Harmonics Analysis Using DFT

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Abstract— **Harmonics analysis is established by international standards as well as national norms [6], [7]. There are also definitions and calculus relationships for the indicators considered representative for this non quality aspect of the electric power. Among the harmonics analysis methods, the authors have dealt with application of DFT, given the fact that were to be analyzed acquired waves experimentally, using data acquisition boards. Experiences in using DFT, difficulties and highlighted errors as well are materialized in this paper, which is of great use to practitioners in this field. Next, there is made a distinction between the a.c. and d.c. current waves in defining the harmonics indicators, which gives accuracy and expressivity to the harmonic analysis.**

I. Introduction

Analysis of voltage and current waves, regular but non-sinusoidal, has a special importance due to technical and economic consequences of the compensation measures, based on results. Existence of errors in using DFT is constantly reported in the literature, and for some, as alias phenomenon in alternating current (a.c.), the maximum number of identifiable harmonic or phase error, has already formulated explanations and demonstrations.

Highlighting the direct current (d.c.) "alias" phenomenon and errors explanations, in determining indicators of distorting regime for d.c. and a.c. waves, is the first objective of the paper.

Analytical foundation of the raised issues is embodied, when necessary, on a particular signal forms, as the rectified voltage single phase, double alternation.

The ability to determine the analytical characteristics of all harmonics from the wave and making the sum of all harmonics involved in the d.c. alias phenomenon, allowed any checks and identifications, able to conclude on the correctness of the assumptions set.

II. Analytical Basis

A. Basic Rules to Apply DFT

Numerical signal processing and analog-digital conversion led to waveform sampling, according to Fig. 1. Harmonics analysis of real waves, obtained experimentally by oscillograph, acquisition and recording, thus requires, first, identifying the fundamental period T and dividing it into 2p equal parts, so p parts per half cycle, the samples Yk, forming the set values

$$Y_k=y(t_k),\ k\in\{0, 1, 2, ..., (2p\text{-}1)\}, \tag{1}$$

corresponding to the analyzed function $y(t)$, for the times t_k (t was considered as an independent variable; it can be any physical size, for example, length, angle etc.):

$$t_k=\frac{T}{2p}\cdot k,\ where\ \ k\in\{0,1,...,2p-1\}. \tag{2}$$

When using DFT, Fourier coefficients expressions are calculated correctly with the relations [2]:

$$A_N=\frac{1}{p}\sum_{k=0}^{2p-1}Y_k\cdot\sin\left(\frac{N\pi}{p}k\right),\ N\in\{1,(p-1)\};$$
$$B_N=\frac{1}{p}\sum_{k=0}^{2p-1}Y_k\cdot\cos\left(\frac{N\pi}{p}k\right),\ N\in\{1,(p-1)\}, \tag{3}$$

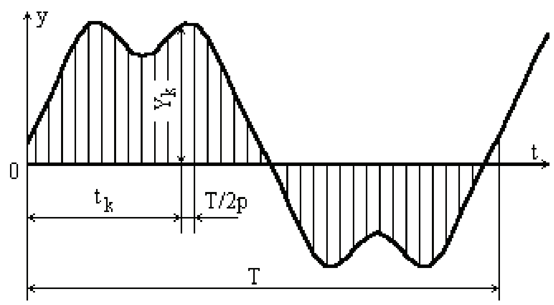

Fig.1. Periodic wave, non-sinusoidal $y(t)$, sampled.

and the continuous component is determined by the relationship

$$Y_0=\frac{1}{2p}\cdot\sum_{k=0}^{2p-1}Y_k \tag{4}$$

where N is the order of harmonics (relation 3).

Development of searched Fourier wave $y(t)$ is presented in the following condensed expression:

$$y(t)=Y_0+\sum_{N=1}^{p-1}Y_N\cdot\sin(N\omega t-\varphi_N), \tag{5}$$

where the harmonics amplitude and phase are determined, respectively, with the following relations:

$$Y_N=\sqrt{A_N^2+B_N^2}\ ; \tag{6}$$

$$\varphi_N=\arcsin(-B_N/Y_N)\cap\arccos(A_N/Y_N), \tag{7}$$

and $\omega=2\pi/T$ is the corresponding fundamental pulsation.

