} = \overline{z} , \qquad (42)$$

$$\overline{v} = k_{vv} \overline{y} , \qquad (43)$$

$$-k_{AZ}\overline{z} = \tau_{y}s\overline{y} + \overline{p_{A}}, \qquad (44)$$

$$\overline{y} = k_{Ay} \overline{p_A} . \tag{45}$$

System's simplified block diagram with transfer function is presented in figure 3.

## **3. CONTROLLER'S TRANSFER FUNCTION**

Observing the simplified model's equation's form, as well as the block diagram in figure 3, one can affirm that the fuel's dosage valve's opening y depends on the engine's speed n, as well as on the slide-valve's spring (6) pre-tension, through the adjustment u.

So, after favorable substituting between the above equations, the system becomes a single-equation one, which has the form:

$$\overline{y} = \frac{(b_1 \mathbf{s} + b_0)\overline{n} - c_1 \mathbf{s}\overline{u}}{a_2 \mathbf{s}^2 + a_1 \mathbf{s} + a_0}, \qquad (46)$$

where

$$a_2 = k_{Ay} \tau_{yC} \tau_y , \qquad (47)$$

$$a_{1} = \tau_{yC} + k_{Ay} (\tau_{y} \rho_{y} - k_{Az} k_{u} k_{uy} \tau_{yC} + k_{v} k_{vy} \tau_{z}), \quad (48)$$

$$a_{0} = \rho_{v} (1 - k_{Ay} k_{Az} k_{vy} k_{v}), \qquad (49)$$

$$b_{1} = k_{Ay} k_{Az} k_{Cz} k_{x1} k_{n} T_{x} , \qquad (50)$$

$$b_0 = k_{Ay} k_{Az} k_{Cz} k_{x1} k_n, \qquad (51)$$

$$c_1 = k_{Ay} k_u \tau_z \,, \tag{52}$$

$$\tau_{vC} = k_{x1}k_C T_x + k_{Cz}\tau_z, \qquad (53)$$

$$\rho_{y} = k_{x1}k_{c} + 1.$$
 (54)

Based on this single equation, one has built the equivalent block diagram in figure 4.

$$\overline{n} \xrightarrow{b_1 s + b_0} \xrightarrow{a_2 s^2 + a_1 s + a_0} \xrightarrow{+} \overline{y}$$

$$\overline{u} \xrightarrow{c_1 s} \xrightarrow{-}$$

Figure 4: Equivalent block diagram with transfer functions

As far as the adjustments are made before the engine becomes fully operational, during the ground tests, one can affirm that the opening y depends only on the engine's speed n, so one can define only one transfer function for the system, the one with respect to the speed,

$$H_{n}(s) = \frac{\overline{y}}{\overline{n}} = \frac{b_{1}s + b_{0}}{a_{2}s^{2} + a_{1}s + a_{0}}.$$
 (55)

Its characteristic polynomial has 2<sup>nd</sup> degree, so the system is a second order one.

Its main part, the actuator, can be described as a first order subsystem

$$H_{ar}(s) = \frac{\overline{y}}{\overline{z}} = \frac{1}{\tau_{ar}s + \rho_{ar}},$$
 (56)

The actuator's time constant  $\tau_{ar}$  and its stability coefficient  $\rho_{ar}$  have the forms:

$$\tau_{ar} = \frac{S_{A}}{\mu_{r} b \sqrt{p_{A0}}} \frac{y_{0}}{z_{0}} \sqrt{\frac{2}{\rho}}, \qquad (57)$$

$$\tau_{ar} = \frac{k_8 \left( k_{9A} + k_{rA} \right)}{\mu_r b S_A \sqrt{p_{A0}}} \frac{y_0}{z_0} \sqrt{\frac{2}{\rho}} \,. \tag{58}$$

### 4. ON CONTROLLER'S STABILITY

One can perform a stability study, using the Routh-Hurwitz criteria, which are easier to apply because of the characteristic polynomial's form. So, the stability conditions are

$$a_{2} = k_{Av} \tau_{vC} \tau_{v} > 0 , \qquad (59)$$

$$a_{1} = \tau_{yC} + k_{Ay} \left( \tau_{y} \rho_{y} - k_{Az} k_{u} k_{uy} \tau_{yC} + k_{v} k_{vy} \tau_{z} \right) > 0 , \quad (60)$$

$$a_{0} = \rho_{v} \left( 1 - k_{Av} k_{Az} k_{vy} k_{v} \right) > 0 .$$
 (61)

The first condition (59) is identical accomplished, the two time constants  $\tau_{yc}$ ,  $\tau_y$ , as well as the coefficient  $k_{Ay}$ , being always positive.

