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THERMAL TRANSFER SIMULATION FOR AN ELECTRICAL RESISTOR

Mitică Iustinian NEACĂ, Andreea Maria NEACĂ

University of Craiova, ineaca@elth.ucv.ro

Abstract **– Starting from the necessity of the functioning analysis of an electrical resistor through which a current is circulating, the paper evaluates the temperature, electrical resistance and power's way of variation. To exemplify it has been chosen a resistor with big range of variation of the temperature between the in off regime and nominal functioning regime. There are also considered the influences that the environment has on the functioning of the resistance. The analysis is based on the modeling of the system with the help of the Matlab-Simulink kit.**

Keywords: *electrical heating, resistance, MATLAB-SIMULINK modelling.*

1. INTRODUCTION

The problem of determining the thermal transfer is treated in the specialty literature especially in the frame of some thermal stabilized stationary systems. This fact is normal from two points of view:

•The systems that produce and use thermal energy are systems relatively slow; they have long time constants and long functioning periods.

•The study of dynamic regime, unstationary, leads to the use of some equations of superior order, which depend in general on many parameters.

If we refer to electrical heating with resistors it is known the fact that a metal conductor, through heating, modifies its electric resistance. Depending on the used material and the temperature that can be touch during its functioning period, the electric resistance can become few times bigger than that from the ambient temperature.

This paper is proposing to analyse the functionality of an electric resistor through which is circulating a current and which suffers an auto-heating process. The use of the kit MATLAB-SIMULINK allowed us modeling the system and, based on the simulations, allowed us to determine the temperature of transient and permanent regime at which the electric resistance functions. It has also permitted to determine other parameters as well.

For analysis we have chosen a system that transforms the electrical energy into thermal energy at which the range of temperature of the resistive element is as large as possible. If we consider the Tungsten filament of an electric lightbulb, its temperature without electrical energy supply is about 300K, and after the connection to the power supply, in permanent regime, the temperature overpasses 2000K. By experiment it has been determined the fact that the filament of a lightbulb of 100W is made of a Tungsten conductor of 2.5m long and a diameter of 0.03mm. It is double-spiral at the beginning, it is reeled around a punch with a diameter of 0.12mm and after that, around a punch having a diameter of 1mm. The step of the two coils is almost double compared with the weight of the whirls; this means 0.06mm and 2mm respectively. It results a filament of 38÷40mm long and 2mm thick.

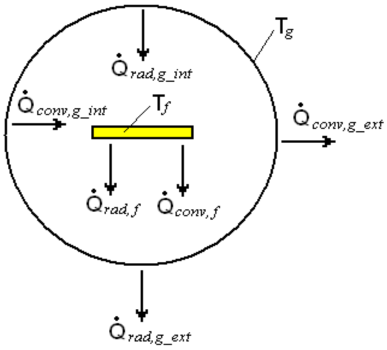

Figure 1: Heat transfer processes

The functioning of such a system, neglecting the fixing part of the lightbulb, is presented in Fig. 1, where:

$\dot{Q}_{rad,f}$ = the thermal flux irradiated by the filament (W);

$\dot{Q}_{conv,f}$ = the power yielded by the filament through convection (W);

$\dot{Q}_{conv,g\_int}$ = the power yielded on the inside by the glass bulb through convection (W);

$\dot{Q}_{rad,g\_int}$ = the thermal flux irradiated on the inside of the glass bulb (W);

$\dot{Q}_{rad,g\_ext}$ = the thermal flux irradiated on the outside of the glass bulb (W);

$\dot{Q}_{conv,g\_ext}$ = power yielded on the outside by the glass bulb through convection (W);

Observations:

1. If we take into account the fact that the glass bulb is vacuumed:

$$\dot{Q}_{conv,f} = \dot{Q}_{conv,g\_\mathbf{int}} = 0$$

2. In fact, the irradiance emitted by the filament arrives on the glass bulb, where, it is divided into:

$Q_{rad,g\_ext} = t \cdot Q_{rad,f}$ - the energy radiated on the outside after the transition of the glass bulb;

$Q_{stoc,g} = a \cdot Q_{rad,f}$ - the energy that realizes the heating of the glass bulb;

$Q_{rad,g\_\mathbf{int}} = r \cdot Q_{rad,f}$ - reflected radiation;

Where: $t + a + r = 1$.

3. Considering the glass bulb having uniform temperature and the filament having the temperature $T_f > T_g$, the radiant energy reflected on the inside comes on the bulb, and we can consider:

$$Q_{rad,g\_\mathbf{int}} = 0 \ ,$$

this means that $r = 0$.