As regards the phase φ_N of N order harmonic, the correct determination should take into account that phases $\varphi_N \geq 0$, located in [0, 2π] interval has to be determined, which requires consideration of the coefficients signs A_N and B_N, for placing phases in the proper quadrant [2].

It should be mentioned that each harmonic is a phasor, so that the complete identification of a harmonic having given the order N, may be done using either the pair of Fourier coefficients (A_n, B_n) or the pair amplitude - phase (Y_N, φ_N).

The theorem referring the maximum determinable order using DFT has been formulated and demonstrated in [2]:

based on 2p samples on a fundamental period, the continuous component, if any, and (p-1) harmonics can be correctly determined, each of them being identified by amplitude and phase.

Applying this theorem is materialized in the way of writing analytically the reconstructed wave, as in relationship (5). Since the identification of the continuous component and (p-1) harmonics are exhausted only (2p-1) conditions from the (2p) available, expressed by equations (1) and (2), only a single information is still utilizable and it is referring the fact that the p order harmonic exists, so it has a nonzero amplitude [2], but it cannot be determined.

B. Phase error

For trigonometric functions as $\sin(N\pi k/p)$ and $\cos(N\pi k/p)$ the generic name of modulation or filtering functions were proposed, last of these functions restoring the contribution of identifying the Fourier coefficients of the N-order harmonic.

If the summation index $k=\overline{1,\,2p}$ in relations (3) (as in some specialized materials) and not in $k=\overline{0,\,(2p-1)}$, then for the Fourier coefficients of the N-order harmonic

$$y_N(x) = Y_N \cdot \sin(Nx - \varphi_N), \tag{8}$$

result the expressions [2]:

$$A_N = Y_N \cdot \cos\left(\varphi_N + \frac{N\pi}{p}\right); \tag{9}$$

$$B_N = -Y_N \cdot \sin\left(\varphi_N + \frac{N\pi}{p}\right), \tag{10}$$

revealing that the calculated phase is greater than the phase φ_N with

$$\Delta\varphi_N = \frac{N\pi}{p}, \tag{11}$$

which represents a phase error.

Noting that the size of the phase error, given by (11), corresponds to the increase of the filter function arguments, $\sin(N\pi k/p)$ and $\cos(N\pi k/p)$ when the summation index is taken $k=\overline{1,\,2p}$, versus the case $k=\overline{0,\,(2p-1)}$, as in relations (3). Appearance and influence of the phase error were analytically and graphically highlighted in [2].

The relations (9) and (10) have also enabled to formulate a theorem of independence between amplitudes values, Y_N, in report with the summation limits, if it covers the same range of summation, corresponding to a period of fundamental [2].

C. P order harmonic

According to Shannon's sampling theorem, sampling a periodic and non-sinusoidal signal must be with a frequency f_e at least two times higher than the frequency of $N_{\max}$ maximum order harmonic, to be evidenced from the spectral analysis of this signal:

$$f_e \geq 2\left(f_1 N_{\max}\right). \tag{12}$$

The frequency representing the half of sampling frequency ($f_e/2$) is called the Nyquist frequency [7], which has the rank p, if it considers (2p) samples over a period of fundamental. Regarding the order $N_{\max}$, Shannon's sampling theorem is expressed as:

$$N_{\max} \leq p. \tag{13}$$

The sign equal, allowed by relation (13), is however questionable in terms of number of distinct conditions that exist, as already previously emphasized.

For the analytical study of the specific identifying of Nyquist frequency harmonic, the Fourier coefficients have been calculated for $N=p$, that is for the harmonic given by the samples series

$$Y_{pk} = Y_p \sin\left(\frac{p\pi}{p}k - \varphi_p\right) = Y_p \sin\left(k\pi - \varphi_p\right),\ k=\overline{0, 2p-1}, \tag{14}$$

for which the Fourier coefficients (3) result as follows:

$$A_p = \frac{1}{p}\sum_{k=0}^{2p-1} Y_p \sin\left(k\pi - \varphi_p\right)\sin k\pi = 0\,; \tag{15}$$

$$B_p = \frac{1}{p}\sum_{k=0}^{2p-1} Y_p \sin\left(k\pi - \varphi_p\right)\cos k\pi = -2Y_p \sin\varphi_p\,, \tag{16}$$

showing that only $B_p \neq 0$ ($A_p = 0$) for $\varphi_p \notin \{0, \pi\}$ case, but both the amplitude and the phase cannot be correctly calculated. The initial phase problem $\varphi_p \notin \{0, \pi\}$ can be solved by de-phasing string samples. Compared with (10), it appears that B_p follows a value twice that which would be normal.

