Annals of the University of Craiova, Electrical Engineering series, No. 33, 2009; ISSN 1842-4805

7TH INTERNATIONAL CONFERENCE ON ELECTROMECHANICAL AND POWER SYSTEMS
October 8-9, 2009 - Iaşi, Romania

WORKING OUT OF ANALYTICAL EXPRESSIONS FOR THE SIMPLIFIED OPERATIVE DEFINITION OF ADDITIONAL LOSSES AT VARIOUS SCENARIOS OF EXPORT

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Abstract – **Because export and consumption schedules vary from a month by a month accounts of stray load losses in networks of state enterprise "Moldelectrica", caused by export of electric energy from the Moldavian state district power station, it is required to carry out monthly.**
Therefore in the given work the task in view to develop the simplified technique of operative definition of losses from export without monthly repeated performance of accounts under programs of the established modes.
The generalized analytical expressions which would allow to carry out accounts of the losses caused by export with maintenance of good accuracy for any scenario of export are with that end in view received. Polynoms from transmission capacities, having in a basis the theory and methods of interpolation functions can be such expressions.

Keywords: *Additional losses of energy, the networks 110÷330 kV.*

1. THE GENERAL FORMULATION OF A PROBLEM

For the problem decision it is necessary to have so-called "branch points", that is the same meanings of stray load losses at various scenarios of export through which there should pass approximating these losses curves. These branch points can be received by performance of accounts under programs of established modes PSS/E for possible scenarios of export in working and the days off.
In work it is executed such 1008 accounts.
Results are presented in таб.1 and they have formed a basis for reception of analytical expressions and for check of accuracy of received results.
In the theory of interpolation functions two cases are possible:
- The first - interpolated function depends only on one independent variable;
- The second - interpolated function depends on several variables.

In the first case analytical expression can be received from a condition that its numerical meaning has coincided precisely with meanings of approximated function in some points named "central". The such concern interpolate polynom of Lagrange, Newton, Gauss, Sterling, etc.
In the second a case interpolating expressions geometrically represent surfaces and to achieve their exact passage through branch points it is practically impossible. Therefore in this case it is necessary to recede from the specified requirement and to demand, that numerical meanings of approximating expression in branch points came nearer as much as possible to numerical meanings of approximated functions, satisfying demanded accuracy. In this case the range of possibilities by selection of approximating functions and their combinations extends. Possibility of selection of approximating functions here turns out, being based on engineering intuition.
In our case approximated function of stray load losses in the Moldavian electric power system depends on four unknown persons caused by export on: Isaccea, Husi, Stinca, Russia (Belarus).

2. MATHEMATICAL ASPECTS OF THE FORMULATION AND THE PROBLEM DECISION

It is required to define some constant factors $a_1,a_1 \ldots a_n$ so that expression:

$$y\left(x^{(1)},x^{(2)} \ldots x^{(n)}\right)=a_0\varphi_0\left(x^{(1)}\right)+a_1\varphi_1\left(x^{(2)}\right)+\ldots+a_n\varphi_n\left(x^{(n-1)}\right) \quad (1)$$

at substitution in it of meanings of capacity $x_i^{(1)},x_i^{(2)},\ldots x_i^{(n)}$ transmitted in one of the specified directions represented stray load losses in the Moldavian electric power system from the given export.
Here $\varphi_0\left(x^{(1)}\right),\varphi_1\left(x^{(2)}\right)\ldots\varphi_n\left(x^{(n)}\right)$ are functions on which approximation is carried out.

The quantity of the equations of type (1) will coincide with quantity of branch points and we will choose its such that exceeded quantity m weight coefficients $a_1, a_1 \ldots a_m$.
Arriving thus for all branch points we will receive n the equations with m in unknown weight coefficients

$$\left.\begin{array}{l} a_0\varphi_0\left(x_1^{(1)}\right)+a_1\varphi_1\left(x_1^{(2)}\right)+\ldots+a_m\varphi_m\left(x_1^{(m-1)}\right)=y_1; \\ a_0\varphi_0\left(x_2^{(1)}\right)+a_1\varphi_1\left(x_2^{(2)}\right)+\ldots+a_m\varphi_m\left(x_2^{(m-1)}\right)=y_2; \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ a_0\varphi_0\left(x_n^{(1)}\right)+a_1\varphi_1\left(x_n^{(2)}\right)+\ldots+a_m\varphi_m\left(x_n^{(m-1)}\right)=y_n, \end{array}\right\} \quad (2)$$

