

## TIME-DOMAIN BASED ACTIVE COMPENSATION STRATEGIES UNDER NONSINUSOIDAL CONDITIONS IN THREE-PHASE THREE-WIRE SYSTEMS

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**Abstract** – This paper deals with the peculiarities involved by the operation under nonsinusoidal voltage conditions on the generation of the reference compensating current in three-phase three-wire shunt active power filters. The total compensation of both current harmonics and reactive power is taken into consideration and the attention is mainly directed to the desired supply current after compensation. Two of the most common time-domain based approaches are analyzed in order to be improved for a proper implementation when the voltage waveform in the point of common coupling is distorted. The former is based on the concepts of the p-q theory of the instantaneous power and involves using the two-axes stationary reference frame to generate the reference compensating current. The latter does not require any reference frame transformation and allows calculating the non-active current in phase coordinates system. The whole active filtering system has been modelled under Matlab-Simulink environment and extensive simulations have been carried out to analyse the influence of the adopted compensation strategy on the filtering performance. The simulation results validate the specific algorithms for reference current generation under nonsinusoidal voltage conditions when two compensation purposes are imposed, either the achievement of a unity power factor or the perfect harmonics cancellation of the current.

**Keywords:** active current, active power filter, non-active instantaneous power theory, nonsinusoidal voltage, p-q theory

### 1. INTRODUCTION

The more and more evident spread of the nonlinear loads in both domestic and industrial area has been led to searching and finding advanced solutions to remove the associated undesired effects on the power supply and neighbouring customers.

Undoubtedly, the use of shunt active power filters (SAPF) based on voltage source inverter structure is one of the most convenient solutions in power quality improvement. The high control flexibility allows researchers to ceaselessly conceive and implement new control algorithms which are adapted to the specific filtering requirements under concrete operating conditions.

As a main principle, starting from a distorted load current ( $i_L$ ), the shunt active power filter is able to

inject such a compensating current ( $i_c$ ) in the point of common coupling (PCC) so that the current drawn from the network ( $i_s$ ) has the desired shape and zero passings (Fig. 1).

$$i_{ck} = i_{Lk} - i_{sk}, \quad k = \overline{A, B, C} \quad (1)$$

Obviously, when the three-phase voltage system is balanced and sinusoidal, the global compensation of current harmonics and reactive power leads to an active power flow to the nonlinear load by absorbing a sinusoidal current from the network which is in phase with the supply voltage. However, the operation under nonsinusoidal conditions does not allow to achieve simultaneously the two major compensation goals. Indeed, by forcing the supply current to have a sinusoidal shape and the same phase as the fundamental component of the voltage, the power factor cannot reach unity value, whereas by forcing the supply current to keep the voltage shape and zero passings, the current harmonics cancelling is not achieved.

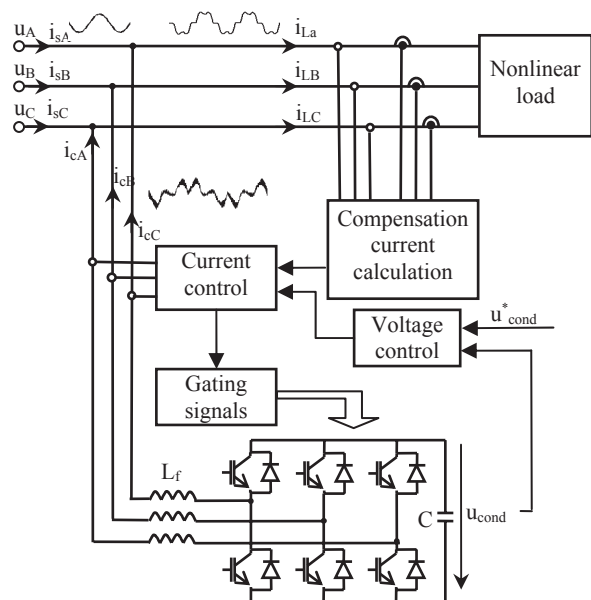


Figure 1: The principle scheme of shunt active power filter system.

In this context, the recent literature in this field provides many opinions and argumentations related to the most appropriate supply current waveform after compensation [1]-[11]. Thus, concerns about compliance with standards and recommendations such as IEEE 519 and IEC 61000 have led to the desire of perfect harmonic cancelation (PHC) [1]-[4]. This is the most common approach in generating the reference compensating current under nonsinusoidal voltage conditions. However, the active filtering system is unable to assure a unity power factor through PHC strategy.

