MODELING AND CONTROL OF HIGH POWER INVERTERS

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Abstract – **In this paper we present an inverter model average in the dq0 coordinates and we determine the transfer functions of the system in an open/closed circuit□The system is the equivalent of a SISO system, with three independent inputs and a single output□Also we plot the frequency characteristics of the system which are useful for determining its control strategies□**

Keywords: high power inverters, modeling, control, transfer functions, frequencies characteristics.

1□INVERTER SWITCHING MODEL

The switched model of the inverter was created to be as similar as possible with the real one. Figure 1 shows the scheme of the switched inverter model and in Table 1 the parameters of the inverter are presented. The fourth conductor (N) permits the control of the neutral current. For the three-phase inverters if the load requires us to connect the null, we utilize the null point of the capacitor filter of the CC connection [1].
In this case, the unbalanced loads or the non-linear single phase loads generate null currents and distortions when passing through zero. When using inverters with four terminals (A, B, C and N), we obtain control over the null and we reduce the zero distortions caused by the control system of the inverter [2].

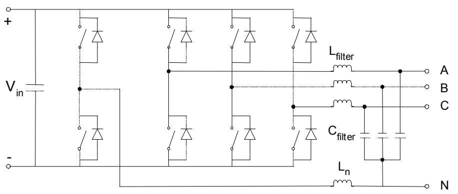

Figure 1. The scheme of the switched inverter model with 4 conductors

Table 1

Parameter	Value
Input/Output	$V_{in} = 6\square\square - 7\square\square \mathrm{V}_{cc}$ $V_{output} = 115\mathrm{V}_{rms}, 4\square\square\mathrm{Hz}$ $P_{output} = 9\square k\mathrm{VA}$
Filters components	$L_{filtru} = 42{,}8\mu\mathrm{H}$ $L_n = 42{,}8\mu\mathrm{H}$ $C_{filtru} = 25\square\mu\mathrm{F}$
Switch parameters	$f_s = 15{,}6k\mathrm{Hz}$

2□INVERTER AVERAGE MODEL

Although the switched inverter model is useful for a physical inverter, it is nonlinear because of the switching. Because of this a linear model must be designed in order to obtain a frequency characteristic which can be easily studied. Also the time required for simulating an average model is significantly lowered compared to the switched model because the simulation size step can be increased (the frequency of the switching doesn't appear in the average model). The average model is the ideal model for designing and developing control systems.
The modeling of the three-phase inverter controller with 3 terminals (A, B, C) is accomplished in the dq reference system. For the modeling of the four-phase inverter controller with 4 terminals (A, B, C and N) a reference rotary frame was used. Figure 2 shows a three-phase average inverter model with four terminals in the dq coordinate system. The passive elements of the model have the values from Table 1. We can observe that when modeling an inverter in the dq coordinates, the three channels are decomposed.

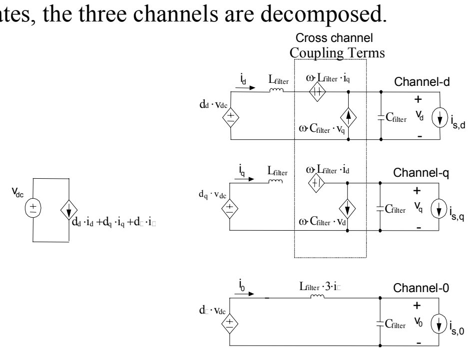

Figure 2. The average model of the inverter with 4 conductors in the dq coordinates

Besides the transient coupling conditions, shown in Figure 2, the system is equivalent to a SISO system, with three independent inputs and a single output. However, the coupling of the d and q channels can be significant in some situations and therefore cannot be ignored. The average model in figure 2 is used for simulating and developing control techniques [3].

3. No-load conditions

The inverters must function with or without load (light) or no-load, whatever their application domain. This condition seems to appear in both cases. First, the power utilized by the load decreases to zero, second, in the event that a malfunction occurs at the output, the inverter will have to work empty. Whatever the reason of the drop of the load, the inverter must be functional.

3.1. Open loop control to output transfer function

First, we consider the cross coupling of the channels as insignificant in the studied model, each channel of the inverter can be considered a simple SISOconvertor [1]

$$\begin{bmatrix} \dot{I}_L \\ \dot{V}_C \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{R_S \cdot C} \end{bmatrix} \cdot \begin{bmatrix} I_L \\ V_C \end{bmatrix} + \begin{bmatrix} \frac{V_{DC}}{L} \\ 0 \end{bmatrix} \cdot D_{d,q,0},$$

$$V_{d,q,0} = \begin{bmatrix} 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} I_L \\ V_C \end{bmatrix} \tag{1}$$

where R_S the balanced phase load, d, q and 0 are are the three channels, L and C is the passive components of the circuit in figure 3.
From the functional equations, the transfer function for the controlled output is:

$$H_v(s) = \frac{V_{DC}}{1 + \left(\frac{L}{Rs}\right) \cdot s + (L \cdot C) \cdot s^2}. \tag{2}$$

