ELASTIC EQUIVALENT SPRING FOR THE ELECTROMECHANICAL CIRCUIT BREAKERS DRIVEN BY ELECTROMAGNETS

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Abstract - The aim of the present paper is to analyze the transient phase of electromagnetic actuators mechanisms of the electrical switching device by two different analytical methods, in order to simplify the determination of the electromechanical parameters of the mobile assembly of the electromechanical circuit breaker by an approximation method named the "*method of the equivalent spring*".

The paper discusses the mathematical modeling of electromagnetic switching devices such as low voltage contactors, breakers or switches devices, considered electromechanical energy conversion systems with two degrees of freedom, one electric and another mechanical, which are valid for Lagrange equations of analytical mechanics.

The value of the parameters on connecting regime can be determined by one or more mathematical systems of differential equations that characterize the transitional electromechanical regime of the direct current electrical switching device.

Equivalent spring method has been developed starting from the equations that characterize the electromechanical mechanism of the electrical switching device, on the basis of energy conservation and the approximation of the mechanical work developed by the electromagnetic actuation with magnetic energy variation between the beginning and the end of transient phase for the connecting regime.

The innovative solution allows to analyze the behavior of the electrical switching device easily than the other methods. It presents this advantage due to the fact that it is necessary only a system of equations, and therefore does not require a compounding of equations systems.

Keywords: *equivalent elastic spring**,** transient regime, nonlinear differential equations systems, resistant characteristic, electrical switching device.*

1. INTRODUCTION

The switching device refers to an electrical or electromechanical system that connect or disconnect a circuit. They represent electrical and mechanical assemblies, used for automatic or non-automatic control, protection, and adjustment, of the electrical installations parameters that have to oversee and ensure the normal transport of electricity from power supply to consumers. The connections between power plants and energy transfer lines, and between electrical networks and industrial consumers are made by using the electrical switching equipment.

As an electrical switching device we choose a contactor based on a direct current electromagnet, with a resistance characteristic, [1], [2].

Based on the duality between the translation and rotation movement of the movable assembly, we chose to treat only the actuator systems with electromagnets drive considering a translational motion of the actuator assembly (the principle is shown in Figure 1). This option has been chosen because the parameters that characterize are more intuitive and more convenient to determine a solution correlation obtained by solving the nonlinear system of differential equations that describe a real model. Also results can be applied to any electromagnetic actuator drive, [3], [4].

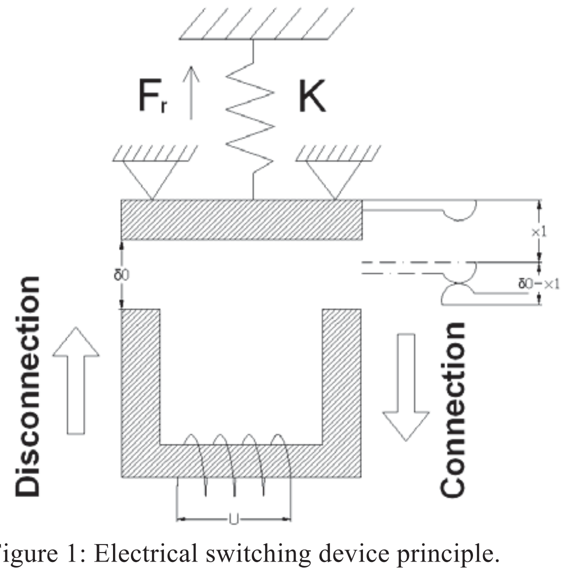

Figure 1: Electrical switching device principle.

We define theoretical the nonlinear systems of differential equations that describe the behavior of an antagonist spring-type resistant characteristic actuator ensemble and propose the numerical analysis method for such equations systems .

The nonlinear differential equations systems proposed, allows the adaptation of the mathematical model to the resistant characteristic "contact" with discontinuity (hopping jump) at the point of reaching the contacts.