On the other hand, when SAPF together with the nonlinear load acts as purely resistive load, the supply current becomes the active component of the distorted load current and, consequently, as the rms current drawn from the network is minimal, the network losses are reduced as much as possible and the power factor is corrected to the unity value [5], [6]. Ideally, the supply current harmonic distortion is only due to the voltage distortion and may be acceptable provided that the latter is within the limits prescribed by standards.

In addition to these approaches, the development of DSP-based control systems made it possible to implement complex control strategies in order to obtain an optimal power factor after imposing constraints on the current harmonic distortion [7]-[9]. Irrespective of active filtering goal, the control system has at first to generate the desired reference currents and then to ensure the accurate tracking of these prescribed currents by the currents provided at the inverter output.

Although there are different techniques for generating reference currents in the frequency domain based on Fourier transformation, working in the time-domain decreases the amount of calculation and provides a better dynamic regime. Thus, in most approaches in literature, either the orthogonal two-axes reference frame (stationary or rotating) [4], [8], [12]-[15] or phase coordinates reference frame [2], [3], [5], [7], [10], [11] are chosen to calculate the compensating reference currents.

This paper is organized as follows. Section 2 presents the specific calculation of the desired supply current in  $\alpha$ - $\beta$  stationary reference frame based on the concepts of p-q theory of the instantaneous power. In the next section, the phase coordinates reference frame is used to calculate the desired supply current by applying the Fryze-Buchholz-Depenbrock theory of the non-active power. Then, the performances of the compensation algorithms, in terms of total harmonic distortion factor of the supply current, the rms current drawn from the network and power factor, are tested by simulation. Finally, some conclusions are drawn in section 5.

## 2. P-Q THEORY OF THE INSTANTANEOUS POWER

In 1983, professor Akagi and his collaborators introduced the p-q theory of the instantaneous power for three-phase three-wire systems [12]. The applicability of this theory has been rapidly found in both original version and modified or extended versions, especially for the control of power compensators [13]-[15].

This theory uses the complex apparent power ( $\underline{s}$ ) which is defined on the basis of voltage and current space vectors ( $\underline{u}$  and  $\underline{i}$ ) as follows:

$$\underline{s} = \frac{3}{2} \cdot \underline{u} \cdot \underline{i}^* = p + jq, \quad (2)$$

where the instantaneous active ( $p$ ) and reactive ( $q$ ) powers are introduced as the real and imaginary parts of  $\underline{s}$ .

Based on (2), the current space vector can be expressed as

$$\underline{i} = (2/3) \cdot (\underline{u} \cdot \underline{s}^*) / |\underline{u}|^2, \quad (3)$$

where  $|\underline{u}|^2$  is the square of the voltage vector modulus.

If the voltage has a sinusoidal shape and  $\underline{i}_s$  is the supply current vector after a total compensation, the power supply provides only the active power  $P$ , i.e.

$$\underline{s}^* = P, \quad (4)$$

where

$$P = \frac{1}{T} \int_{t-T}^t p dt = \frac{1}{T} \int_{t-T}^t \text{Re} \left\{ (3/2) \cdot \underline{u} \cdot \underline{i}_L^* \right\} dt. \quad (5)$$

Under these conditions, the current drawn from the network has the meaning of active component of the distorted load current, that is

$$\underline{i}_s = \underline{i}_{La} = 2/3 \cdot \frac{\underline{u} \cdot P}{|\underline{u}|^2}. \quad (6)$$

The significant drawback in generating the reference current under nonsinusoidal voltage conditions by using (6) is related to the resulting supply current shape which is nonsinusoidal and different from the voltage shape. In [4], the proposed solution is that, before transformation from phase coordinates system to stationary  $\alpha$ - $\beta$  reference frame which is specific to p-q theory, the voltage to be filtered through a low pass filter. In this way, the supply current will be sinusoidal after compensation.

On the other hand, the supply current can be forced to be the actual active component of the load current by keeping the voltage shape and zero passings, if the square of the voltage vector modulus in (6) is replaced by its average value [16]. Thus, expression (6) becomes

$$i_s = i_{La} = \frac{2}{3} \cdot \frac{P}{\frac{1}{T} \int_{t-T}^t |u|^2 dt} \cdot \underline{u}. \quad (7)$$

### 3. THEORY OF INSTANTANEOUS NON-ACTIVE POWER IN PHASE COORDINATES SYSTEM

Making evident the active component of a nonsinusoidal current which has the same waveform and phase as the supply voltage and ensures only the active power transfer was Fryze's idea. In 1931, he proposed the following current decomposition for single-phase circuits [17]:

- the active current,

$$i_a(t) = \frac{P}{\|u\|^2} \cdot u(t), \quad (8)$$

- the non-active current,

$$i_n(t) = i(t) - i_a(t), \quad (9)$$

where  $\|u\|$  is the rms value of the voltage.