Expression (2) can be replaced with a low-pass filter of the second degree of the form:

$$H_v(s) = \frac{H_{v0}}{1 + \frac{s}{Q \cdot \omega_0} + \left(\frac{s}{\omega_0}\right)^2}, \tag{3}$$

$$H_{v0} = V_{DC}, \tag{4}$$

$$\omega_0 = \frac{1}{\sqrt{L \cdot C}}, \tag{5}$$

$$Q = R_S \cdot \sqrt{\frac{C}{L}}. \tag{6}$$

When running without a load ($R_S \to \infty$), the resonance frequency of the output filter is not damped and the quality factor aims to infinity. This is the ideal case of the output filter, but it does not represent the real physical system. Subsequent to adding the parasite resistance in the model of the system, as shown in figure 3, the resonance frequency of the output filter will be easy to dampen with its own parasite resistance when working without a load. The R_L and R_C resistances were chosen with a value of $10\ m\Omega$. Because of the coordinate transformations R_L from the null channel will be twice the inductance of the ESR filter (equivalent resistance in series) [3], [6].

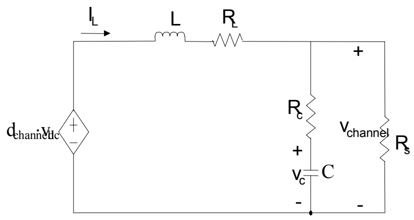

Figure 3. The average model in the dq0 coordinates including parasite resistance

From the model described earlier the following functional equations result [3]:

$$\begin{bmatrix} \dot{I}_L \\ \dot{V}_C \end{bmatrix} = \begin{bmatrix} -\frac{R_L}{L} - \frac{R_S \cdot R_C}{L \cdot (R_S + R_C)} & -\frac{R_S}{L \cdot (R_S + R_C)} \\ \frac{1}{C} - \frac{R_C}{C \cdot (R_S + R_C)} & -\frac{1}{C \cdot (R_S + R_C)} \end{bmatrix} \cdot \begin{bmatrix} I_L \\ V_C \end{bmatrix} + \begin{bmatrix} \frac{V_{DC}}{L} \\ 0 \end{bmatrix} \cdot D_{d,q,0}.$$

$$V_{d,q,0} = \begin{bmatrix} \frac{R_S \cdot R_C}{R_S + R_C} & \frac{R_S}{R_S + R_C} \end{bmatrix} \cdot \begin{bmatrix} I_L \\ V_C \end{bmatrix} \tag{7}$$

From the set of functional equations the transfer functions for the controlled output results:

$$H_v(s) = \frac{H_{v0} \cdot \left(1 + \frac{s}{\omega_z}\right)}{1 + \frac{s}{Q \cdot \omega_0} + \left(\frac{s}{\omega_0}\right)^2}, \tag{8}$$

$$H_{v0} = V_{DC} \cdot \left(\frac{R_S}{R_S + R_L}\right), \tag{9}$$

$$\omega_Z = \frac{1}{C \cdot R_C}, \tag{10}$$

$$\omega_0 = \sqrt{\frac{R_S + R_L}{L \cdot C \cdot R_S + L \cdot C \cdot R_C}}, \tag{11}$$

$$Q = \frac{\sqrt{(L \cdot C \cdot R_S + L \cdot C \cdot R_C) \cdot (R_S + R_L)}}{L + C \cdot (R_S \cdot R_L + R_S \cdot R_C + R_L \cdot R_C)}. \tag{12}$$

The earlier definitions illustrate two aspects of the control when the parasite resistance is included and also of the control over the transfer function at the output. Adding the equivalent resistance in series for the filter capacitor (ESR) results in a zero at high frequency (expression (8)); also, the capacitor-inductor combination dampens the resonance frequency, even without a load. Figure 4 shows the transfer functions at the controlled output for the dq0 inverter channels with a light load.

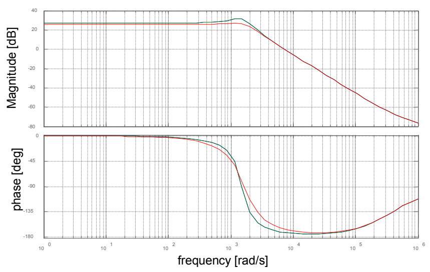

Figure 4. Frequency characteristics of the transfer functions at the controlled output of the system for the d, q, 0 channels

The resonance frequency of the 0 channel is half of that of the d and q channels because the equivalent induction of the 0 channel is four times higher than that of the d and q channels. Therefore the necessity of utilizing control systems for each of the four channels.