2. ANALYSIS OF THE RESISTANT CHARACTERISTIC

To demonstrate the above statements we compared the results obtained by numerical analysis with those using equivalent spring method.
For the beginning we must analyze the transient phase for connection and this purposes that we must solve the nonlinear systems of differential equations that characterize the conservative electromechanical system (neglecting the losses), without considering the magnetic dispersion flux and the expansion of the magnetic field lines in the air gap.
At the same time it must be highlight the possibility of some damped oscillations appearance at the end of the mobile armature course of the electromagnet.
The proposed numerical method is used to define the electromechanical system behavior during the connecting transient regime for the equivalent resistant spring case. It is also consider the real case of the normal resistant characteristic for the electrical switching device with discontinuity corresponding to the touch of the contact parts.
Comparative results obtained using the specified methods allow to identify opportunities for optimization of the whole drive by correlation of the resistant forces characteristic with the electromechanical characteristic of the electromagnet. The validity of the results obtained by solving the non linear differential equations systems using the numerical method has been confirmed by experimental data obtained on the basis of connection time measurement of the direct current electrical switching device, obtaining acceptable deviations between the results.
Accounting the influence of the dispersion magnetic flux and of the expansion of the magnetic field lines in the air gap zone (connecting process), we must highlight the reduction of the calculated connection time. The difference between the analytical obtained time and the one obtained by direct measurements, must situate in a maximum difference of 20%.
The analysis results indicate that in the moment of contacts opening there is the possibility of appearance of movable ensemble oscillations. This thing indicates that the contacts are prematurely exposed to the electric arc action as a result of these repeated separations.
The analysis of the transient phase on the electrical switching device with contacts operated by electromagnets is difficult due to the specific resistant characteristic pattern (hopping jump at reaching contacts).
The resistant characteristic of the electrical switching device presented in Figure 2 shows two levels characterized by different mechanical parameters. The first level is corresponding to the movement of the movable armature from the largest air gap δ_0 to the contacts touching point, and it is characterized by the antagonist spring constant k_1, initial force F_0 and the final force F_1; the second level, is corresponding to the to the movable armature movement between air gap x_1 and minimum air gap, being characterized by the elastic constant of the spring compression of the contacts k_2, respectively by the initial force F_2 and final one F_M.
For each of the two presented areas of the resistant characteristic, are corresponding two different mechanical parameters. The analysis in dynamic phase of the switching devices basically requires to solve two nonlinear systems of differential equations of the form (1), for each system will be imposed continuity conditions for both sides of the movable armature position corresponding to the interaction of the contact parts.
This method is quite laborious and the evaluation of the switching device behavior under dynamic phase with classical methods of determination by the evolution of parameters is hard to follow. For using such a method are necessary calculating performance units and appropriate programs.
The innovative method is proposing to define a fictitious resistance characteristics as an equivalent elastic spring type which leads to a reduction of computation and also can provide global information about the behavior in a more comfortable way regarding the dynamic regime of the electrical switching device electromechanical system as closer to the corresponding real resistant characteristic , [5], [6].
The equivalent elastic spring resistant characteristic type has been chosen because the mathematical model able to describe the switching device under dynamic regime is closer to the one which describe a characteristic of an electromagnet with an antagonist spring. Equivalence criterion for defining of the elastic spring constant is considered to be a conservation of mechanical work type.
The condition of equality between mechanical work received and the one disposed by the mobile armature is expressed by the equality of areas: $A_{123} = A_{3456}$. Physically means that the mechanical work that would develop antagonistic forces corresponding to the equivalent elastic spring characteristic, must be equal to the work developed by the antagonistic forces that define the real resistant characteristic during the transient regime. The electrical switching device evolution in dynamic regime can be made just by solving a single system of differential equations [7], [8].
In the case of the connection process, the variables which must be determined by solving the differential equations systems will be the magnetic flux $\Phi(t)$ and electromagnet armature movement $x(t)$.

