

AIRCRAFTS' MOVE STABILISATION USING OPTIMAL CRITERIA AND LINEAR STATE OBSERVERS

Mihai LUNGU

*Avionics Department, University of Craiova, Faculty of Electrotechnics,
Blv. Decebal, No.107, Craiova, Dolj, ROMANIA
Lma1312@yahoo.com*

Abstract – Using a synthesis algorithm of a linear state observer one estimates the state vector for longitudinal and lateral move of an aircraft (whose wings are or aren't affected by elastic deformations). Also one presents the estimation of a flexible aircraft's state vector. Gain matrix of the observer is calculated using the method of poles' positioning.

Keywords: *optimal, stabilization, observers, Liapunov.*

1. INTRODUCTION

Observers are used for estimation of flying object state vector using its input and output signals. Measuring some of the state vector's components (output vector's components) one reduces sensors' number especially the number of sensors for reach less or difficult measurable state variables (e.g. elastic deformations) [1], [2], [3].

First observer structures have been developed by Luenberger in 1966, 1971, 1979. The observers are used both for systems with known inputs and unknown inputs systems; their project belongs to different researchers (Bhatta-Charyya, 1978; Chen and Patton, 1999; Darouach, 1994; Hostetter and Meditch, 1973; Hou and Müller, 1992; Hui and Zak, 1993 and 2005; Kudva, 1980; Kurek, 1983; Wang, 1975; Yang and Wilde, 1988; Krzminski and Kakzorek, 2005 and so on).

Recent observers are obtained for systems with some unknown inputs or for systems with subsystems whose dynamic is unknown. For such systems one may use reduced order observers. Unknown inputs may be disturbances, errors and so on.

The estimator (filter) Kalman – Bucy [4], [5], [6] is the best observer for stochastic systems' state estimation. In case of nonlinear systems with partial known inputs or with unknown subsystems adaptive observers have been made; these observers use neural networks [7], [8], [9], [10], [11].

Recent researches deal with project of observers for the intern delay systems [12], [13], [14], based on Liapunov theory; observers' project means matrices inequalities solve. Linear state observers are used for x state estimation for linear system using measured

variables vector y , the input vector or some components of this vector.

2. STRUCTURE OF THE LINEAR STATE OBSERVER

Flying object's dynamic is described by state equations

$$\begin{aligned}\dot{x} &= Ax + Bu, \\ y &= Cx,\end{aligned}\tag{1}$$

where x is the state vector, u – the input vector, y – the output vector, A, B, C – matrices; let's consider x_0 – the vector of stationary state variables' values. The second equation (1) is the equation of the measurement system. One projects an optimal observer which gives an estimated state vector $\hat{x} \rightarrow x$. Linear state observer is modeled by equation [1]

$$\dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}),\tag{2}$$

where y has the form from the second equation (1) and \hat{y} is the estimated output vector,

$$\hat{y} = C\hat{x}.\tag{3}$$

Taking into account equations (1) and (3), equation (2) becomes

$$\dot{\hat{x}} = (A - LC)\hat{x} + Bu + LCx\tag{4}$$

and, with Bu from state equation (1) and error $e = \hat{x} - x$, one obtains

$$\dot{e} = \bar{A}e, \bar{A} = A - LC.\tag{5}$$

Matrix L of the observer may be calculated so that \bar{A} – stabile matrix.