For different types of glass, the absorption coefficient A varies within the range of [0.04÷0.11]. This coefficient depends on the wave-length of the radiation. Taking into account that the radiation emitted by the filament is maximum for wave-lengths of about 1.4µm, it can be considered a medium absorption coefficient a =0.08. It results the value of the penetration coefficient: t =0.92.

2. MATHEMATICAL DESCRIPTION

The functionality of the filament is described by the following set of equations:

$$i_{bulb} = \frac{u_{bulb}}{R_{bulb}(T)} \qquad (1)$$

$$P_{bulb} = u_{bulb} \cdot i_{bulb} \qquad (2)$$

$$P_{bulb} = P_{rad} + P_{stored} \qquad (3)$$

where:

- P_{bulb} represents the power dissipation of a bulb;
- P_{rad} represents the power radiation; ($P_{rad} = \dot{Q}_{rad,f}$);
- P_{stored} represents the power stored in the filament;

At the glass bulb level, we have the following equations:

$$P_{rad} = P_{rad,g\_ext} + P_{conv,g\_ext} + t \cdot P_{rad} + P_{stored,g} \qquad (4)$$

where:

- P_{rad,g_ext} = represents the power radiated on the outside by the glass bulb;
- P_{conv,g_ext} = represents the power yielded through convection by the glass bulb;
- $P_{stored,g}$ = represents the power transformed into thermal energy in the glass bulb;

It has to be remarked the fact that while the processes described by the equations (1)÷(3) are fast, the process defined by the equation (4) is much more slow, being characterized by a bigger time constant when the glass bulb is heating. This fact explains the gradual growth of the total radiation emitted by the lightbulb until a maximum value, characteristic of the long period regime. We talk about electromagnetic radiation within the whole spectrum (not only within the light spectrum), the glass bulb being warmed around the temperature of 100°C.

$$P_{rad} = C_1 \cdot \left(T_f^4 - T_g^4\right) \qquad (5)$$

$$P_{stored,f} = \frac{d}{dt}\left(W_{stored,f}\right) = C_{Tf} \cdot \frac{dT}{dt} \qquad (6)$$

$$P_{rad,g\_ext} = C_2\left(T_g^4 - T_{amb}^4\right) \qquad (7)$$

$$P_{conv,g\_ext} = \alpha \cdot S_g \cdot \left(T_g - T_{amb}\right) \qquad (8)$$

$$P_{stored,g} = \frac{d}{dt}\left(W_{stored,g}\right) = C_{Tg} \cdot \frac{dT}{dt} \qquad (9)$$

The constants that are in the equations from are determined in the following way:

$$P_{rad} = C_1 \cdot \left(T_f^4 - T_g^4\right) \qquad (10)$$

where:

- $\varepsilon_W \cong 0{,}3$ represents the degree of blackness of the filament around the functioning temperature;
- $c_n = 5{,}67 \cdot 10^{-8}$ W/m^{2}K^4 represents radiation constant of the black corp. (Stefan - Boltzmann);
- $S_f = \pi \cdot d \cdot l = \pi \cdot 0{,}03 \cdot 10^{-3} \cdot 2{,}5 = 235{,}6 \cdot 10^{-6}$ m^2 re-presents the radiant surface of the Tungsten wire from which is made the filament;

$$C_{Tf} = \frac{\pi \cdot d^2}{4} \cdot l \cdot \gamma_W \cdot c_{TW} = 4{,}84 \cdot 10^{-3} \text{ J/K} \qquad (11)$$

where:

- $\gamma_W = 19{,}3 \cdot 10^3$ Kg/m^3 = the density of the Tungsten;
- $c_{TW} = 142$ J/Kg·K = specific thermal capacitance of Tungsten;

$$C_2 = \varepsilon_g \cdot c_n \cdot S_g \cong 3{,}95 \cdot 10^{-10} \text{ W/K}^4 \qquad (12)$$

where:

- $\varepsilon_g \cong 0.87$;
- $S_g \cong 0.8 \cdot 10^{-2}$ m^2 – the radiate surface of the glass bulb;

$$\alpha = 0.7 \cdot \left(T_g - T_{amb}\right)^{1/3} \qquad (13)$$

(The above equation is simulated and calculated dynamically)

$$C_{Tg} = V_g \cdot \gamma_g \cdot c_{Tg} \cong 7.5 \text{ J/K} \qquad (14)$$

where:

- $V_g \cong 0.4 \cdot 10^{-5}$ m^3 – the volume of glass used when constructing the bulb;
- $c_{Tg} = 750$ J/Kg·K – the specific thermal capacitance of the glass;
- $\gamma_g = 2.5 \cdot 10^3$ Kg/m^3 – the density of the glass;

3. MODELING AND SIMULATION

The model designed in order to simulate the functioning of the lightbulb is presented in Fig.2.