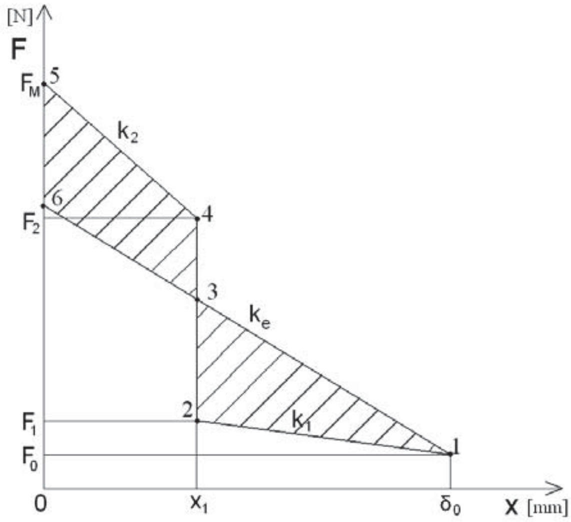

Figure 2: The resistant characteristic of the electrical switching device.

For the electrical switching device actuated by electromagnets with a discontinuous real resistant characteristic, (hopping jump), nonlinear systems of differential equations that describe the dynamic phase when the contact parts interact are written taking into account the following simplifying assumptions:

1) The magnetic dispersion flux and the magnetic field lines expansion in the air gap have been neglected;
2) The Joule effect energy losses from the magnetic core have been neglected;
3) the reduced mass of the electromechanical ensemble is considered constant throughout the movement of the movable armature;
4) The friction energy losses have not been considered.

The equations system for the first part of the characteristic which are corresponding to the movable armature movement between the maximum air gap δ_0 and contacts touching point x_1, is:

$$\begin{cases} \dfrac{d^2x(t)}{dt^2} + \Omega_1(x(t)+b) = \dfrac{\phi^2}{2\mu_0 Sm} \\ \dfrac{d\phi(t)}{dt} + \dfrac{R}{L}(d_0 - x(t) + a)\phi(t) = \dfrac{U(t)}{N} \end{cases} \quad (1)$$

$$\Omega_1 = \frac{k_1}{M} \quad (2)$$

M means the reduced mass of the whole electromechanical ensemble with initial conditions:

$$\phi_0^{(1)} = \sqrt{2\mu_0 SF_0} \;\; ; \; v_0^{(1)} = 0 \quad (3)$$

To describe the movement of the electromagnet movable armature from the air gap x_1 to minimum air gap we used the next system:

$$\begin{cases} \dfrac{d^2x(t)}{dt^2} + \Omega_2(x(t)+b) = \dfrac{\phi^2}{2\mu_0 Sm} \\ \dfrac{d\phi(t)}{dt} + \dfrac{R}{L}(d_0 - x(t) + a)\phi(t) = \dfrac{U(t)}{N} \end{cases} \quad (4)$$

$$\Omega_2 = \frac{k_2}{M} \;,\; \phi_0^{(2)} = \phi_{final}^{(1)} \;;\; v_0^{(2)} = v_{final}^{(1)} \quad (5)$$

$\Phi_0^{(1)}$ and $v_0^{(1)}$ are the initial magnetic flux and the initial speed for the second equation system;
$\Phi_0^{(2)}$ and $v_0^{(2)}$ are the initial magnetic flux and the initial speed for the second equation system.
The contactor parameters are shown in the below table:

Parameter		Value
Coil voltage supply	U	220 [V]
Maximum air gap	δ_0	0.008 [m]
Coil resistance	R	1616 [Ω]
Core section	S	0.00138 [m^2]
Reduced mass	M	1 [Kg]
Number of turns of the coil	N	27800
Elastic spring constant	k_1	1000
Elastic spring constant	k_2	4700
Movement through the contact	x_1	0.003 [m]
Equivalent air gap	a	0.003 [m]
Initial force(k_1)	F_0	10 [N]
Final force(k_1)	F_1	15 [N]
Initial force(k_2)	F_2	40 [N]
Final force(k_2)	F_M	54 [N]

Table 1: The parameters of the contactor.

