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AKAGI'S P-Q THEORY AND ACTIVE FILTERING UNDER NON-SINUSOIDAL VOLTAGE

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Abstract **– This paper presents an analysis of Akagi's p-q theory for active filtering under really non-sinusoidal voltage. So, it is ascertained that compensating of alternative parts of instantaneous active and reactive powers establishes a sinusoidal current in the parallel active filter. But this is true under sinusoidal voltage only. The mathematical demonstration is found and it is verified on few practical examples. For really non-sinusoidal voltage (total harmonic distortion factor less 10%), the difference between Akagi's active current and the true active current is very small. A modified definition of the active current is proposed for the operation under non-sinusoidal voltage conditions. The DC motor and full control rectifier driving system is used as study case.**

Keywords: *Active filter, non sinusoidal.*

1. INTRODUCTION

Making evident the components of the current, especially the active one, is an old concern of researchers in straight connection with the necessity of substantiation of performant methods for improving power factor. Recent efforts have been concentrated on the development and control of active power filters. Thus, when total compensation is expected, the active filter has to provide the current vector

$$\underline{i}_F = \underline{i}_L - \underline{i}_a , \tag{1}$$

where $\underline{i}_L$ and $\underline{i}_a$ are the load current vector and its active component.

The development of active filtering techniques has rendered topical the theory of instantaneous complex powers. This theory was first introduced, as a unitary concept, by V. Nedelcu [1], [2] and it was developed by other authors who used it for grounding certain active filtering techniques [3].

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So, the p-q theory of the instantaneous power developed by Akagi and his coauthors [4], provides the mathematical foundations for the active filters control and it is about to become a means of identification and analysis of powers under non sinusoidal current and/or voltage operation [5], [6], [7]. This theory uses the complex space vector theory and introduces the concepts of instantaneous active power and instantaneous reactive power.

2. THE THEORY OF APPARENT INSTANTANEOUS COMPLEX POWER AND THE ACTIVE FILTERING

If $\underline{u}$ and $\underline{i}$ are the space phasors of the supply voltages of the distortion load and distorted three-phased current, defined in the following matrices [2]

$$[i] = \frac{2}{3}[A] \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix},$$

$$[u] = \begin{bmatrix} u_d \\ u_q \end{bmatrix} = \frac{2}{3}[A] \cdot \begin{bmatrix} u_a \\ u_b \\ u_c \end{bmatrix}, \tag{2}$$

$$[A] = \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \dfrac{\sqrt{3}}{2} & -\dfrac{\sqrt{3}}{2} \end{bmatrix},$$

the instantaneous apparent complex power results as

$$\underline{s} = p + jq = \frac{3}{2}\left[u_d i_d + u_q i_q + j\left(-u_d i_q + u_q i_d\right)\right]. \tag{3}$$

The direct (P and Q) and alternating ($p_{\sim}$ and $q_{\sim}$) components can be outlined in the instantaneous active and reactive powers, p and q:

$$\begin{aligned} p &= P + p_{\sim} \\ q &= Q + q_{\sim} \end{aligned} . \tag{4}$$

The average values P and Q result from

$$\begin{aligned} P &= \frac{1}{T}\int_{t-T}^{t} p dt \\ Q &= \frac{1}{T}\int_{t-T}^{t} q dt \end{aligned} . \tag{5}$$

So, the instantaneous apparent complex power can be written as

$$\underline{s} = P + p_{\sim} + j(Q + q_{\sim}) . \tag{6}$$

H. Akagi proposed the compensation of the alternating components, respectively the calculation of the reference currents in the active filter based on the relation resulting from developing the current in (3) [3], [4],

$$\underline{i}_L = i_d + ji_q = \frac{2}{3}\frac{1}{u_d^2 + u_q^2}\underline{u}\cdot\underline{s}^*, \tag{7}$$

where $\underline{s}^*$ is the conjugate of $\underline{s}$.
Thus, if $\underline{s}_F$ is the apparent instantaneous complex power corresponding to the active filter, the current absorbed by this is:

$$\underline{i}_F = i_{Fd} + ji_{Fq} = \frac{2}{3}\frac{1}{u_d^2 + u_q^2}\underline{u}\cdot\underline{s}_F^*, \tag{8}$$