Here $n>m$.
Systems, in which the number of the equations exceeds number of unknown persons, are called as redefined. For them it is impossible to find such meanings of unknown persons, in our case $a_0, a_1 \ldots a_m$ which strictly would satisfy to system (2). Therefore the problem is put to find such meanings of factors $a_0, a_1 \ldots a_m$ for which the sum of squares of a difference of the left and right parts of system (2) would be the least.
Method allowing solving a problem thus is called as a least-squares method.
Its essence consists in the following.
Let a difference between the left part (2) after substitution in functions $\varphi_i(x_i)$ and losses in Moldavian electric power system y_i at export x_i it is equal ε. Then it is possible to write down:

$$\left.\begin{array}{l} a_0\varphi_0\left(x_1^{(1)}\right)+a_1\varphi_1\left(x_1^{(2)}\right)+\ldots+a_m\varphi_m\left(x_1^{(m-1)}\right)-y_1=\varepsilon_1; \\ a_0\varphi_0\left(x_2^{(1)}\right)+a_1\varphi_1\left(x_2^{(2)}\right)+\ldots+a_m\varphi_m\left(x_2^{(m-1)}\right)-y_2=\varepsilon_2; \\ \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\ a_0\varphi_0\left(x_n^{(1)}\right)+a_1\varphi_1\left(x_n^{(2)}\right)+\ldots+a_m\varphi_m\left(x_n^{(m-1)}\right)-y_m=\varepsilon_n, \end{array}\right\} \quad (3)$$

Let's consider as the decision (2) those of approach x_i for which the sum of squares $\sum_{i=1}^{n}\varepsilon_i^2$ will be the least.
Let's make the sum of squares $\sum_{i=1}^{n}\varepsilon_i^2$

$$\begin{aligned} Q &= \varepsilon_1^2+\varepsilon_2^2+\ldots+\varepsilon_n^2= \\ &= \left(a_0\varphi_0\left(x_1^{(1)}\right)+a_1\varphi_1\left(x_1^{(2)}\right)+\ldots+a_m\varphi_m\left(x_1^{(m-1)}\right)-y_1\right)^2+ \\ &+ \left(a_0\varphi_0\left(x_2^{(1)}\right)+a_1\varphi_1\left(x_2^{(2)}\right)+\ldots+a_m\varphi_m\left(x_2^{(m-1)}\right)-y_2\right)^2+\ldots+ \\ &+ \left(a_0\varphi_0\left(x_n^{(1)}\right)+a_1\varphi_1\left(x_n^{(2)}\right)+\ldots+a_m\varphi_m\left(x_n^{(m-1)}\right)-y_m\right)^2 \end{aligned} \quad (4)$$

The sum (4) will have the least meaning at equality to zero of derivatives $\frac{\partial Q}{\partial a_0}, \frac{\partial Q}{\partial a_1} \ldots \frac{\partial Q}{\partial a_m}$.
These derivatives will be:

$$\begin{aligned} \frac{\partial Q}{\partial a_0} &= 2\varphi_0\left(x_1^{(1)}\right)\left(a_0\varphi_0\left(x_1^{(1)}\right)+a_1\varphi_1\left(x_1^{(2)}\right)+\ldots+a_m\varphi_m\left(x_1^{(m-1)}\right)-y_1\right)+ \\ &+2\varphi_0\left(x_2^{(1)}\right)\left(a_0\varphi_0\left(x_2^{(1)}\right)+a_1\varphi_1\left(x_2^{(2)}\right)+\ldots+a_m\varphi_m\left(x_2^{(m-1)}\right)-y_2\right)+\ldots+ \\ &+2\varphi_0\left(x_m^{(m-1)}\right)\left(a_0\varphi_0\left(x_n^{(1)}\right)+a_1\varphi_1\left(x_n^{(2)}\right)+\ldots+a_m\varphi_m\left(x_n^{(m-1)}\right)-y_m\right)=0; \end{aligned} \quad (5)$$