3. NUMERICAL ANALYSIS METHOD OF THE CONNECT TRANSIENT REGIME

Using the Matchcad 14 software, the solutions of the equation systems were approximated using numerical methods. This method consist in the replacement of the derivatives with finite difference expressions of the derived measurements at mesh intervals " h " equal to 0.00001 sec (the equations form are indicated below). To establish the characteristics of the first area it was necessary a total of 2750 intervals.

$$\begin{pmatrix} x_{i+2} \\ \phi_{i+1}\end{pmatrix} := \begin{bmatrix} \left[\left[\frac{(\phi_i)^2}{2\cdot\mu\cdot S\cdot M}\right] - \frac{k_1\cdot(x_i+0.002)}{M}\right]\cdot h^2 + 2\cdot x_{i+1} - x_i \\ \left[\frac{U\cdot h}{N} - \frac{R\cdot\phi_i\cdot(d-x_i+a)\cdot h}{\mu\cdot S\cdot N^2} + \phi_i\right]\end{bmatrix} \qquad (6)$$

The connection time is indicated in seconds and is given by the product of number of intervals " i " and interval size " h ".

The initial conditions for the first equations systems are:

- the connection process begins when the force produced by the magnetic flux reaches a greater value than the maximum resistance force;
- the initial speed is set to 0.

In Figure 3 is indicate the connection time determined by an appropriate analytical method for the first system of equations which correspond to the interval from the maximum air gap δ_0 to the reaching of the contacts x_1. As can be seen, for a movement of 5 mm, the characteristic is not linear, the first $2*10^{-3}$ *s* has a movement almost null. Once exceeded this value the ensemble increases its speed exponential.

As mentioned above the second unknown with a major importance in the behavior of the electrical switching device is the magnetic flow. The minimum value of the flux for which the force exerted by the direct current electromagnet is equal or it exceed the antagonist spring resistance force is $1862*10^{-4}$ *Wb*. The evolution of the magnetic flow tends to be linear (grow in direct proportion to the time value) as shown in Figure 4.

For determination of the speed parameter for the first interval we use this formula:

$$v_i = \frac{x_{i+1} - x_i}{h} \qquad (7)$$

Variation of the armature speed and the force exerted by the electromagnet for the first interval are indicated in the Figure 5, and Figure 6.

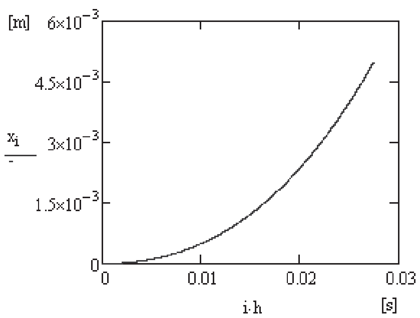

Figure 3: Variation of the air gap versus time for the first interval.

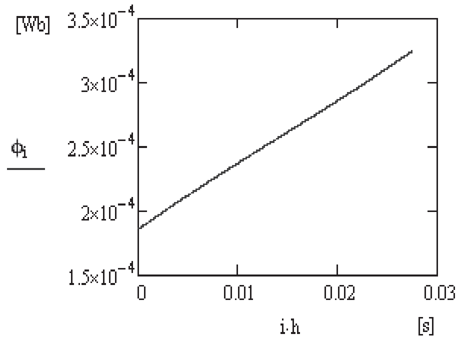

Figure 4: Variation of the magnetic flux versus time for the first interval.

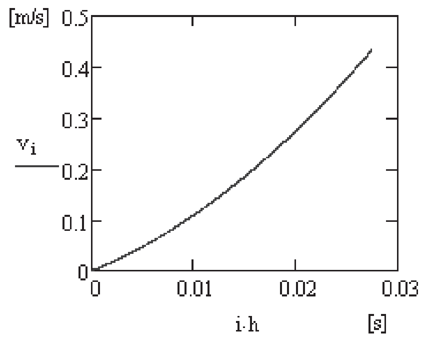

Figure 5: Variation of the speed versus time for the first interval.

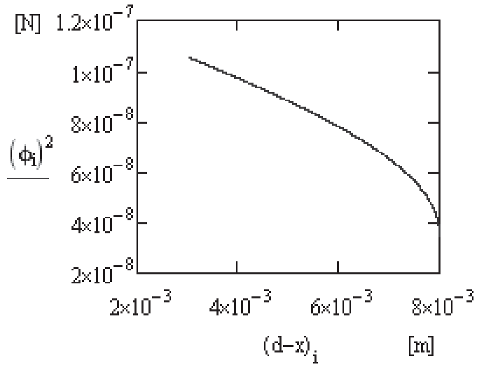

Figure 6: Electromagnet force versus air gap for the first interval.