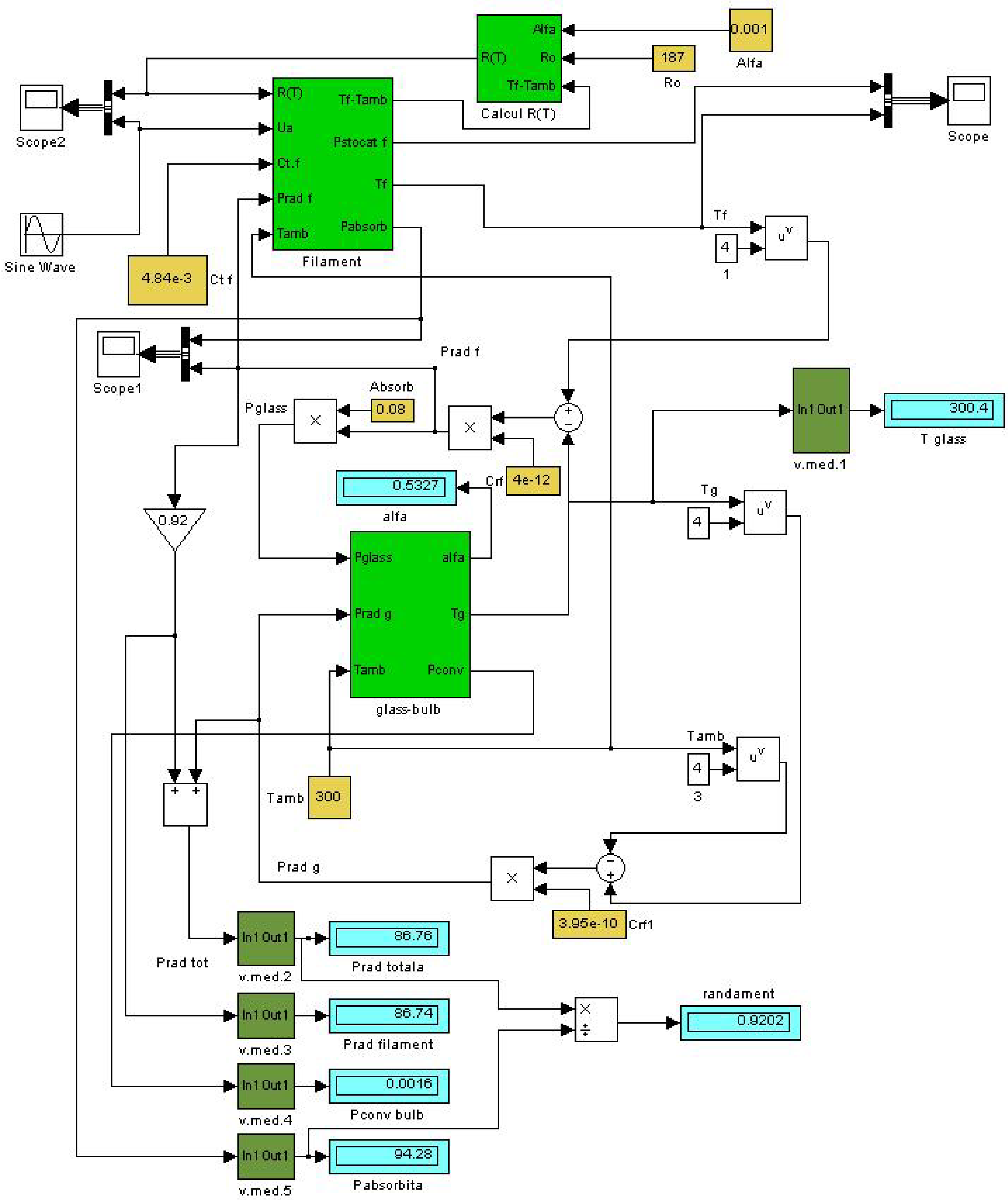

Figure 2: The model of the lightbulb

Taking into account the fact that within the processes that appear, we use very different time constants, it was necessary, once, to simulate it at very short time constants, to present the thermal transient processes specific to the filament, and secondary, the simulation with long time constants, to make possible the study of the heating process of the glass bulb.