and the current absorbed from the network is

$$\underline{i}_s = \underline{i}_L - \underline{i}_F . \tag{9}$$

Proposing the compensation of the alternating components of the powers and the reactive power required by the distortion load, the apparent complex power of active filter is

$$\underline{s}_F = p_{\sim} + j(Q + q_{\sim}) . \tag{10}$$

Using the relations (6), (7), (8) and (10), relation (9) becomes

$$\underline{i}_s = \frac{2}{3}\frac{\underline{u}}{u_d^2 + u_q^2}P . \tag{11}$$

This is just the active current component in accordance with Akagi's theory. But in accordance with Fryze's, Shepherd's and Zakikhani's and many others authors opinions, the active current must have the same shape as the voltage [8], [9], [10], [11]. It means that the square of voltage space vector modulus must be constant.

3. CORRECT INTERPRETATION OF P-Q THEORY

Starting from expression (11), the following current space-vectors can be defined.
1. The active current vector ($\underline{i}_a$), whose components are

$$i_{ad} = \frac{2}{3}\cdot\frac{u_d}{|\underline{u}|^2}P , \tag{12}$$

$$i_{aq} = \frac{2}{3}\cdot\frac{u_q}{|\underline{u}|^2}P . \tag{13}$$

2. The reactive current vector ($\underline{i}_r$), whose components are

$$i_{rd} = \frac{2}{3}\cdot\frac{u_q}{|\underline{u}|^2}Q , \tag{14}$$

$$i_{rq} = -\frac{2}{3}\cdot\frac{u_d}{|\underline{u}|^2}Q . \tag{15}$$

3. The supplementary useless current vector on account of $p_{\sim}$ ($\underline{i}_{sp}$), whose components are

$$i_{spd} = \frac{2}{3}\frac{u_d}{|\underline{u}|^2}p_{\sim} , \tag{16}$$

$$i_{spq} = \frac{2}{3}\frac{u_q}{|\underline{u}|^2}p_{\sim} . \tag{17}$$

4. The supplementary useless current vector on account of $q_{\sim}$ ($\underline{i}_{sq}$), whose components are

$$i_{sqd} = \frac{2}{3}\frac{u_q}{|\underline{u}|^2}q_{\sim} , \tag{18}$$

It is also possible to define the total supplementary useless current vector ($\underline{i}_s$) as a sum of the two supplementary useless current vectors. Thus, its components are

$$i_{sd} = \frac{2}{3}\frac{u_d p_{\sim} + u_q q_{\sim}}{|\underline{u}|^2} , \tag{19}$$

$$i_{sq} = \frac{2}{3}\frac{u_q p_{\sim} - u_d q_{\sim}}{|\underline{u}|^2} . \tag{20}$$

But, in order to obtain an active current whose waveform has the same shape as the supply voltage, we propose the replacement of square of voltage space vector modulus with the square of its rms value [12], i.e.

$$U^2 = \frac{1}{T}\int_{t-T}^{t}|\underline{u}|^2 dt . \tag{21}$$

After this replacement, the new active current components are

$$i_{ad} = \frac{2}{3} \cdot \frac{u_d}{U^2} P, \quad (22)$$

$$i_{aq} = \frac{2}{3} \cdot \frac{u_q}{U^2} P. \quad (23)$$

It is obvious that the use of expression (22) and (23) for the active current calculation makes this current keep the voltage waveform.

4. THE CASE STUDIES

We will analyze two case studies: A) the three phase symmetrical system with strong distorted voltage containing 1st and 5th harmonics and symmetrical resistive load; B) three phase bridge full controlled rectifier which supplies a DC motor.

4.1 Three Phase System with Non-Sinusoidal Voltages and Symmetrical Resistive Load

As first case study we have chosen a three phase symmetrical system with strong distorted voltage to serve as a model to active current calculation [9]. A three-phase balanced resistive load with R = 2Ω is supplied by a three-phase non-sinusoidal voltage system as follows:

$$u_A = \sqrt{2}(100 \sin \omega t + 50 \sin 5\omega t),$$

$$u_B = \sqrt{2}(100 \sin(\omega t - 2\pi/3) + 50 \sin 5(\omega t - 2\pi/3)), \quad (24)$$

