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CASCADE CONTROL SYSTEM OF THREE PHASE SHUNT ACTIVE FILTERS

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Abstract **– This paper focuses on the control system for the output current and voltage across the DC capacitor of a three-phase shunt active filter. A classical PI-PI cascade control solution is taken into consideration. The design of both inner current loop and outer voltage loop is based on Modulus–Optimum criterion. It is pointed out that this tuning approach provides a good phase margin. In order to finalise the voltage controller tuning, the passband of the unity feedback system is imposed. Simulations were performed under Matlab-Simulink environment and the results are presented to show the effectiveness of such tuning technique.**

Keywords: *Active filters, Control system, PI controller, Tuning, Modulus Optimum Criterion.*

1. INTRODUCTION

In recent years, shunt active power filters (SAPF) based on a voltage source inverter structure have been widely studied and developed as a solution to harmonic current pollution problems. They improve the power quality by injecting compensating currents into the power system based on calculated reference currents. Different complex approaches have been investigated to improve the performance tracking [1], [2]. However, to facilitate low-cost analogue control, cascade control of shunt active filter via simple and robust PI controllers is a viable solution [3].

The active power filters performance depends on the modulation technique of the static converter. Among the various pulse width modulation (PWM) techniques, the sinusoidal one seems to be frequently used because of its simplicity of implementation. Besides, in the context of closed loop control, the performances of the SAPF with sinusoidal modulation in terms of total harmonic distortion in source current and DC-bus utilization are close to space vector modulation technique performances [4].

As the modulus optimum method for optimization of regulators is applied with good performances in a wide variety of cases in the control field [5], [6], [7], the PI controller parameters can be tuned according to absolute value optimum criterion. The gain and phase margin approach is also available for high performance and robustness requirement [8].

2. STRUCTURE OF THE CONTROL SYSTEM

In this study, the cascade control is composed of two control loops with the voltage loop outside the inner current loop (Fig. 1).

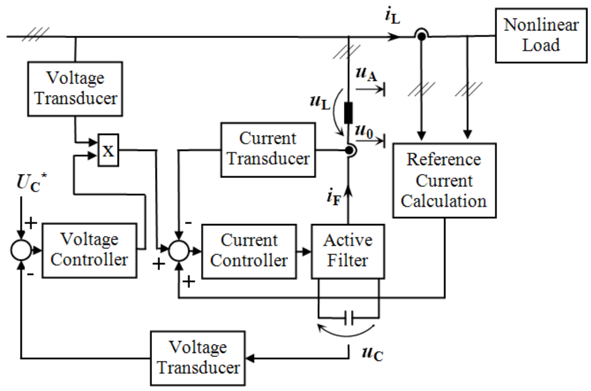

Fig.1 - Single-line block diagram of the control system

The two essential parameters to be controlled are the inverter output current and the DC-bus voltage at the inverter input from the viewpoint of active power balance. Therefore, two PI controllers are to be designed, a current one and a voltage one.

In the associated block diagram of Fig. 2, the following transfer functions are pointed out: G_{Cu} – transfer function of the voltage controller; G_{Ci} - transfer function of the current controller; G_{Fi} – first partial transfer function of the active filter, from the modulating voltage signal to the output current; G_{Fu} - second partial transfer function of the active filter, from the output current to the DC-bus voltage; G_{Ti} - transfer function of the current transducer; G_{Tu} - transfer function of the voltage transducer.

3. SAPF TRANSFER FUNCTIONS

As it was previously specified, the active filter intervenes in the block diagram of Fig. 2 by two transfer functions [9].

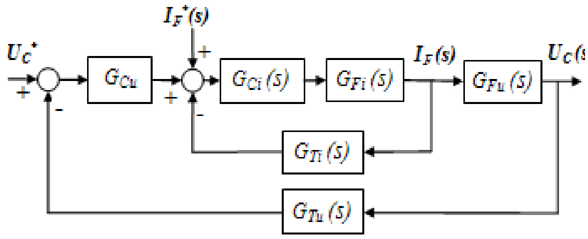

Fig.2 - Block diagram of closed-loop active filter

3.1. First partial transfer function of the active power filter

Supposing that the filter output voltage (u_0) is constant and equal with its average value (U_{0av}) during a period of the carrier signal, then u_0 can be expressed as a function of the reference voltage u_m in the Laplace domain, i.e.