For a better analyze of the processes of long period, there had been implemented blocks which show medium values on certain time segments set differently, in function of the simulation time. With their help it had been determined a series of data, part of them being presented in Table 1.

Table 1

Period of time	0.5	1	5	10	25	50	100	200	300	400	500
T glass	300.4	300.9	304.8	309.6	322.7	345.3	369.5	387.3	391.5	391.8	392.1
alpha	0.537	0.686	1.188	1.49	1.984	2.406	2.8	3.028	3.071	3.08	3.081
Prad filament	86.74	86.74	86.74	86.74	86.52	86.75	86.74	87.3	86.57	87.06	87.51
Total Prad	86.76	86.78	86.95	87.17	87.61	88.84	90.47	92.42	92	92.54	93.02
Bulb Prad	0.001	0.004	0.045	0.114	0.360	0.774	1.43	1.961	2.076	2.098	2.103
P absorbed	94.28	94.28	94.28	94.28	94.05	94.26	94.34	94.89	94.14	94.62	95.12
Radiation efficiency	92%	92%	92.2%	92.4%	93.1%	94.2%	95.8%	97.4%	97.7%	97.8%	97.8%
Total emission efficiency	92%	92%	92.2%	92.5%	93.5%	95.0%	97.4%	99.4%	99.9%	99.9%	~100%

The analyze of Table 1 shows clearly that the heat of the glass bulb is made with a big time constant. The temperature is stabilized around the value of 392.3K (after 500s). The temperature characteristic of the glass bulb is presented in Fig.3.

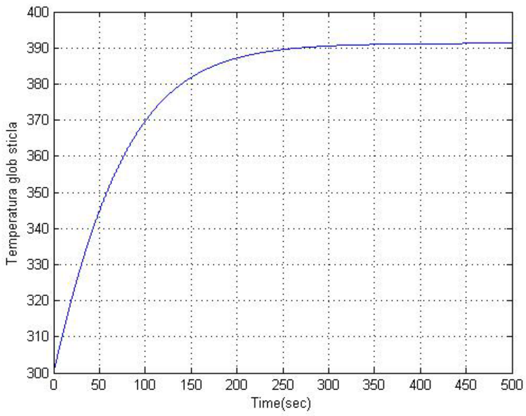

Figure 3: The temperature characteristic of the glass bulb

In Fig.4 is presented the graphical variation of the total radiation (filament radiation and glass bulb radiation). It can be observed a growth with almost the same time constant. The data analyze from the table shows the fact that after a time, big enough, the efficiency of the radiation is almost 100%. This demonstrates a good use within the frame of electro-thermal equipments, where light radiance and also thermal radiance are necessary.

In Fig.5, Fig.6 and Fig.7 are presented the results of the simulation for a time of 1000 times shorter than the one studied in the previous case. There can be observed the radiations with double frequency (100Hz) of all dimensions (in Fig.5 is presented the source tension with red color, with the frequency of 50Hz).