3.2. Open loop control to inductor current transfer function

Utilizing the equations in (7) and adding the parasite resistance of the filter, the transfer function of the control circuit for the current in the inductor can be obtained with the following expressions:

$$H_i(s) = \frac{H_{i0} \cdot \left(1 + \frac{s}{\omega_z}\right)}{1 + \frac{s}{Q \cdot \omega_0} + \left(\frac{s}{\omega_0}\right)^2}, \tag{13}$$

$$H_{i0} = V_{DC} \cdot \left(\frac{1}{R_S + R_L}\right), \tag{14}$$

$$\omega_Z = \frac{1}{C \cdot R_S + C \cdot R_C}, \tag{15}$$

$$\omega_0 = \sqrt{\frac{R_S + R_L}{L \cdot C \cdot R_S + L \cdot C \cdot R_C}}, \tag{16}$$

$$Q = \frac{\sqrt{(L \cdot C \cdot R_S + L \cdot C \cdot R_C) \cdot (R_S + R_L)}}{L + C \cdot (R_S \cdot R_L + R_S \cdot R_C + R_L \cdot R_C)}. \tag{17}$$

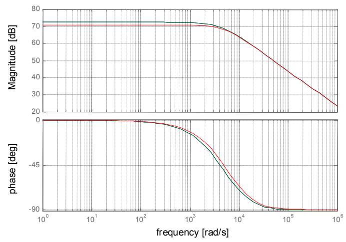

Figure 5. Frequency characteristics of the transfer function for the d, q and 0 channels

From the above expressions: the quality factor Q and the resonance frequency ω_0 of the system, they are identical with the ones from the system in which the parasite resistance was neglected [4].

The amplification H_{i0} (14) and the ω_z (15) zero change with the load resistance. Frequency characteristics for the transfer function (13) are presented in figure 6.

3.3. Transfer functions with cross-channel coupling

In principle, the effects of the cross-coupling of the channels are insignificant in the resulting transfer functions. This permits each channel, d, q and 0 to be reduced to a secondary SISO system.

The cross-coupling of the channels is insignificant, but it must be taken into consideration. Adding the effects of the cross-coupling, the d, q and 0 channels combine forming a 4th degree system, with two inputs and two outputs. In principle the transfer functions (8) and (13) result from this, a supplementary pair of poles and zeros. Figures 6 and 7 present the results of coupling the d and q channels.

The transfer function of the 0 channel remains unaffected because this channel is completely uncoupled from the d and q channels.

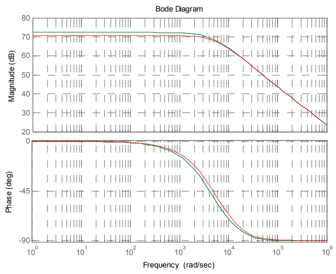

Figure 6. Frequency characteristics for the transfer functions of the controlled output when coupling the terms form the d, q and 0 channels

Figure 6a shows the block scheme in Simulink built upon the equations in (1).

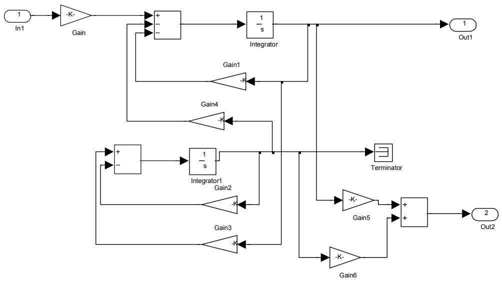

Figure 6a. Block scheme in Simulink of the inverter

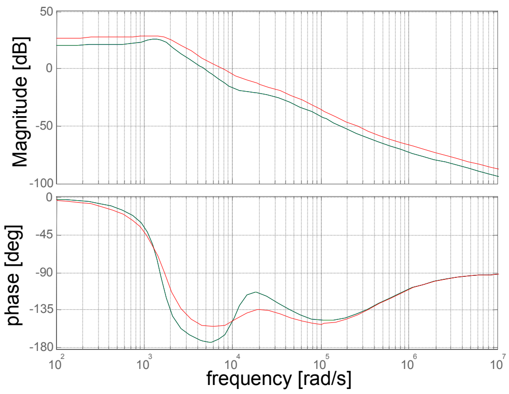

Figure 7 Frequency characteristics of the transfer functions of the current in the control-inductor when coupling the terms form the d, q and 0 channels

The bandwidth represents the difficulty of extending the control range over the resonance frequency with a traditional PID compensator, especially when the delay caused by the digital implementation is high. It would be possible to negate the poles directly when running without load utilizing a set of imaginary zeroes in the compensator.
But, at high load, the poles of the filter almost reverse and cannot be directly negated by the imaginary zeroes in the compensator. Therefore we can see the necessity of using adapters for the control circuit in order to negate the poles of the filter in all work conditions [5].

CONCLUSIONS

In this paper is presented an inverter model average in the dq0 coordinates and we determine the transfer functions of the system in an open/closed circuit. No-load, unbalanced loading and non-linear loading each have unique characteristics that negative influence the performance of the inverter. Because of the low control bandwidth of high power inverter, harmonic distortions due to non-linear loads can be a significant problem in inverter-fed power systems.

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