$$\begin{aligned} \frac{\partial Q}{\partial a_1} &= 2\varphi_1\left(x_1^{(1)}\right)\left(a_0\varphi_0\left(x_1^{(1)}\right)+a_1\varphi_1\left(x_1^{(2)}\right)+\ldots+a_m\varphi_m\left(x_1^{(m-1)}\right)-y_1\right)+ \\ &+2\varphi_1\left(x_2^{(1)}\right)\left(a_0\varphi_0\left(x_2^{(1)}\right)+a_1\varphi_1\left(x_2^{(2)}\right)+\ldots+a_m\varphi_m\left(x_2^{(m-1)}\right)-y_2\right)+\ldots+ \\ &+2\varphi_1\left(x_m^{(m-1)}\right)\left(a_0\varphi_0\left(x_n^{(1)}\right)+a_1\varphi_1\left(x_n^{(2)}\right)+\ldots+a_m\varphi_m\left(x_n^{(m-1)}\right)-y_m\right)=0; \end{aligned}$$

… … … … … … … … … … … …

Easy to notice that (5) it is possible to write down:

$$\begin{bmatrix} \varphi_0\left(x_1^{(1)}\right) & \varphi_0\left(x_2^{(1)}\right) & \ldots & \varphi_0\left(x_1^{(1)}\right) \\ \varphi_1\left(x_1^{(2)}\right) & \varphi_1\left(x_2^{(2)}\right) & \ldots & \varphi_1\left(x_2^{(2)}\right) \\ \ldots & \ldots & \ldots & \ldots \\ \varphi_m\left(x_1^{(m-1)}\right) & \varphi_m\left(x_2^{(m-1)}\right) & \ldots & \varphi_m\left(x_n^{(m-1)}\right) \end{bmatrix}\times , \quad (6)$$

$$\times\left\|\begin{bmatrix} \varphi_0\left(x_1^{(1)}\right) & \varphi_1\left(x_1^{(2)}\right) & \ldots & \varphi_m\left(x_1^{(m-1)}\right) \\ \varphi_0\left(x_2^{(1)}\right) & \varphi_1\left(x_2^{(2)}\right) & \ldots & \varphi_m\left(x_2^{(m-1)}\right) \\ \ldots & \ldots & \ldots & \ldots \\ \varphi_0\left(x_n^{(1)}\right) & \varphi_1\left(x_n^{(2)}\right) & \ldots & \varphi_m\left(x_n^{(m-1)}\right) \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \\ \ldots \\ a_m \end{bmatrix}-\begin{bmatrix} y_1 \\ y_2 \\ \ldots \\ y_m \end{bmatrix}\right\|=\begin{bmatrix} 0 \\ 0 \\ \ldots \\ 0 \end{bmatrix}$$

Or in the generalized form:

$$[\varphi]^T[\varphi][a]-[\varphi]^T[y]=[0], \quad (7)$$

Where $[\varphi]^T$ - there is transposed matrix $[\varphi]$.
Thus, the redefined system (3) is led to certain system

$$\begin{bmatrix} \varphi_0\left(x_1^{(1)}\right) & \varphi_0\left(x_2^{(1)}\right) & \ldots & \varphi_0\left(x_n^{(1)}\right) \\ \varphi_1\left(x_2^{(2)}\right) & \varphi_1\left(x_2^{(2)}\right) & \ldots & \varphi_1\left(x_n^{(2)}\right) \\ \ldots & \ldots & \ldots & \ldots \\ \varphi_m\left(x_1^{m-(1)}\right) & \varphi_m\left(x_2^{(m-1)}\right) & \ldots & \varphi_m\left(x_n^{(m-1)}\right) \end{bmatrix}\times$$

$$\times\begin{bmatrix} \varphi_0\left(x_1^{(1)}\right) & \varphi_1\left(x_1^{(2)}\right) & \ldots & \varphi_m\left(x_1^{(m-1)}\right) \\ \varphi_0\left(x_2^{(1)}\right) & \varphi_1\left(x_2^{(2)}\right) & \ldots & \varphi_m\left(x_2^{(m-1)}\right) \\ \ldots & \ldots & \ldots & \ldots \\ \varphi_0\left(x_n^{(1)}\right) & \varphi_1\left(x_n^{(2)}\right) & \ldots & \varphi_m\left(x_n^{(m-1)}\right) \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \\ \ldots \\ a_m \end{bmatrix}= \quad (8)$$