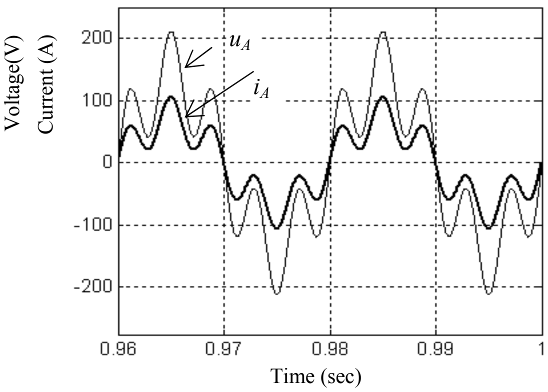

Fig.1. Non-sinusoidal phase voltage and current waveforms

$$u_C = \sqrt{2}(100 \sin(\omega t + 2\pi/3) + 50 \sin 5(\omega t + 2\pi/3)).$$

The waveforms of phase voltage and supply current are both non-sinusoidal but they are in-phase (Fig. 1). As it can be seen, the active current calculated with

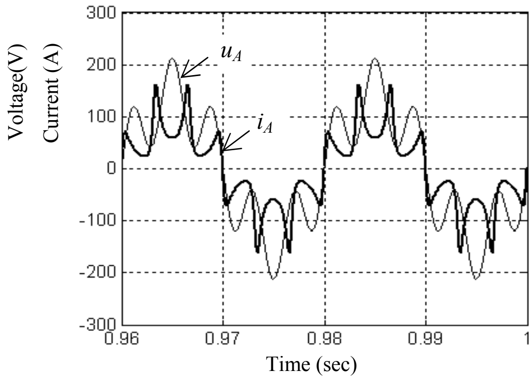

Fig 2. Non-sinusoidal phase voltage and active current calculated with (12)

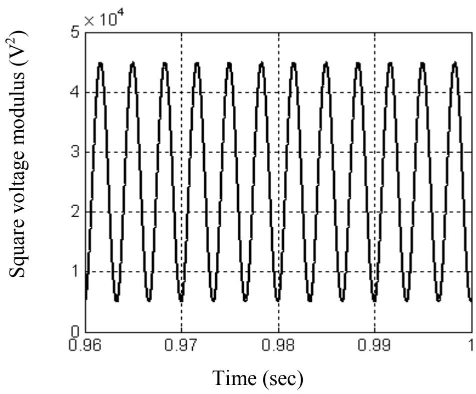

Fig 3. Evolution of the square of voltage vector

(12) is substantially different in shape compared to the phase voltage even for a resistive load (Fig. 2).

This situation occurs always under non-sinusoidal voltage conditions. Indeed, the square of voltage vector modulus $|\underline{u}|^2$ in active current components definition is time-dependent (Fig.3).

In this case study, the square of voltage vector modulus can be expressed even by analytical way as

$$|\underline{u}|^2 = 5000(5 - 4\cos\omega t). \quad (25)$$

More suggestive is the voltage space vector modulus locus that is not a circle and it is very strong distorted (Fig. 4).

The active current obtained by proposed relation (22) is identical with the network current because the load is purely active. In this case, the filtration is not necessary and the active filter current is zero.

This result in such a simple case study allows us to conclude that the active current components defined by (12) and (13) are not useful for reference current calculation in active filtering if the voltage is strong distorted.

Indeed, if the compensation is achieved by a parallel active filter and its reference current is distorted related to the supply voltage, the rms value of the

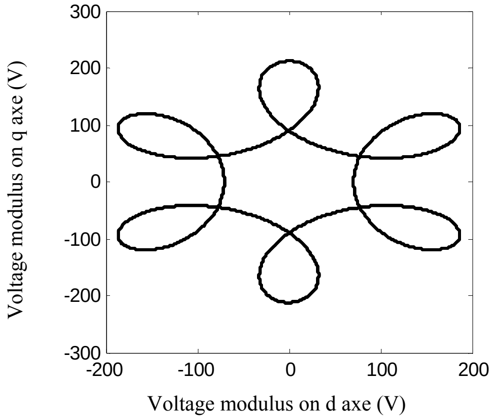

Fig 4. Evolution of the voltage vector modulus locus

supply current is higher than the initial load current even if this new current provides the necessary active power, removes the AC component of the instantaneous active power and has the same phase with the voltage. For example, in this case study, the rms initial load current is exceeded by about 30% after compensation (Fig. 5). Consequently, it is not a better solution.