The second condition leads to a relation between the two time constants,  $\tau_{ur}$ ,  $\tau_z$ , which has the form

$$\alpha \tau_{ar} + \beta \tau_z + \gamma > 0; \qquad (62)$$

this inequality represents, in a co-ordinate system  $(\tau_{ar} - \tau_z)$ , a semi-plan (see figure 5), limited by the line of equation  $\alpha \tau_{ar} + \beta \tau_z + \gamma = 0$ , which co-efficient have the forms

$$\alpha = 1 + \frac{S_c p_a}{4k_{cpx}}, \qquad (63)$$

$$\beta = \frac{S_1 S_A p_a p_{C0}}{2(k_6 + k_7)k_8 k_{r_2} z_0^2},$$
(64)

$$\gamma = \frac{S_c^2 p_a \left[ \left( k_{9_A} + k_{rA} \right) - k_4 S_A z_0 \right]}{4 \left( k_{9_A} + k_{rA} \right) k_8 k_{rz} k_{cpx} v_0} \,. \tag{65}$$

This line has a negative slope and intersects the axis

in 
$$A\left(-\frac{\gamma}{\beta},0\right)$$
 and  $B\left(0,-\frac{\gamma}{\alpha}\right)$ .

The last condition (61) leads to

$$k_{Ay}k_{Az}k_{yy}k_{v}(k_{c}k_{x1}+1)-k_{c}k_{x1}<1, \qquad (66)$$

equivalent to

$$\left(1 + \frac{S_{c}p_{a}}{4k_{cpx}}\right)\left[1 - \tau_{ar}\frac{k_{4}k_{7}k_{rz}^{2}}{k_{8}(k_{6} + k_{7})(k_{9A} + k_{rA})}\frac{z_{0}}{y_{0}}\right] > 0,(66')$$

which represents, as figure 5 shows, a semi-plan limited by a line of equation

$$\tau_{ar} = \frac{k_8 (k_6 + k_7) (k_{9A} + k_{rA})}{k_4 k_7 k_{rz}^2} \frac{y_0}{z_0}, \qquad (67)$$

intersecting the vertical axis of the co-ordinate system in figure 5 in C  $\left(0, \frac{k_8 (k_6 + k_7) (k_{9.4} + k_{r.4})}{k_4 k_7 k_{rz}^2} \frac{y_0}{z_0}\right)$ .

So, one has revealed the maximum value of the actuator's time constant, above which the controller becomes unstable.

According to the specific values of the co-efficient, two situations can exist, as figures 5.a and 5.b shows, each one representing a specific shape of the stability domains.

If 
$$(\tau_{ar})_{\max} \ge \frac{\gamma}{\alpha}$$
, the stability domain is shown by the



Figure 5: Controller's stability domains

figure 5.a. Otherwise, if  $(\tau_{ar})_{max} < \left|\frac{\gamma}{\alpha}\right|$ , the stability

domain is the one in figure 5.b. In this case, the stability domain was displaced to the right, the intersection between the two lines, (62) and (67), is D, which has as significance the minimum value of the time constant  $\tau_z$ , that means the minimum value for stability,

$$\left(\tau_{z}\right)_{\min} = \frac{1}{\beta} \left[\gamma - \alpha \left(\tau_{ar}\right)_{\max}\right]. \tag{68}$$

The condition for non-periodic stability,

$$a_1^2 - 4a_2a_0 > 0, (69)$$

gives a more complicate form, but can offer some new conditions for the actuator's time constant choosing, even in the pre-design stage.

## CONCLUSIONS

One has studied an engine's acceleration controller, which operates as fuel's flow rate's growing speed limiter, based on the engine's speed information.

Controller's non-dimensional mathematical model, linearised and simplified, leaded to a second order transfer function.

The stability studies, which are realized based on the algebraic criteria Routh-Hurwitz, are showing the stability domains, depending on the actuator's time constant.

Furthermore they are offering, indirectly, some predesign information concerning the actuator's geometry and performance, starting from imposed or choice time constants and stability constants of the involved actuators.

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