$$U_0(s) = U_{0av}(s) = \frac{U_c}{2U_{t\max}} \cdot U_m(s)\,, \tag{1}$$

where U_c and U_{tmax} denote the DC-bus voltage and the amplitude of triangular carrier signal.
Then, considering that the time origin corresponds to the moment when the A phase-voltage, $u_A = U_A \sin(100\pi \cdot t)$, passes through zero becoming positive, the Kirchhoff's voltage law at the filter output in the Laplace domain allows to express the first partial transfer function of the active filter,

$$G_{Fi}(s) = \frac{I_F(s)}{U_m(s)} = \frac{U_c}{2L \cdot U_{t\max} \cdot s} = \frac{1}{K_{Fi} \cdot s}\,, \tag{2}$$

where

$$K_{Fi} = \frac{2L \cdot U_{t\max}}{U_c}\,. \tag{3}$$

3.2. Second partial transfer function of the active power filter

Let us consider a small variation Δu_c of the voltage across the capacitor around its average value U_{cm}. Hence, the power corresponding to the energy stored in capacitor has to correspond to the whole power in the connecting point of the filter to the network, that is to the apparent power. Thus, the Laplace transform of the above condition allows expressing the desired transfer function,

$$G_{Fu}(s) = \frac{U_c(s)}{I_F(s)} = \frac{3U_A}{\sqrt{2}C \cdot U_{cm} \cdot s} = \frac{1}{K_{Fu} \cdot s}\,, \tag{4}$$

where

$$K_{Fu} = \frac{\sqrt{2}C \cdot U_{cm}}{3U_A}\,. \tag{5}$$

4. CURRENT CONTROLLER TUNING

In the block diagram of the current loop (Fig. 3), it is assumed that the dynamic behavior of the transducer-current filter can be approximated by a first-order transfer function:

$$G_{Ti}(s) = \frac{K_{Ti}}{1 + T_{Ti} \cdot s} \tag{6}$$

A classical proportional-integral structure is adopted for the current controller, i.e.

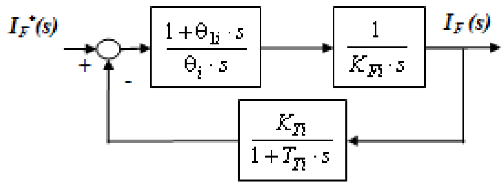

Fig.3 - Block diagram of the current loop

$$G_{Ci}(s) = \frac{1 + \theta_{1i} \cdot s}{\theta_i \cdot s} \tag{7}$$

To use the modulus optimum tuning criterion, the open-loop transfer function is written in the following form:

$$G_{di}(s) = G_{Ci}(s) \cdot G_{Fi}(s) \cdot G_{Ti}(s) = \frac{1 + \theta_{1i} \cdot s}{T^2 \cdot s^2 \cdot (1 + T_{Ti} \cdot s)}\,, \tag{8}$$

where

$$T^2 = \frac{\theta_i \cdot K_{Fi}}{K_{Ti}}\,. \tag{9}$$

As regards the transfer function of the closed-loop unity feedback system, it can be written as:

$$G_i(s) = \frac{1 + \theta_{1i} \cdot s}{T^2 \cdot T_{Ti} \cdot s^3 + T^2 \cdot s^2 + \theta_{1i} \cdot s + 1}\,. \tag{10}$$

After expressing the square of the above transfer function modulus, the simple condition of canceling the denominator coefficients which contain differences leads to the following relations:

$$\theta_{1i} = \sqrt{2}T\,; \qquad T = 2\sqrt{2}T_{Ti}\,. \tag{11}$$

Taking into account (9) and (11), the two parameters of current controller are expressed:

$$\theta_{1i} = 4T_{Ti}\ ; \qquad \theta_i = 8K_{Ti} \cdot T_{Ti}^2 / K_{Fi}\ . \qquad (12)$$

Thus, the transfer function of the current controller is given by:

$$G_{Ci}(s) = \frac{K_{Fi}}{2K_{Ti} \cdot T_{Ti}} \left(1 + \frac{1}{4T_{Ti} \cdot s}\right) \qquad (13)$$

Accordingly, the open-loop and close-loop unity feedback transfer functions given by (8) and (10) become:

$$G_{di}(s) = \frac{1 + 4T_{Ti} \cdot s}{8T_{Ti}^2 \cdot s^2 \cdot (1 + T_{Ti} \cdot s)} \qquad (14)$$

$$G_i(s) = \frac{1 + 4T_{Ti} \cdot s}{1 + 4T_{Ti} \cdot s + 8T_{Ti}^2 \cdot s^2 + 8T_{Ti}^3 \cdot s^3} \qquad (15)$$