With notation $z = \hat{y} - y$ and taking into account equations (1) and (3) one results

$$z = Ce.\tag{6}$$

If \bar{A} is a Hurwitz matrix, for every matrix $Q_c > 0$ and $Q_c = Q_c^T$, there is a matrix $P > 0$ which verifies

Liapunov equation

$$\bar{A}^T P + P \bar{A} + Q_c = 0. \quad (7)$$

Matrix P from Liapunov function

$$V(t) = e^T(t) P e(t) \quad (8)$$

assures achievement of condition $\dot{V}(t) > 0$ and Liapunov equation (7) assures the achievement of the stability condition ($\dot{V}(t) < 0$). By derivation of function (8) and having in mind (5) and (7) one results

$$\begin{aligned} \dot{V}(t) &= e^T P \dot{e} + \dot{e}^T P e = e^T P \bar{A} e + (\bar{A}^T e)^T P e = \\ &= e^T (P \bar{A} + \bar{A}^T P) e = -e^T Q_c e < 0. \end{aligned} \quad (9)$$

In figure 1 the block diagram of the optimal command system for the aircraft's move is presented. It consist of the following sub-systems: dynamic model of the flying object, measurement system, linear state observer and optimal command subsystem (gain matrix K). In this figure $w = v = 0$.

3. NUMERIC EXAMPLES

Let's consider the case of the longitudinal move of A. The equation which describes A's move is

$$\begin{bmatrix} \Delta \dot{V}_x \\ \Delta \dot{\alpha} \\ \Delta \dot{\theta} \\ \Delta \dot{\omega}_y \end{bmatrix} = \begin{bmatrix} -0.007 & 0.012 & -9.81 & 0 \\ -0.128 & -0.54 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0.065 & 0.96 & 0 & -0.99 \end{bmatrix} \begin{bmatrix} \Delta V_x \\ \Delta \alpha \\ \Delta \theta \\ \Delta \omega_y \end{bmatrix} + \begin{bmatrix} 0 \\ -0.04 \\ 0 \\ -12.5 \end{bmatrix} \delta_p. \quad (10)$$

and the measurement system is defined by $y = Cx$,

with $C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \end{bmatrix}$; the measured state variables are α and θ ; hence $y = [\theta \ 9]$, $9 = \theta - \alpha$ (the trajectory's slope). Matrix L is calculated using the method of poles' positioning (one imposed the eigen values of matrix $\bar{A} = (A - LC)$ of the closed loop

observer). The observer's poles are chosen such that their minimum real part is 5÷10 times smaller than maximum real part of the control system's poles (longitudinal dynamic of the aircraft)

$$|\operatorname{Re}\{\lambda_{\min}\}_{\text{observer}}| > 5 \div 10 |\operatorname{Re}\{\lambda_{\max}\}_{\text{system}}|. \quad (11)$$

One imposed the following values of the matrix \bar{A} : -60.15, -2.84, -18.90, -15.11; it results matrix

$$L^T = 10^3 \cdot \begin{bmatrix} 1.45 & 0.06 & 0.07 & 0.92 \\ -0.26 & -0.04 & -0.02 & -0.40 \end{bmatrix}. \quad (12)$$

Before calculus of L , the program presented in Appendix verifies if pair (A, C) is observable and pair (A, B) is controllable using instructions **OBSV** and **CTRB**; one obtained two matrices with rank 4; because the A is a quadratic matrix with 4 lines and 4 columns, one results that the model of the longitudinal move is controllable and observable.

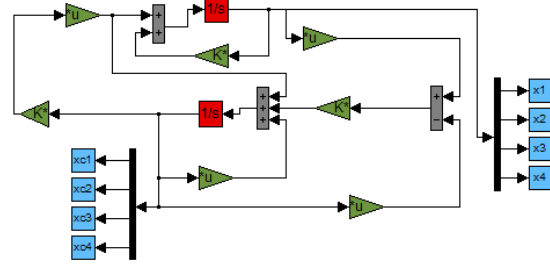


Figure 2: Matlab/Simulink model for the block diagram from figure 1

Matlab/Simulink model for the block diagram from figure 1 is the one from fig.2. Using this model, one calculates estimation errors for the state variables using observer described by equation (4) without command law ($K = 0$); these errors are presented in fig.3. With the estimated state vector one achieves