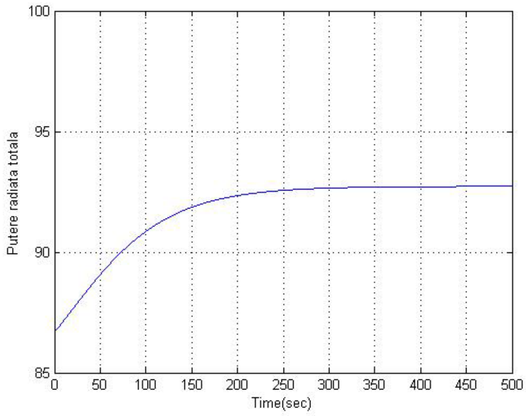

Figure 4: The total radiation

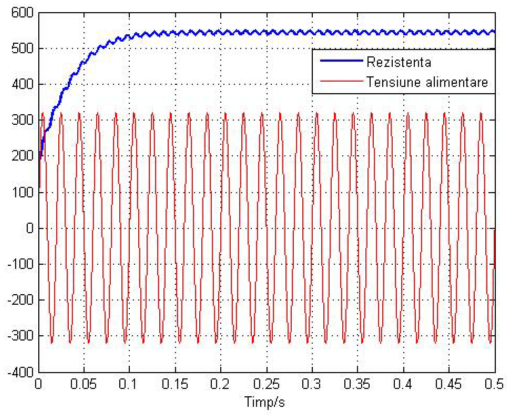

Figure 5: The resistance and the tension supply

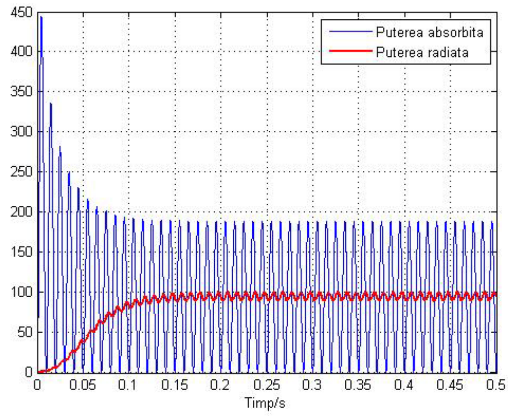

Figure 6: Absorbed power and radiated power

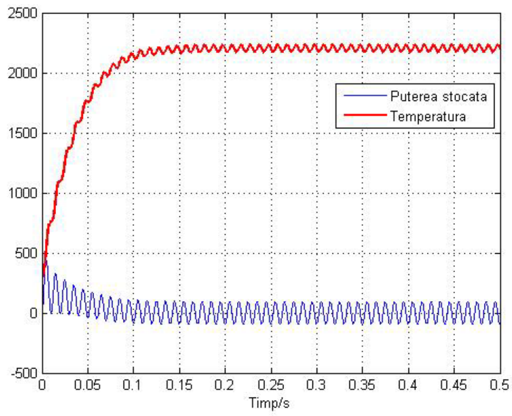

Figure 7: Stored power and temperature

In the same way it can be observed the similar variation of the resistance of the filament (the blue curve in Fig.5), and its temperature (the red curve in Fig.7). In Fig.7 it is represented with blue the variation of the power stored in the filament. In Fig.6 are presented the power absorbed from the supply (blue) and the power radiated by the filament (red).

4. CONCLUSIONS

The model designed in MATLAB-SIMULINK permits a strict simulation of the functionality of the electric lightbulb.

The analyze of the presented graphics leads to the following conclusions:

- The phenomena from the lightbulb can be divided into fast and slow phenomena, and the difference between the time constants are 1:1000.
- When the temperature of the filament grows form 300K to an oscillate value around 2200K, the resistance of the filament is almost triple, and it stabilizes at a medium value of 543Ω.
- The emitted powers have sinusoidal variation with the double frequency compared with the one of the power supply and implicitly, the emitted light presents intensity oscillations with the frequency of 100Hz.
- The power radiated by the filament has small oscillations around the value of 94W, while the stored power oscillates around 0W, with the role to maintain constant the temperature of the filament(at stabilized regime over 500s). This fact takes to the transformation of the whole energy delivered by the supplying source into total radiated energy and energy yielded through convection. In this way the efficiency of the total energy emission touches the value of 100%.

In another step, the realized model had been used to simulate another type of light bulb, having the filament made of a tungsten conductor, with a 0.02mm diameter, reeled around punches of 0.08mm and 0.8mm respectively.

Taking into consideration the whirls' steps of 0.04mm and 1.5mm respectively, it results a wire from which it is realized the filament long of 1.11m.

The simulations realized in this case took us to the following conclusions:

- The filament's temperature grows more, reaching a stabilization point at 2750K, but this fact is causing a faster wear of the light bulb.
- The resistance grows more, as well, and reaches a stabilization point at 660Ω.
- The power absorbed from the network and radiated is smaller (approximately 77W).
- Taking into consideration the Wien's law, we can obtain a shifting of the maximum point of radiation from $\frac{2897.8}{2200} = 1.317\ \mu m$ to $\frac{2897.8}{2750} = 1.054\ \mu m$, without entering into the visible spectrum.

References

[1] F.P.Incropera, D.P.DeWitt, T.L.Bergman, A.S.Lavine: *Fundamentals of Heat and Mass Transfer*; John Wiley &Sons, Inc., 2007

[2] A.Bejan, A.D.Kraus: Heat Transfer Handbook; John Wiley &Sons, Inc., 2003

[3] H.Czichos, M.Hennecke: HÜTTE - *Das Ingenieurwissen*; Springer-Verlag Berlin Heidelberg, 2008

[4] S.T.Karris; *Introduction to Simulink with Engineering Applications*; Orchard Publications, 2008

[5] Bürmen A. ; *An Introduction to ICAP/4*; The International Workshop, Rosenheim, 2000

[6] M.Kalechman; *Practical Matlab Applications for Engineers*; CRC Press, Taylor & Francis Group, 2009