As the terms containing T_{Ti}^2 and T_{Ti}^3 are very insignificant compared with 1, the transfer function (15) becomes:

$$G_i(s) \approx 1\ . \qquad (16)$$

5. VOLTAGE CONTROLLER TUNING

A PI controller is also adopted to control the voltage across the capacitor. In the forward path of the block diagram (Fig.4), due to passing to unity feedback, there is the inverse of the current transducer transfer function. The forward transfer function can be expressed as follows:

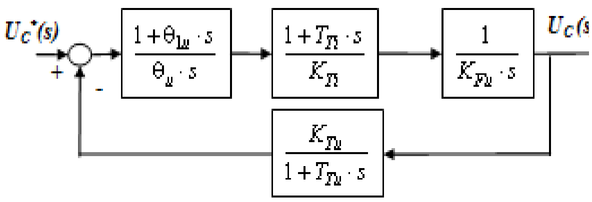

Fig.4 - Block diagram of the DC-bus voltage loop

$$G_{du}(s) = \frac{(1 + \theta_{1u} \cdot s) \cdot (1 + T_{Ti} \cdot s) \cdot K_{Tu}}{K_{Ti} \cdot K_{Fu} \cdot \theta_u \cdot s^2 \cdot (1 + T_{Tu} \cdot s)}, \qquad (17)$$

where θ_{1u} and θ_u are the parameters to be determined. If the time constants of the current and voltage transducers are supposed to be equals, (17) becomes:

$$G_{du}(s) = \frac{1 + \theta_{1u} \cdot s}{T_u^2 \cdot s^2}\ ; \qquad T_u^2 = \frac{K_{Ti} \cdot K_{Fu} \cdot \theta_u}{K_{Tu}} \qquad (18)$$

Then, the closed-loop unity feedback transfer function is given by:

$$G_u(s) = \frac{1 + \theta_{1u} \cdot s}{1 + \theta_{1u} \cdot s + T_u^2 \cdot s^2} \qquad (19)$$

and the square of its modulus can be expressed as:

$$M_u^2(\omega) = \frac{1 + \theta_{1u}{}^2 \cdot \omega^2}{1 + \omega^2 \cdot \left(\theta_{1u}{}^2 - 2T_u^2\right) + \omega^4 \cdot T_u^4}\ . \qquad (20)$$

Canceling the denominator term which contains a difference in (20) gives the condition:

$$\theta_u = \frac{K_{Tu}}{2K_{Ti} \cdot K_{Fu}} \theta_{1u}^2\ . \qquad (21)$$

It must be noticed that the above condition imposes the loop phase-margin. Thus, the open-loop transfer function, in the frequency domain, can be arranged as

$$G_{du}(j\omega) = -\frac{1 + j\omega \cdot \theta_{1u}}{\omega^2 \cdot T_u^2} = -\frac{1 + j(\omega/\omega_1)}{(\omega/\omega_{01})^2}, \qquad (22)$$

where

$$\omega_1 = 1/\theta_{1u}\ ; \quad \omega_0 = 1/T_u\ . \qquad (23)$$

Moreover, by introducing the cut-off pulsation ω_c, the associated phase margin φ_m is given by:

$$\varphi_m = arctg\left(\frac{\omega_c}{\omega_1}\right) \approx 65.5^0\ . \qquad (24)$$

In order to find the second relation required in the voltage controller design, the passband of the unity feedback system is imposed. So, the square modulus of the closed-loop unity feedback transfer function given by (20) can be written as:

$$M_u^2(\omega) = 4 \cdot \frac{1 + (\omega/\omega_1)^2}{4 + (\omega/\omega_1)^4}\ . \qquad (25)$$

But, as the magnitude response remains within $\sqrt{2}$ of its maximum value inside the passband, the passband frequency is obtained:

$$f_p = \frac{\omega_1}{\pi} \sqrt{\frac{1}{2}\left(\sqrt{5} - 1 + \sqrt{4 - \sqrt{5}}\right)}\ . \qquad (26)$$

Therefore, the time constants in the voltage controller transfer function can be expressed as follows:

$$\theta_{1u} \approx 0.36/f_p\ ; \quad \theta_u \approx 64.95 \cdot 10^{-2} \cdot \frac{K_{Tu}}{K_{Ti} \cdot K_{Fu}} \cdot \frac{1}{f_p^2} \qquad (27)$$

6. CONTROL SYSTEM PERFORMANCES

To test the performances of the control system, the block diagram in the Fig. 2 has been implemented under Matlab-Simulink environment.