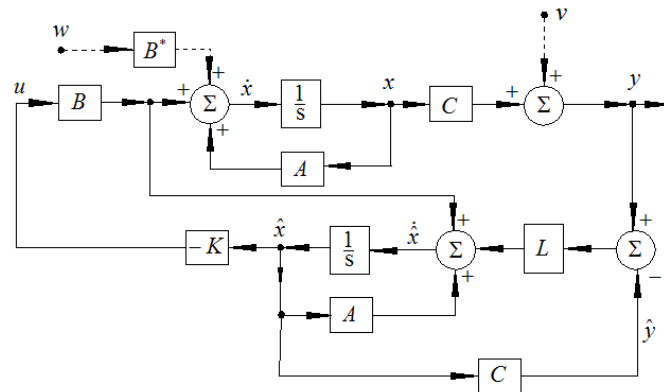


Figure 1: Block diagram of the optimal command system for the aircraft's move with state observer

the command of the longitudinal move ($u = -K\hat{x}$, with K obtained using algorithm ALGLX [15])

$$K = [2.225 \quad -0.316 \quad -10.736 \quad -1.428]. \quad (13)$$

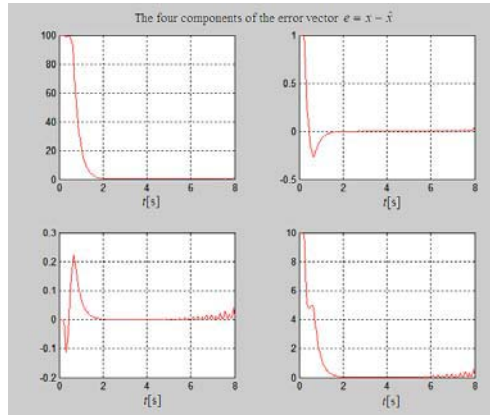


Figure 3: Estimation errors of the state variables ($u = 0$)

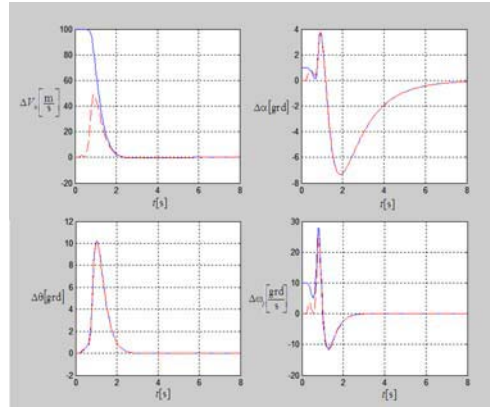


Figure 4: State variables $x_i(t)$ and $\hat{x}_i(t)$ for $u = -K\hat{x}$

Using again Matlab/Simulink model from fig.2 one represents the time characteristic from fig. 4; these express state variables $x_i(t)$ – with blue continuous line and $\hat{x}_i(t)$ – with red dashed line; it is obvious that $\hat{x}_i(t) \rightarrow x_i(t) \rightarrow 0$ in a short period of time.

For the longitudinal move of a deformable aircraft, with state vector

$$x^T = [\Delta V_z \quad \Delta \omega_y \quad \dot{\lambda}_1 \quad \dot{\lambda}_2 \quad \dot{\lambda}_3 \quad \dot{\lambda}_4 \quad \dot{\lambda}_5 \quad \dot{\lambda}_6 \quad \lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \lambda_4 \quad \lambda_5 \quad \lambda_6 \quad \delta_e \quad \delta_{pi} \quad \delta_{pe}]$$

and $u = \delta_{pi,c}$, with command matrices A, B and matrix C of the measurement system [1]