$$=\begin{bmatrix} \varphi_0\left(x_1^{(1)}\right) & \varphi_0\left(x_2^{(1)}\right) & \ldots & \varphi_0\left(x_n^{(1)}\right) \\ \varphi_1\left(x_2^{(2)}\right) & \varphi_1\left(x_2^{(2)}\right) & \ldots & \varphi_1\left(x_n^{(2)}\right) \\ \ldots & \ldots & \ldots & \ldots \\ \varphi_m\left(x_1^{m-(1)}\right) & \varphi_m\left(x_2^{(m-1)}\right) & \ldots & \varphi_m\left(x_n^{(m-1)}\right) \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ \ldots \\ y_m \end{bmatrix}.$$

The system (8) has dimension $m\times m$, i.e. is defined and dares one of known methods.
In work there were tested various polynoms for various combinations of export and self-consumption of SE "Moldelectrica". In total 1008 accounts under program PSS/E have been executed.

Mode	Isaccea	Husi	Stinca	Russia	*The working day*						*The day off*					
					23:00-7:00			*7:00-23:00*			*23:00-7:00*			*7:00-23:00*		
					PSS/E	TELME		PSS/E	TELME		PSS/E	TELME		PSS/E	TELME	
1	0	0	0	50	-0,3	-0,345	-0,3	-0,4	-0,361	-0,4	-0,3	-0,344	-0,3	-0,4	-0,356	-0,4
2	0	0	0	150	-0,8	-0,846	-0,8	-0,9	-0,862	-0,9	-0,8	-0,845	-0,8	-0,9	-0,857	-0,9
3	0	0	0	250	-1,2	-1,179	-1,2	-1,2	-1,195	-1,2	-1,2	-1,178	-1,2	-1,2	-1,189	-1,2
4	0	0	0	350	-1,3	-1,342	-1,3	-1,4	-1,359	-1,4	-1,3	-1,341	-1,3	-1,4	-1,353	-1,4
5	0	0	0	450	-1,3	-1,338	-1,3	-1,4	-1,354	-1,4	-1,3	-1,337	-1,3	-1,3	-1,348	-1,3
…																
247	150	15	35	50	2,5	2,480	2,5	2,5	2,521	2,5	2,5	2,478	2,5	2,5	2,508	2,5
248	150	15	35	150	1,9	1,883	1,9	1,9	1,924	1,9	1,9	1,881	1,9	1,9	1,910	1,9
249	150	15	35	250	1,6	1,579	1,6	1,6	1,620	1,6	1,6	1,577	1,6	1,6	1,607	1,6
250	150	15	35	350	1,6	1,569	1,6	1,6	1,610	1,6	1,6	1,567	1,6	1,6	1,597	1,6
251	150	15	35	450	1,9	1,853	1,9	N/A	N/A	N/A	1,9	1,851	1,9	N/A	N/A	N/A

As a numerical example we will define losses from energy transit to Isaccea taking into account self-consumption of SE "Moldelectrica".

Losses from export we will approximate a polynom

$$\Delta\Delta P = A\,P_{isaccea} + B\;P_{MCE} + C\,P_{isaccea}^{4}\,. \quad (9)$$

Data on transit and losses we will accept by results of account of modes from tab. 1.

Initial data are presented to tab. 2.

Table 2

Export to Isaccea	MD Consumption	Losses from export under program PSS/E	The results received by approximation on (9)
60	377	0,4	0,413
60	569	0,4	0,414
120	569	1	1,015
30	366	0,2	0,198
150	506	1,5	1,497
90	506	0,7	0,677

Let's substitute export and consumption of SE "Modelectrica" in (9) and we will receive a matrix of system of the redefined equations and a matrix of the right parts in a kind

$$[K]=\begin{bmatrix} 60 & 377 & 1.296\times10^{7} \\ 60 & 569 & 1.296\times10^{7} \\ 120 & 569 & 2.074\times10^{8} \\ 30 & 366 & 8.100\times10^{5} \\ 150 & 506 & 5.063\times10^{8} \\ 90 & 506 & 6.571\times10^{7} \end{bmatrix}, \quad (10)$$

$$[L]=\begin{bmatrix} 0.4 \\ 0.4 \\ 1 \\ 0.2 \\ 1.5 \\ 0.7 \end{bmatrix}. \quad (11)$$