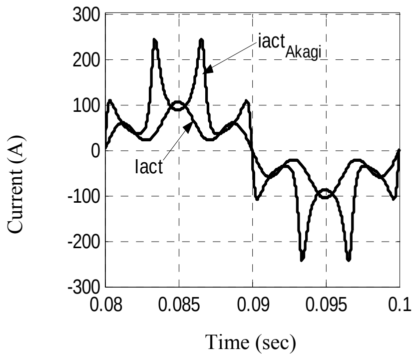

Fig 5. The shapes of the Akagi's active current and the proposed active current

4.2 The DC Motor and Full Controlled Bridge Rectifier System

Let us consider the DC motor and full controlled bridge rectifier system supplied by a D/Y transformer. Two situations will be analysed:

1. The active filtration is done in the secondary of transformer where the voltage is distorted by rectifier commutation;
2. The active filtration is done at the network side, where the voltage is practically not distorted.

4.2.1 The active current in the secondary of the transformer

In the secondary of the transformer, the phase current has the well-known waveform that outlines the effect of the limited value of the filtering inductivity. Moreover, the phase voltage is distorted by the rectifier commutations (Fig. 6).

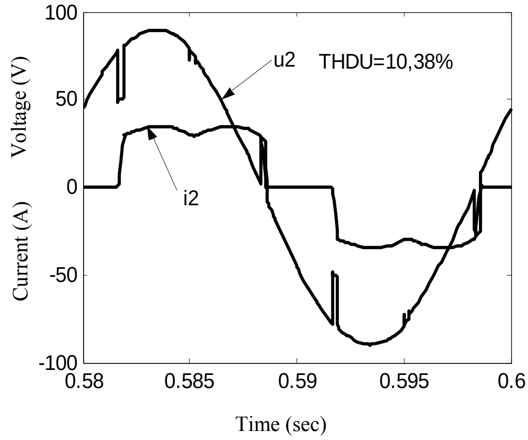

Fig 6. The voltage and current in the secondary of the transformer

As it can be seen, the current and voltage contain the significant harmonics of $6n \pm 1$ order (Fig. 7 and Fig. 8).

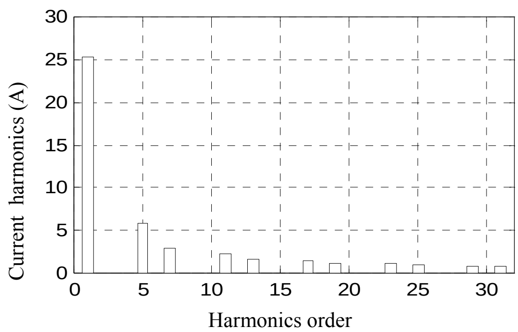

Fig 7. The harmonics of the load current

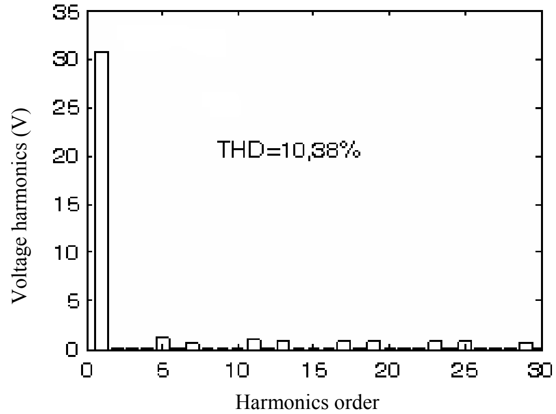

Fig 8. The harmonics of the secondary voltage

The total harmonics distortion factor value is of 31% for the load current and of 10.38% for the supply voltage.
The active current and the corresponding filter current calculated by Akagi's relation (12) are pointed out in Fig. 9. It can be seen that the Akagi's active current shape and the voltage shape are different (Fig. 9).