In conclusion, the Nyquist frequency harmonic is not determinable, having at the most the possibility to prove that it exists, when the coefficient $B_p \neq 0$, which ends the demonstration of theorem that refers to the fixed maximum order of harmonics, when using DFT.

D. Undetectable harmonic translation at DFT

1) AC „alias" phenomenon

The translating phenomenon of the higher order harmonics, $N > p$, from composition of a periodic wave, has been mentioned [7], but only in [2] has been analytically justified. This consists to identify as lower order harmonic, $K<p$, of the M-order harmonic, with $M>p$. This phenomenon, known in literature as an alias effect or retreat (or even redefine) effect, distorts or "falsifies" identifying harmonics of a periodic and non-sinusoidal wave.

Analytical justification to the possibility of occurrence of the alias effect and how its expression is based on consideration of a string samples of an rank M>p harmonic, as

$$Y_{Mk} = Y_M \sin\left(\frac{M\pi}{p}k - \varphi_M\right),\ k = \overline{0, 2p-1}, \qquad (17)$$

for which the question is whether it can occur in discrete Fourier analysis, as an N-order harmonic.

Proceed with the determination of Fourier coefficients A_N and B_N with relations (3), for the samples string given by (17). Although it is known that for a harmonic $M \neq N$ and $M<N$, the coefficients A_N and B_N are null, is searching for those particular cases, when the coefficients can be non-zero. After elementary transformations, the relations for the coefficients A_N and B_N are obtained in this case in the form:

$$A_N = \frac{Y_M}{2p}\left\{\left[\sum_{k=0}^{2p-1}\cos\frac{(M-N)k\pi}{p} - \sum_{k=0}^{2p-1}\cos\frac{(M+N)k\pi}{p}\right]\cos\varphi_M + \right.$$
$$\left. + \left[\sum_{k=0}^{2p-1}\sin\frac{(M-N)k\pi}{p} - \sum_{k=0}^{2p-1}\sin\frac{(M+N)k\pi}{p}\right]\sin\varphi_M\right\} \quad ; \qquad (18)$$

$$B_N = \frac{Y_M}{2p}\left\{\left[\sum_{k=0}^{2p-1}\sin\frac{(M-N)k\pi}{p} + \sum_{k=0}^{2p-1}\sin\frac{(M+N)k\pi}{p}\right]\cos\varphi_M - \right.$$
$$\left. - \left[\sum_{k=0}^{2p-1}\cos\frac{(M-N)k\pi}{p} + \sum_{k=0}^{2p-1}\cos\frac{(M+N)k\pi}{p}\right]\sin\varphi_M\right\} \quad . \qquad (19)$$

If $M>N$ and ($2p$) divides the numbers of the form ($M \pm N$), which is ($2p$) | ($M \pm N$), Fourier coefficients, calculated with equations (18) and (19) may be non-zero, with one following forms:

- if (2p)|(M-N),

$$A_N = Y_M \cos\varphi_M;\quad B_N = Y_M \sin\varphi_M \quad ; \qquad (20)$$

- if (2p)|(M+N),

$$A_N = -Y_M \cos\varphi_M;\quad B_N = -Y_M \sin\varphi_M \quad , \qquad (21)$$

analytical confirming the possibility of developing alias effect [2].

Therefore, the condition that a $M>p$ order harmonic to appear as a $N<p$ order harmonic, when is using ($2p$) samples to an analyzed wave period, is expressed analytically by the relationship

$$(2p) \mid (M \pm N), \qquad (22)$$

that is, the condition that the integers ($M \pm N$) to divide by ($2p$).

Because of undetectable harmonic translation phenomenon, accuracy of the Discrete Fourier Analysis, of a string samples may be questioned. To eliminate analysis errors of a periodic and non-sinusoidal waves with DFT are proposed in [2] algorithms, but the most effective measure can be considered as limiting the maximum harmonics frequency of the signals acquired through a low pass filter, before the waves to be submitted to the acquisition card.

The phenomenon of identifying a higher harmonic, undetectable ("invisible") as a lower order harmonic was also highlighted in the case of harmonic waves with a complex composition, when was noted that the phenomenon of undetectable harmonic translation is accompanied by the composition of an alias harmonic (translated) with the lower harmonic ($N>p$), existing in the actual analyzed wave.