The transposed matrix looks like

$$[K]^T=\begin{bmatrix} 60 & 60 & 120 & 30 & 150 & 90 \\ 377 & 569 & 569 & 366 & 506 & 506 \\ 12960000 & 12960000 & 207360000 & 810000 & 506250000 & 65610000 \end{bmatrix}. \quad (12)$$

Product of transposed matrix $[K]^T$ on $[K]$ gives

$$[K]^T[K]=\begin{bmatrix} 5.31\times10^{4} & 2.575\times10^{5} & 1.083\times10^{11} \\ 2.575\times10^{5} & 1.436\times10^{6} & 4.199\times10^{11} \\ 1.083\times10^{11} & 4.199\times10^{11} & 3.039\times10^{17} \end{bmatrix}. \quad (13)$$

Product of the transposed matrix on a matrix of the right parts gives

$$[K]^T[L]=\begin{bmatrix} 60 & 60 & 120 & 30 & 150 & 90 \\ 377 & 569 & 569 & 366 & 506 & 506 \\ 12960000 & 12960000 & 207360000 & 810000 & 506250000 & 65610000 \end{bmatrix}\times$$

$$\times\begin{bmatrix} 0.4 \\ 0.4 \\ 1 \\ 0.2 \\ 1.5 \\ 0.7 \end{bmatrix}=\begin{bmatrix} 462 \\ 2.134\times10^{3} \\ 1.023\times10^{9} \end{bmatrix} \quad (14)$$

Thus we will receive simultaneous equations

$$\begin{bmatrix} 5.31\times10^{4} & 2.575\times10^{5} & 1.083\times10^{11} \\ 2.575\times10^{5} & 1.436\times10^{6} & 4.199\times10^{11} \\ 1.083\times10^{11} & 4.199\times10^{11} & 3.039\times10^{17} \end{bmatrix}\begin{bmatrix} A \\ B \\ C \end{bmatrix}=\begin{bmatrix} 462 \\ 2.134\times10^{3} \\ 1.023\times10^{9} \end{bmatrix}, \quad (15)$$

Having solved which we will receive

$$\begin{bmatrix} A \\ B \\ C \end{bmatrix}=\begin{bmatrix} 0.0713350598\ \ 54683 \\ -0.0000451358\ \ 90448 \\ 0.0000000008\ \ 8842487 \end{bmatrix}. \quad (16)$$

Having substituted factors *And, In, With* from (16) in a polynom (9) for the export presented to tab.2 have received the sizes of losses resulted for comparison in last column of tab.2. The results received in a yielded example testify to working capacity of the specified technique and maintenance of good accuracy.

By the given technique approximating polynoms for

definition of additional losses in SE “Moldelectrica” from export of energy to Romania and Russia (Belarus) have been received.
Let's notice, that structure of polynoms and its factors, providing accuracy of results have been received for the following limits of export co-ordinated with the customer:
Export to:

Isaccea $1 \leq P_{tranzit} \leq 130$ MW;

Husi $1 \leq P_{tranzit} \leq 30$ MW;

Stinca $1 \leq P_{tranzit} \leq 50$ MW;

Russia (Belarus) $1 \leq P_{tranzit} \leq 400$ MW.

At these data the good accuracy received on polynoms at consumption in SE “Moldelectrica” no more 800 MW is provided.
We pay attention, that at excess of the specified limits of export or consumption SE “Moldelectrica” accuracy of results worsens, and for these modes new researches be required.

3. APPROXIMATING POLYNOMS

1. Export separately to Isaccea, Husi, Stinca, Russia (Belarus)

$$\Delta\Delta P = A \cdot P_{ME} + B \cdot P_i + C \cdot P_i^2, \quad (17)$$

Where P_{ME} here and more low consumption SE “Moldelectrica”;

P_i - Export to one of the specified directions.

As an example for a polynom (17), at export to Stinca, we result factors:
A = 0,37865372747E-04;
B =-0,68142892335E-02;
C = 0,71125739629E-03.
Approximating polynoms for a mode of two variables are graphically presented on fig. 1 - 2.