$$A = \begin{bmatrix} -0.680 & -3.28 & -0.04 & -0.01 & -0.02 & -0.02 & -0.02 & 0.05 & -0.64 & -0.65 & -1.71 & 4.5 & -1.36 & 1.85 & -2.3 & -190 & -30.1 \\ -0.55 & -1.17 & 0.04 & -0.04 & -0.33 & -0.32 & -0.12 & 0.25 & -0.11 & -1.68 & -10.3 & -5.7 & -4.94 & 9.11 & -576 & -2500 & -440 \\ -1.49 & 0.11 & -0.99 & -0.02 & -0.15 & 0.51 & 0.02 & -0.26 & -29.9 & -4.09 & -16.6 & -40.6 & -5.84 & -6.69 & 3400 & 1420 & 260 \\ 0.05 & -0.09 & 0.01 & -0.47 & -0.07 & -0.06 & -0.01 & 0 & 0.27 & -124 & -0.87 & -1.58 & -0.16 & 1.51 & -133 & -292 & -63.9 \\ 0.05 & -0.85 & 0.05 & -0.07 & -1.29 & -0.41 & -0.03 & -0.06 & 1.4 & -0.65 & -193 & -15.4 & -0.98 & -3.15 & -1420 & -2570 & -582 \\ -1.11 & -0.99 & 0.08 & -0.04 & -0.27 & -1.09 & -0.18 & 0.44 & -0.2 & -0.68 & -9.37 & -243 & -7.18 & 17 & 804 & -2530 & -625 \\ -0.2 & -0.24 & 0.06 & -0.02 & -0.05 & -0.11 & -0.86 & 0.12 & 0.23 & -0.75 & -1.3 & -3.23 & -306 & 6.08 & -13.1 & -653 & -179 \\ 0.46 & 0.42 & -0.11 & 0.02 & 0.04 & 0.18 & 0.1 & -1.15 & -0.12 & 2.01 & 1.5 & 4.5 & 4.77 & -352 & -411 & 1120 & 323 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B^T = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 75 \quad 0]$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

choosing eigen values of matrix \bar{A} conform equation (11)

$$\begin{aligned} & -6.11 + 1.21i, -6.11 - 1.21i, -0.16, -0.025 + 0.054i, \\ & -0.025 - 0.054i, -0.028 + 0.186i, -0.028 - 0.186i, -0.021 + 0.17i, \\ & -0.021 - 0.17i, -0.01 + 0.1i, -0.01 - 0.1i, -0.02 + 0.15i, \\ & -0.02 - 0.15i, -0.02 + 0.13i, -0.02, -0.13i, -0.3, -0.37, \end{aligned}$$

one obtains characteristics $x_i(t)$ and $\hat{x}_i(t)$ (blue color respectively red color) for $u = 0$ (fig. 5) and errors $e_i(t) = x_i(t) - \hat{x}_i(t)$ (fig. 6) for matrix K calculated with the same algorithm ALGLX [15].

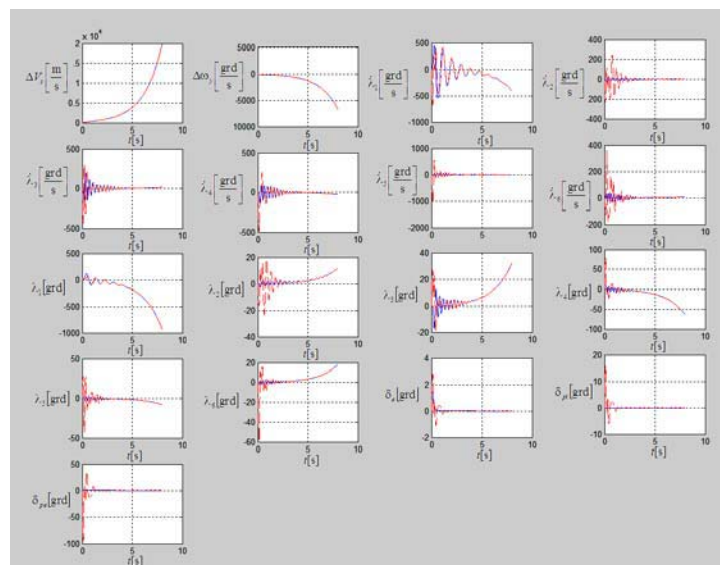
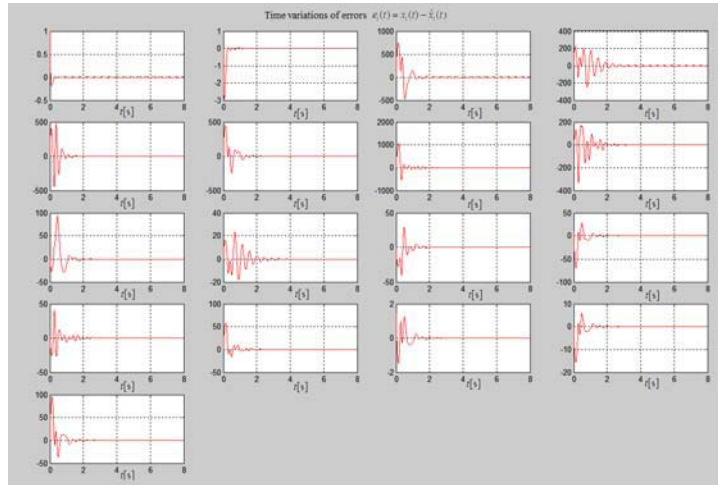