A significant contribution in extending Fryze's definitions for poly-phase circuits has been brought by Buchholz who, in 1950, introduced the so called collective instantaneous and rms values of currents ( $i_\Sigma$  and  $\|i_\Sigma\|$ ) and voltages ( $u_\Sigma$  and  $\|u_\Sigma\|$ ) through the following expressions [18]:

$$i_\Sigma = \sqrt{\sum_{k=1}^m i_k^2}; \quad u_\Sigma = \sqrt{\sum_{k=1}^m u_k^2}, \quad (10)$$

$$\|i_\Sigma\| = \sqrt{\sum_{k=1}^m \|i_k\|^2}; \quad \|u_\Sigma\| = \sqrt{\sum_{k=1}^m \|u_k\|^2}, \quad (11)$$

where  $m$  is the number of phases.

In 1993, starting from Fryze's idea and making use of Buchholz's definitions, Depenbrock introduced the Fryze-Buchholz-Depenbrock (FDB) theory that defines the active current in each phase [19],

$$i_{ka} = G \cdot u_k, \quad (12)$$

where  $G$  is the equivalent conductance,

$$G = P_\Sigma / \|u_\Sigma\|^2, \quad (13)$$

defined as a function of the collective active power  $P_\Sigma$ , which is the average value of the collective instantaneous power  $p_\Sigma$ ,

$$P_\Sigma = \sum_{k=1}^m u_k i_k. \quad (14)$$

Thus, from (12) and (13), the active phase currents are expressed as

$$i_{ka} = \frac{P_\Sigma}{\|u_\Sigma\|^2} \cdot u_k. \quad (15)$$

Then, the non-active phase currents ( $i_{kn}$ ) to be compensated are expressed by

$$i_{kn} = i_k - i_{ka}. \quad (16)$$

The original FDB theory as well as modified versions of this theory have been successfully applied for the calculation of the desired current after compensation under nonsinusoidal conditions [1]-[3], [10].

In a bid to provide a sinusoidal current after compensation, many authors consider that the meaning of  $u_k$  in (15) and (11) is the fundamental component of phase voltage [2], [3], [10], [11]. Therefore, in this case, the expression of the desired supply current is

$$i_{ka} = \frac{P_\Sigma}{\sum_{k=1}^m \|u_{1k}\|^2} \cdot u_{1k}. \quad (17)$$

It can be noted that, the generation of a compensating current in order to draw a supply current with the same shape as the distorted supply voltage is a little-discussed issue in phase coordinates system [6].

### 4. PERFORMANCES OF THE ALGORITHMS OF REFERENCE CURRENT GENERATION UNDER NONSINUSOIDAL VOLTAGE CONDITIONS

The analysis of the influence of the adopted current generation strategy on the supply current, powers flow and power factor has been performed by modelling and simulation under Matlab-Simulink environment in the case study of a controlled rectifier supplied by a nonsinusoidal three-phase voltage system (Fig. 2).

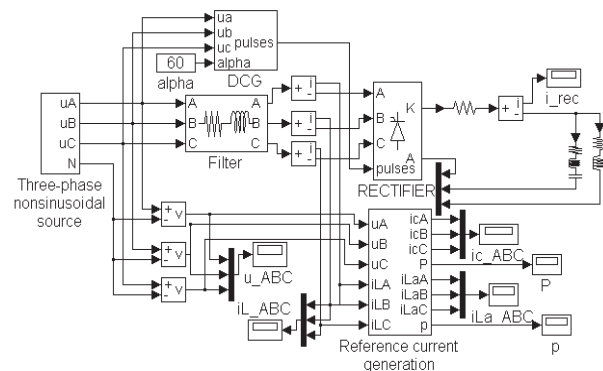


Figure 2: Simulink model of the three-phase active filtering system.

The two algorithms are implemented in a dedicated block which provides the reference compensating currents and the resulting supply currents after compensation are obtained by processing the three-phase voltage and load current systems.