The approximation of the second equations system leads to the time determination of the interval between air gap x_1 to minimum air gap characterized by the compression spring constant k_2, the initial strength F_2 and final F_M.

The initial conditions of the second equation system are:

- the magnetic flux obtained by approximating the first equation system is equal with the initial magnetic flux for the second;
- the speed value at the final of the first stage is equal with the initial speed value of the second stage.

The speed determination for the second phase of the transient regime was made using this relation:

$$v_j = \frac{x_{j+1} - x_j}{h} \qquad (8)$$

To view the entire connection resistant characteristic were overlapped on a single chart the two equations systems of. As can be seen in Figure 7 the connection transient process for the electrical switching device ends in t_{ca}=0.039 s. The movement of the electromagnet armature to the minimum air gap, without separation of the contacts suggests that exist an oscillation with the effect of downforce variation on the contacts. This can lead to negative phenomena, increases the probability of local melt phenomena of the contacts thus shorten their life. The value of the maximum magnetic flux, obtained at the final of the transient regime is presented in Figure 8.

Comparing to the experimental time, the connection time approximated using the analytical method is smaller due to the neglect of the dispersion magnetic flux, friction, etc..

The magnetic flux dispersion influence is modifying the electric switching device actuation by lowering the useful magnetic flux at the electromagnet air gap, thus lead to a decrease of the actuation electromagnetic force of the movable ensemble.

As much as the contacts are approaching, the force of the electromagnet (Figure 9) is decreasing.

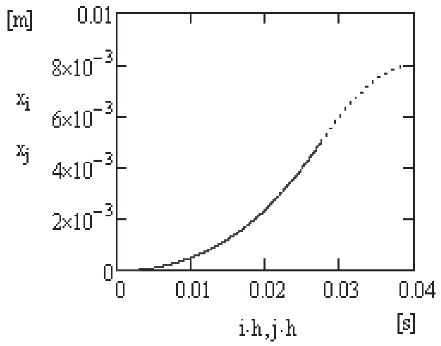

Figure 7: Variation of the air gap versus time for the entire transient regime.

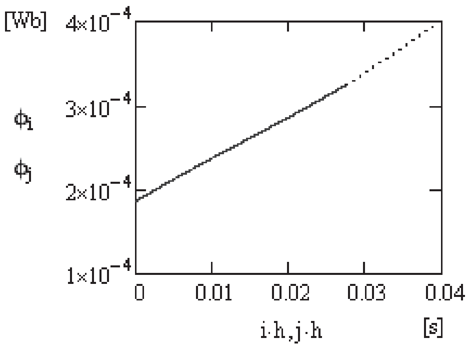

Figure 8: Variation of the magnetic flux versus time for the entire transient regime.

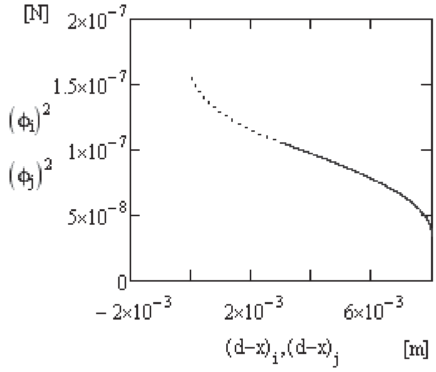

Figure 9: Electromagnet force versus air gap for the entire transient regime.

4. EQUIVALENT SPRING METHOD FOR THE CONNECT TRANSIENT REGIME

For this method it was necessary to build graphical the resistance characteristic of electrical switching device as indicate in Figure 2, in order to determinate the slope, when the two areas (A_{123} and A_{3456}) are equal. For the graphic construction in order to benefit of a very good precision, the AutoCAD 2007 software has been used. The equivalent spring constant value is provided by the line slope *"m"* which in our case is k_e = 3876.