Figure 5: State variables $x_i(t)$ and $\hat{x}_i(t)$ for the longitudinal move of an elastic deformable aircraft ($u = 0$)

Figure 6: Estimation errors $e_i(t) = x_i(t) - \hat{x}_i(t)$ for $u = -K\hat{x}$ (longitudinal move)

Let's consider now the case of lateral move of an aircraft described by equation

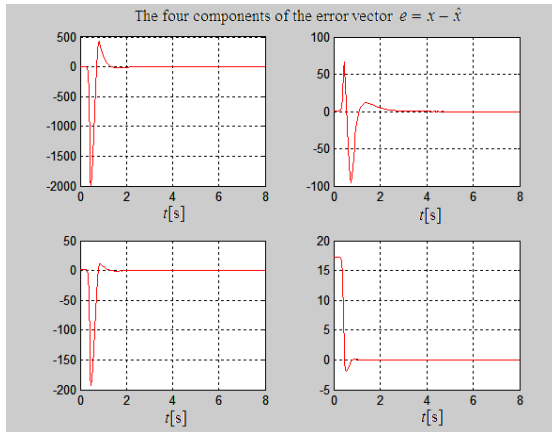
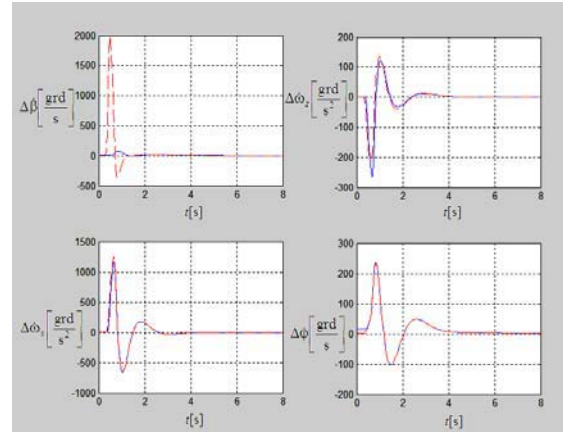
$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{\omega}_z \\ \Delta\dot{\omega}_x \\ \Delta\dot{\varphi} \end{bmatrix} = \begin{bmatrix} -0.0558 & -0.9968 & 0.0802 & 0.0415 \\ 0.598 & -0.115 & -0.0318 & 0 \\ 0.305 & 0.388 & -0.465 & 0 \\ 0 & 0.0805 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta\omega_z \\ \Delta\omega_x \\ \Delta\varphi \end{bmatrix} + \begin{bmatrix} 0.0073 & 0 \\ -0.475 & 0.123 \\ 0.153 & 1.063 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_d \\ \delta_r \end{bmatrix} \quad (14)$$