The simulation parameters are: $U_A = \sqrt{2} \cdot 220$ V,

$U_{cm} = 700\,\text{V}$, L = 14 mH, $U_{t\max} = 10\,\text{V}$, K_{Ti} = 0.2, K_{Tu} = 0.0125, $T_{Ti} = T_{Tu} = 10^{-5}$ s. In addition, the determined parameters in the control loops for f_p = 20 Hz are: $\theta_{1u} = 0.018\,\text{s}, \theta_u = 0.003\,\text{s}, \theta_{1i} = 4\cdot 10^{-5}\,\text{s}$, $\theta_i = 3.73\cdot 10^{-6}\,\text{s}$, $K_{Fi} = 4.28\cdot 10^{-5}\,\text{s}$ and $K_{Fu} = 3.3\cdot 10^{-3}\,\text{s}$.

In order to charge the DC-bus capacitance, a ramp voltage of 700 V is applied and then, after 0.3 seconds, the current to be compensated is applied too.

In first case study, the external reference current generator provides a current which is a superposition of harmonics of orders 5, 7, 11 and 13. As it can be seen in Fig. 5, the voltage response has an overshoot of 5.8% and then it tracks its reference value.

As regards the current loop behavior, Fig. 6 shows the good performance of current tracking characterized by a average square error of 3.22 A during a period of the reference current.

In the second case study, the current to be compensated is provided by a uncontrolled rectifier according to so-called p-q theory [10], [11].

The response of the current loop compared with the reference current (Fig. 7) illustrates a low average square error of 1.14 A which confirms the very good behavior of the control system.

After an overshoot of 5.8%, the DC-bus voltage practically keeps its reference value (Fig. 8). It must be noticed that the modelling of the active power filter through transfer functions do not allow taking into consideration the real instantaneous PWM control, so that some additional oscillations are expected.

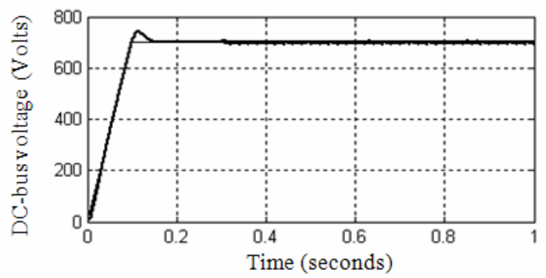

Fig.5 - DC-bus voltage response in case of an imposed harmonic superposition in the reference current

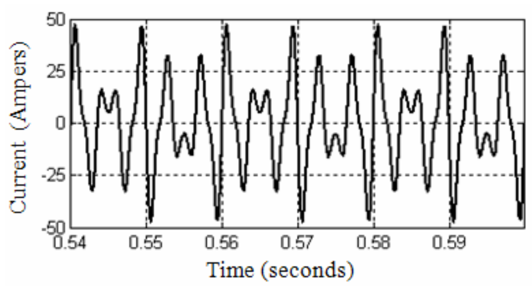

Fig.6 - Current loop response in case of an imposed harmonic superposition in the reference current

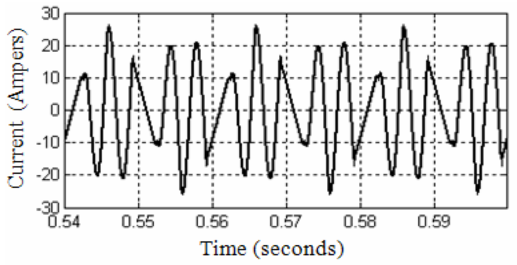

Fig.7 - Current loop response in case of non controlled rectifier load

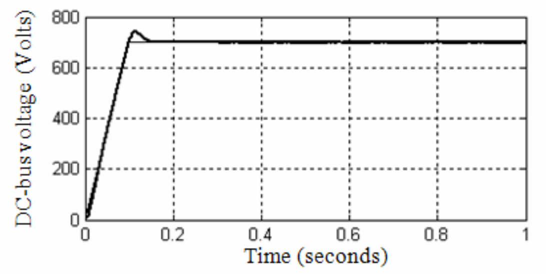

Fig.8 - DC-bus voltage response in case of non controlled rectifier load

However, it can be considered that a good response concerning average value of DC-bus voltage is a warrant of the performant tuning of controllers.

7. CONCLUSIONS

The control system of the active power filter is composed of an inner current loop and an outer voltage loop, both of them based on conventional PI controllers. In the tuning process of PI current controller, the inverter switches and their commutations are neglected and the whole tuning is done according modulus optimum criterion.

It is shown that the modulus optimum criterion gives a relationship between the PI voltage controller parameters which imposes a good loop phase-margin. In addition, to tune the DC-bus voltage controller, the passband of the unity feedback system is imposed.

The simulations performed using the Matlab-Simulink environment illustrate a very good behavior of the control system regarding reference tracking.

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