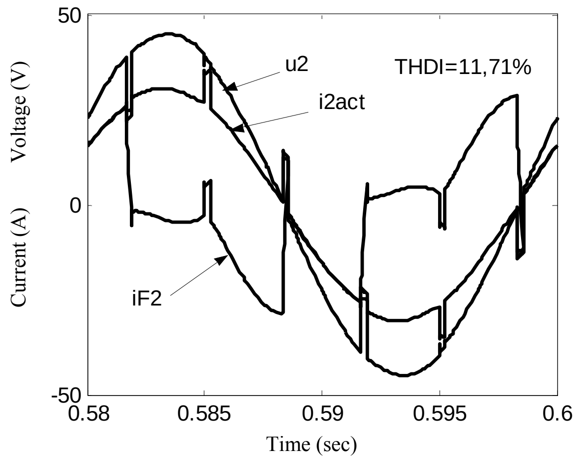

Fig 9. The voltage, the Akagi's active current and the active filter current shapes

Hence, the total harmonics distortion factor of the active current is of 11.71% which is higher than the voltage total harmonics distortion factor.
By proposed relations (22) and (23), the active current and the voltage have the same shape and the total harmonics distortion factors are the same i.e. 10.38% (Fig. 10). Certainly, the active filter current is different according to active current calculation method.

In fact, the Akagi's active current has a

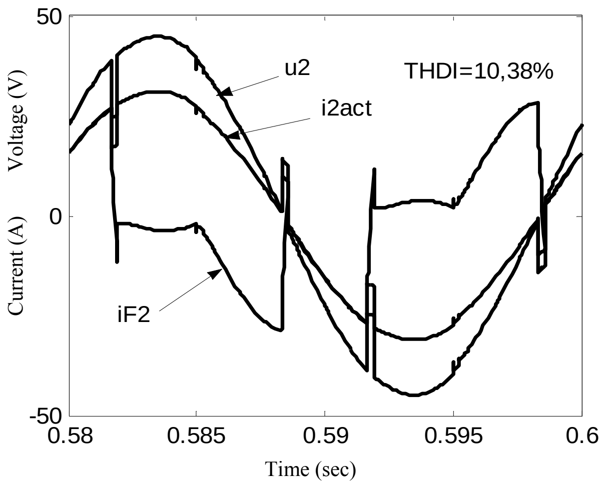

Fig 10. The voltage, the proposed active current and the active filter current shapes

supplementary distortion determined by thyristor commutations from another phases (Fig. 9). Thus, in this current, the harmonics of 7^{th}, 13^{th}, 19^{th}, 23^{th} and 29^{th} orders are higher (Fig. 11 and 12).

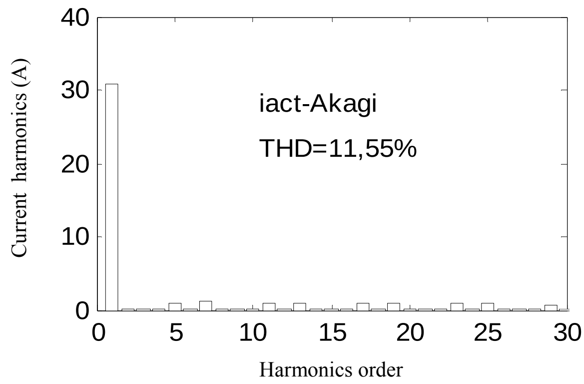

Fig 11. The harmonics of the Akagi's active current

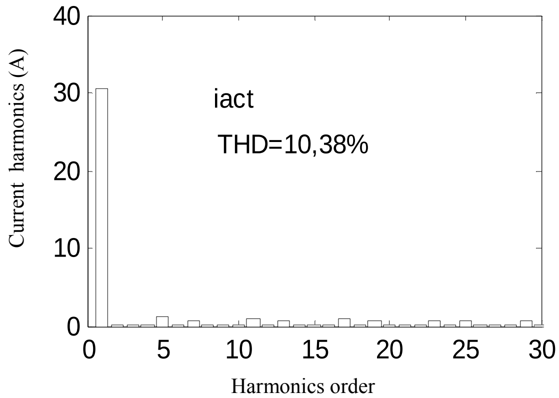

Fig 12. The harmonics of the proposed active current

4.2.2 The active current to the network

The voltage at the network side is sinusoidal (Fig. 13) and the line current has the well-known waveform and significantly contains $6n \pm 1$ order harmonics which are the same as those of the current in the secondary of the transformer (Fig. 7).