Let's notice, that factors *And, In, With* are various for each direction of export and protection in TELME.
2. Export to Husi-Stinca and Isaccea-Russia

$$\Delta\Delta P = A \cdot P_{ME} + B \cdot P_i + C \cdot P_i^2 + D \cdot P_j + E \cdot P_j^2, \quad (18)$$

Here:

In I variant: P_i - export to Husi, P_j - Stinca;

In II variant P_i - export to Исакчу, P_j - export to Russia.

3. Export to Isaccea - Husi - Stinca

$$\Delta\Delta P = A \cdot P_{ME} + B \cdot P_i + C \cdot P_i^2 + D \cdot P_j + E \cdot P_j^2 + \\ + F \cdot P_k + G \cdot P_k^2 + H \cdot P_e + L \cdot P_e^2, \quad (19)$$

Here: P_i - Export to Isaccea;

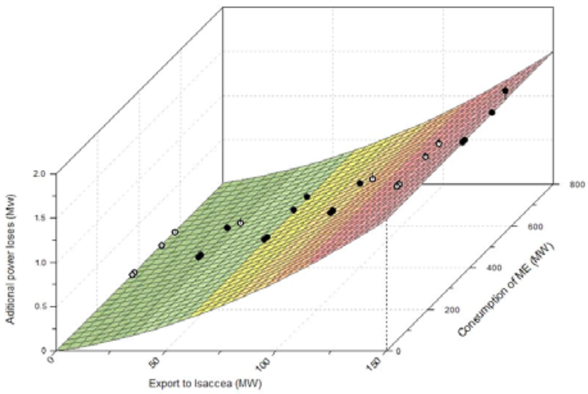

Fig. 1. Graphic representation of export to Isaccea.

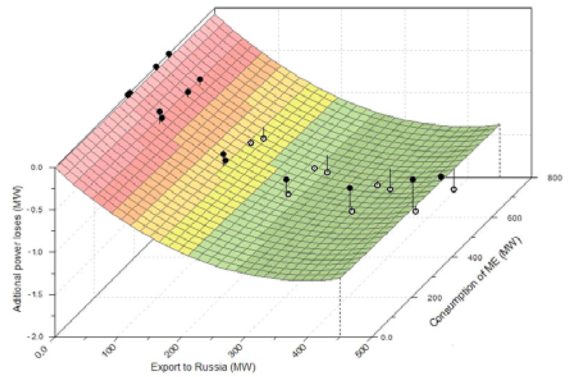

Fig. 2. Graphic representation of export to Russia.

P_j - Export to Husi;

P_k - Export to Stinca.

4. Export to Isaccea - Husi - Stinca - Russia

$$\Delta\Delta P = A \cdot P_{ME} + B \cdot P_i + C \cdot P_i^2 + D \cdot P_j + E \cdot P_j^2 + \\ + F \cdot P_k + G \cdot P_k^2 + H \cdot P_e + L \cdot P_e^2, \quad (20)$$

Here P_e - export to Russia (Belarus).

For using of the dispatcher program TELME in which have been sewn up and protected from extraneous access, except developers, factors of polynoms (17-20) has been made. Comparative results of accounts of the losses executed under program PSS/E and TELME for various scenarios of export of capacity from the Moldavian state district power station are presented in tab. 2.
The program interface is presented on fig. 3.
Comparison of results shows good accuracy of the round data received on TELME.
Work of the dispatcher with program TELME is reduced to a choice of one of 9 modes of export and to introduction of sizes of exported capacities.

4. CONCLUSIONS

Was approximating polynoms are received and the program is made, allowing to define with comprehensible accuracy additional losses in the Moldavian electric power system, caused by export of energy from the Moldavian state district power station to Romania and Russia (Belarus).

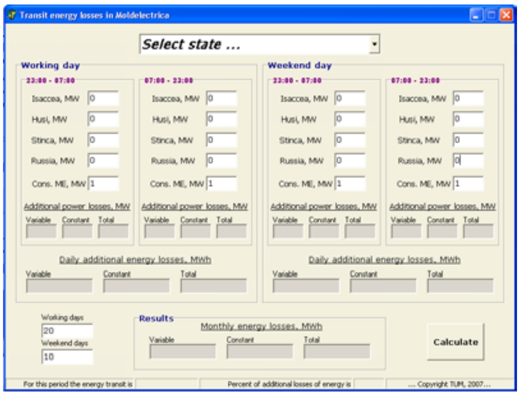

Fig. 3. The interface of program TELME

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