The determination of equivalent spring slope can be done by using several methods such as:

- the determination of the line equation y = mx+b where *m* is the slope;
- the determination of the slope in a two-dimensional cartesian coordinate system.

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} \qquad (9)$$

The initial conditions of the equations system are:

- the connection process begins when the force produced by the magnetic flux reaches a greater value than the maximum resistance force;
- the initial speed is set to 0.

The determination of the connection parameters of the electrical switching device using the innovative method involves the approximation of a single equations system.

In this case the time for the connection transient regime using the equivalent elastic spring method is t_{ce} =0.44 *s* and for the complete analytical method is t_{ca} = 0.039 *s*. This means that the difference between the two result is less then 20%.

In Figure 10 is presented the connection transient regime that corresponds for the entire interval, from the maximum air gap to touch the contacts.

As can be see in the Figure 11, Figure 12, Figure 13 the magnetic flux, the speed and the force produced by the electromagnet have similar forms and values as the first method.

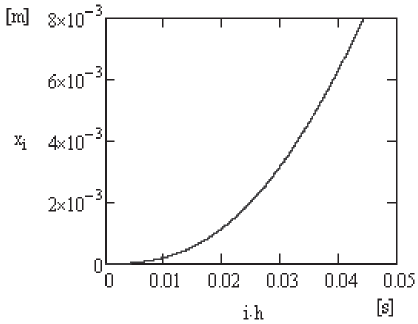

Figure 10: Variation of the air gap versus time for the entire transient regime.

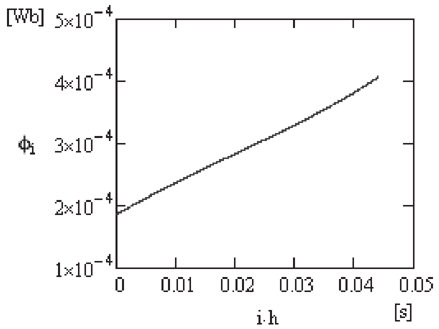

Figure 11: Variation of the magnetic flux versus air gap for the entire transient regime.

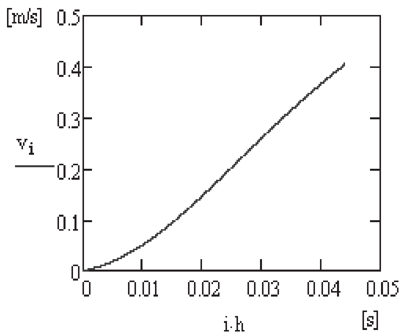

Figure 12: Variation of the speed versus air gap for the entire transient regime.

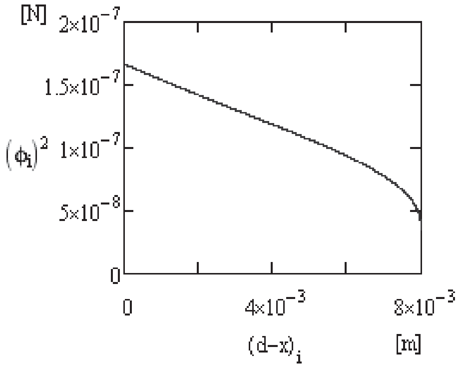

Figure 13: Electromagnet force versus air gap for the entire transient regime.

5. CONCLUSIONS

This paper proposes two mathematical methods to study the transient regime of the direct current electrical switching device with non null start flux, considered like an electromechanical energy conversion system with two freedom degrees described by a non-linear differential equations system. It defines the time evolution during electromagnets' driving, of some parameters which characterize the assembly behavior (the movable armature displacement and dynamic magnetic flux) which allow the estimation of the dynamic electromechanical characteristic.

It compares the results of the transient characteristics obtained by applying the full analytical method, to the one using the method of equivalent elastic spring. Among the advantages of this method can be mentioned:

- decreases the amount of calculation (needs solving just one equation system);
- the system parameters can be more easily determined;
- the influence of the parameters over the transient regime can be easily evaluate;
- using the electromagnetic force analysis in the dynamic regime and the equivalent spring characteristic we can optimize the ensemble in such way to reduce the oscillations.

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