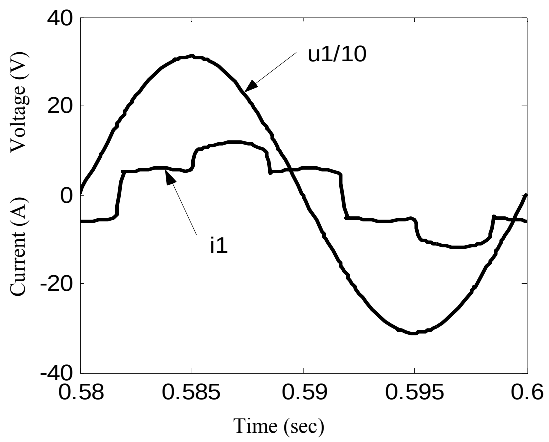

Fig 13. The voltage and current shapes in the primary of the transformer

As it can be seen in Fig. 14, the proposed active current and the active current defined by Akagi are identical because the voltage is sinusoidal and its space vector modulus is constant. In this case, the active filter current contains all the load current harmonics.

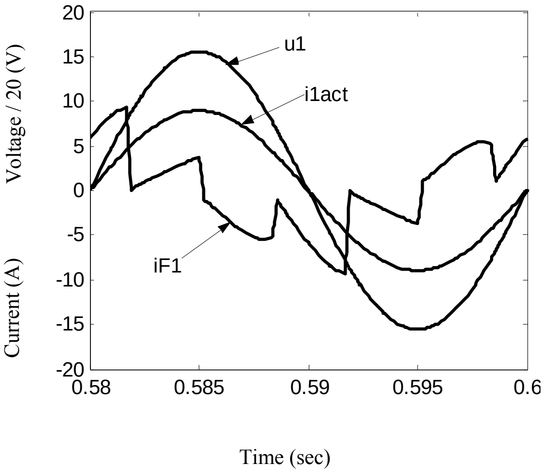

Fig 14. Phase voltage and proposed active current waveforms to the network

As regards the proposed reactive component of the current, it has a sinusoidal shape like the active component, but shifted by 90° behind the voltage, as expected (Fig. 15).

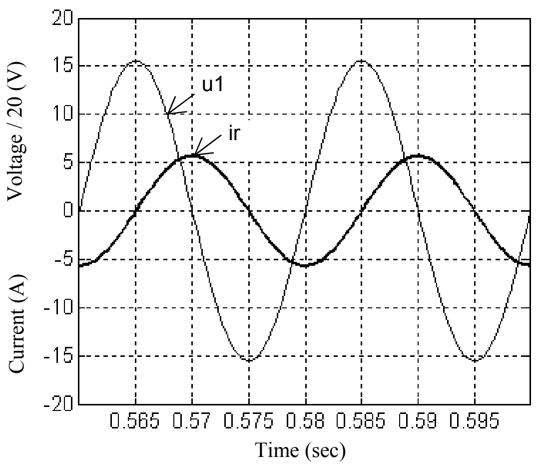

Fig 15. Phase voltage and proposed reactive current waveforms

5. CONCLUSIONS

The subject of the paper is a discussion on the active current extraction in three-phase, three-wire systems based on the so called "p-q theory". For total compensation (harmonics and reactive power compensation), the active current of the load is necessary. Instead of Akagi's active current used in active filter control, we have proposed a new relation which can be used under non-sinusoidal voltage.

Undoubtedly, the proposed active current based on the active power and on the voltage vector is useful in the calculation of the reference current for active power filters.

Thus, when total compensation is expected, the reference current requires only the load current and its active component.

The analysis of some case studies has shown that the active current which was used by Akagy is useful for compensation reasons only if the supply voltages are sinusoidal. A modified definition of the components of the current is proposed for the operation under non-sinusoidal voltage conditions. This propose makes possible to apply the "p-q theory" for active filter current calculation, under non-sinusoidal voltage.

However, under real distortion voltage conditions, when the total voltage distortion factor is less than 10%, the Akagi's active current can be used because the difference between this current and the true active current is very small. Undoubtedly this current is not the true active current, but, after filtering, the line current is a little distorted as the voltage.

At the same time, we can't affirm that the proposed active current is the true active current because we verified it only on a particular example.

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