The total harmonic distortion factor (*THD*) of the supply voltage is of 4.9% (Fig. 3) which is below the limit of 5% imposed by IEEE 519 standard. As regards the distorted load current, it is characterized by  $THD=34.4\%$  and the rms value is of 13.2A (Fig. 4). The associated power factor is of 0.43.

By implementing expression (6) provided directly by the p-q theory (Fig. 5), the compensated current has a harmonic distortion factor of 5% which is close to that of voltage, but the current and voltage waveforms are different even they have the same phase as the fundamental components (Fig. 6). Nevertheless, the rms supply current is greatly diminished (5.76A) and the power factor is greatly increased (0.99). A unity power factor associated with a more pronounced decrease in rms current (5.7A) is obtained by implementing expression (7) (Fig. 7). As expected, this approach leads to a compensated current whose shape is identical to that of voltage (Fig. 8).

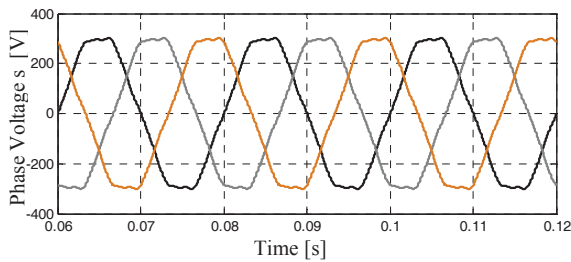


Figure 3: Three-phase nonsinusoidal voltage system.

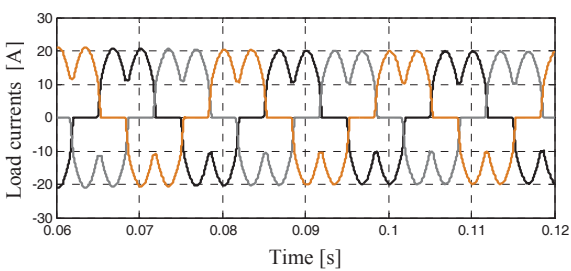


Figure 4: Distorted load currents and their harmonic spectrum.

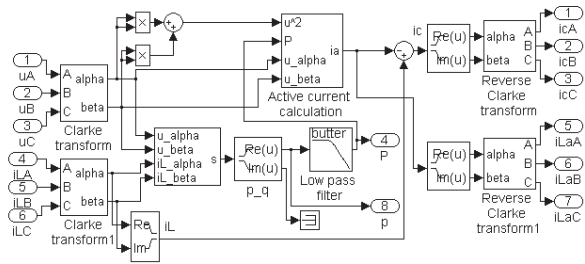


Figure 5: Matlab-Simulink implementation of expression (6) for reference current generation.

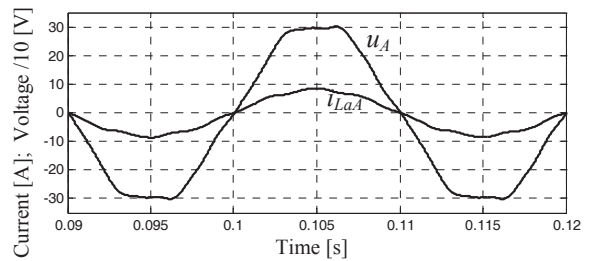


Figure 6: Phase voltage and supply current waveforms after compensation by implementing expression (6) according to original p-q theory.

Similar results are obtained by working in phase coordinates system through implementation of expression (15) (Fig. 9). It is pointed out that the structure of current reference block is much simpler when the phase coordinates system is chosen (Fig. 9) instead of the two-axes coordinates system (Fig. 7).

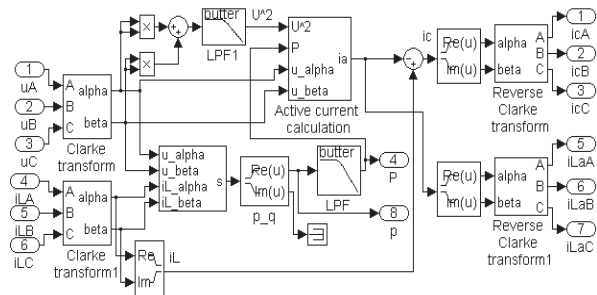


Figure 7: Matlab-Simulink implementation of expression (7) for reference current generation.