with $C = [0 \ 0 \ 0 \ 1]$, choosing eigen values of matrix \bar{A} conform to (11), $-66.27, -3.97, -10.36, -1.58$, one obtained matrix

$$L^T = 10^4 \cdot [1.21 \ -0.05 \ 0.11 \ 0.008]. \quad (15)$$

Using the same Matlab/Simulink model from figure 2, one represents time characteristics from figure 7 (estimation errors of the observer's state variables for $K = 0$) and time characteristics from figure 8 ($x_i(t)$ – with blues continuous line and $\hat{x}_i(t)$ – with red dashed line);

$$K = \begin{bmatrix} -7.63 & -19.94 & -1.37 & 7.93 \\ -8.55 & 13.79 & 4.60 & 5 \end{bmatrix}. \quad (16)$$

Figure 7: Estimation errors of the state variables ($u = 0$) for the lateral move of the aircraftFigure 8: State variables $x_i(t)$ and $\hat{x}_i(t)$ of the lateral move's optimal command system

4. APPENDIX

% Longitudinal move n=4

```
clear all; run prog2sec; close all;
clear A; clear B; clear C; clear D;
clear e; clear ee; clear R; clear r1;
clear contor;
```

% Matrices of the system

```
A=[-0.007 0.012 -9.81 0;-0.128 -0.54 0 1;
    0 0 0 1;0.065 0.96 0 -0.99];
```

```
B=[0;-0.04;0;-12.5];
```

```
C=[0 0 1 0;0 -1 1 0];
```

```
n=size(A,1);
```

```
m=size(B,2);
```

```
s=size(C,1);
```

```

D=zeros(s,m);
sys=ss(A,B,C,D);
% Initial state (x0)
x0=[100;1;0;10];
% Initial state of the observer (xc0)
xc0=zeros(n,1);
% Eigen values of the closed loop system (A-B*K)
p=eig(A-B*K);
% Eigen values of the observer (q)
for j=1:length(p)
    R(j)=real(p(j));
    I(j)=imag(p(j));
    RR=R*5; II=I;
    ee(j)=RR(j)+II(j)*i;
    q=ee';
end
% Controllability and observability calculus matrices
OB=obsv (A, C);
CO=ctrb (A, B);
r1=rank(OB);
r2=rank(CO);
L=place(A',C',q);
% Determination of matrix L using poles' positioning method
L=L';
% Observer matrices' calculus
Aobs=A-L*C;
Bobs=[B L];
Cobs=C;
Dobs=zeros(n,s+m);
KK=K;
% Matlab model simulation (K=0)
K=zeros(m,n);
sim('schobs2');
subplot(221);
plot(t,x1-xc1,'r');grid;
subplot(222);
plot(t,x2-xc2,'r');grid;
subplot(223);
plot(t,x3-xc3,'r');grid;
subplot(224);
plot(t,x4-xc4,'r');grid;
% Block diagram simulation with controller (K=K)
K=KK;
sim('schobs2');
h=figure;
subplot(221);
plot(t,x1,'b',t,xc1,'r--');grid;
subplot(222);
plot(t,x2,'b',t,xc2,'r--');grid;
subplot(223);
plot(t,x3,'b',t,xc3,'r--');grid;
subplot(224);
plot(t,x4,'b',t,xc4,'r--');grid;

```

5. CONCLUSIONS

The paper presents the structure of a linear state observer and this structure is then used for estimation of the state vector for longitudinal and lateral move of an aircraft (using a Matlab algorithm made by the author). Gain matrix of the observer is calculated using the method of poles' positioning. The observer's poles are chosen such that their minimum real part is $5 \div 10$ times smaller than maximum real part of the control system's poles (longitudinal dynamic of the aircraft). One calculates estimation errors for the state variables using observer without command law ($K=0$) or with it ($u = -K\hat{x}$, with K obtained using algorithm ALGLX).

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