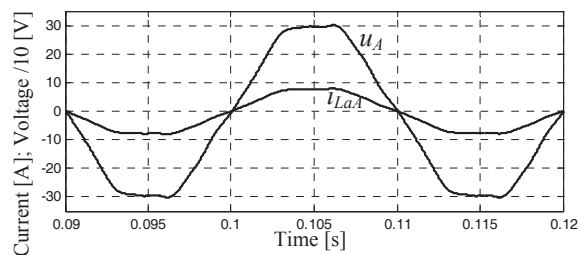


Figure 8: Phase voltage and supply current waveforms after compensation by implementing expression (7)

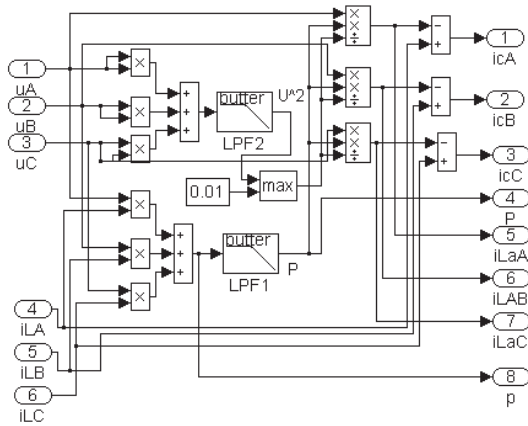


Figure 9: Matlab-Simulink implementation of expression (15) for reference current generation.

But, if unity power factor on the fundamental components together with total harmonic cancelling is the compensation goal, the implementation of expression (17) leads to a sinusoidal supply current (Fig. 10) whose rms value is of 5.73A, and the global power factor of about 0.995 is a little away from the unity value. As the inputs of the reference current calculation block in Fig. 9 are the fundamental components of the phase voltages, their separation is required as a preliminary stage.

However, the filtering performance of a real system essentially depends on the extent to which the current control system manages to handle the accurate tracking of the reference current. Moreover, the high frequency switching of the inverter devices can be found in the waveform of the current injected into PCC, even if a proper designed coupling filter is used.

For instance, by a hysteresis band control of 0.5A and a coupling inductance of 5mH, the implementation of expression (7) leads to a current THD increasing from 4.9% to 9% (Fig. 11).

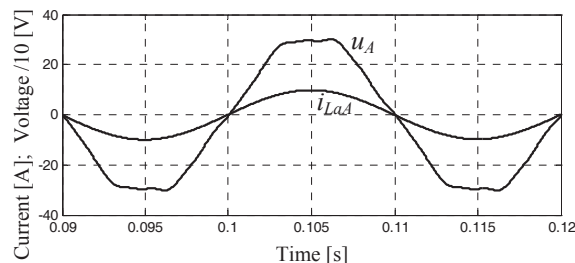


Figure 10: Phase voltage and supply current waveforms after compensation by implementing expression (15) in accordance with the instantaneous non-active power theory.

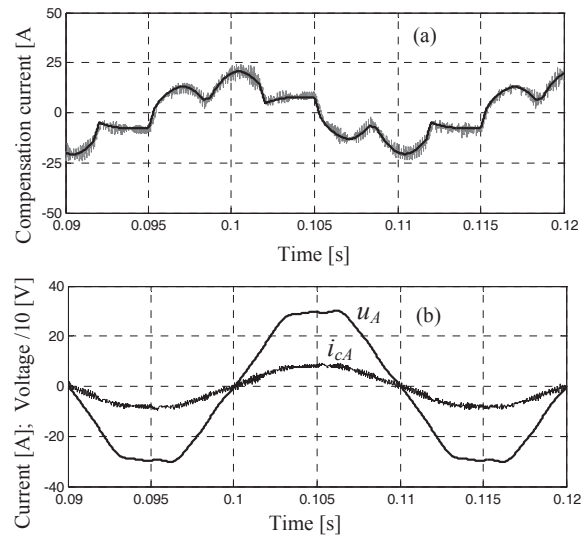


Figure 11: Reference compensating current (in black) and compensating current (in gray) - (a); phase voltage and supply current after compensation - (b) when expression (7) is used.

## 5. CONCLUSIONS

This paper analyzes the possible use of both p-q theory of the instantaneous power and instantaneous non-active power theory for reference current generation when total compensation is achieved through a three-phase three-wire shunt active filtering system under nonsinusoidal voltage conditions.

It is pointed out that appropriate changes in both theories can lead to the generation of such a compensating current which has the effect of either perfect harmonic cancellation of the supply current or unity power factor.

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