Whether N-order harmonic given by (8) and M-order harmonic, translated, which appears still the N-order harmonic, written as:

$$y_M(x) = Y_M \cdot \sin(Nx - \varphi_M), \qquad (23)$$

will determine the resulting wave y_N' as the amount of the two:

$$y_N'(x) = Y_N \cdot \sin(Nx - \varphi_N) + Y_M \cdot \sin(Nx - \varphi_M). \qquad (24)$$

Performing possible calculus in (24), for the resultant wave is determined expression:

$$y_N'(x) = Y_N' \cdot \sin(Nx - \varphi_N'), \qquad (25)$$

where the resultant wave amplitude Y_N' is given by:

$$Y_N' = \sqrt{Y_N^2 + Y_M^2 + 2Y_N Y_M \cos(\varphi_N - \varphi_M)}, \qquad (26)$$

and the resultant wave phase match condition

$$\varphi_N' = \left\{\arcsin\frac{Y_N \sin\varphi_N + Y_M \sin\varphi_M}{Y_N'}\right\} \cap \left\{\arccos\frac{Y_N \cos\varphi_N + Y_M \cos\varphi_M}{Y_N'}\right\}. \qquad (27)$$

2) D.C. "alias" phenomenon

The DFT application for harmonic analysis to a periodic signal, which contains a practically infinite number of harmonics, as the rectified voltage signal type, single phase, double alternation, shown in Fig. 2, has revealed an error in the determination of all components [3].

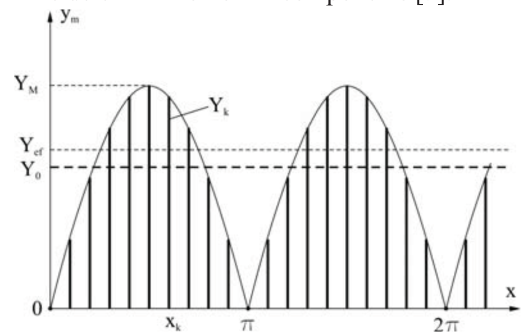

Fig. 2. Rectified voltage, single phase, double alternation

In addition, was highlighted a decrease of determination errors, in relation to the values calculated by applying the Continuous Fourier Transform (CFT), as the number ($2p$) of samples per period is higher. The phenomenon can also manifest to developments through DFT, of waves with a finite number of harmonics, but having a large number of $N > p$ order harmonics and some even of $N > (2p)$.

As the analytical form of the shown signal (Fig. 2) is known [3] its Fourier development has the form:

$$y(x) = \frac{2Y_M}{\pi}\left[1 + \sum_{N=1}^{\infty} \frac{(-1)^{N-1} - 1}{N^2 - 1}\cos(Nx)\right], \quad (29)$$

which highlight that exists only even harmonics.

Continuous component Y_0 of analyzed signal has the size

$$Y_0 = \frac{2Y_M}{\pi} = 0{,}63662 \cdot Y_M, \quad (30)$$

scale represented on the graph in Fig. 2.

The question is "for what rank, *N*, consecutive samples of it have the same values, identical to the initial sample value?" Condition to be satisfied, is written, in this case as:

$$\sin\left(N\frac{\pi}{p} - \varphi_N\right) = -\sin\varphi_N, \quad (33)$$

which shows that for the any order

$$N = 2p\ell, \ \ell \in \{1, 2, 3, ...\}, \quad (34)$$

of harmonics samples that are constantly sizes, so appear "d.c. alias", contributing to redefine the value of continuous component.

To continuing, the contribution of all harmonics, with orders given by equation (34), to determine the continuous component will be estimated. Therefore, the calculated continuous component Y_{0c}, where is also included the c.c. alias effect, is given by the relationship:

$$Y_{0c}(p) = \frac{2Y_M}{\pi}\left[1 - 2\sum_{\ell=1}^{\infty}\frac{1}{(2p\ell)^2 - 1}\right]. \quad (35)$$

Applying in the right sum of relationship (35) the following formula

$$\sum_{n=1}^{\infty}\frac{1}{n^2 - a^2} = \frac{1}{2a}\left(\frac{1}{a} - \pi \cdot ctga\pi\right), \ a \notin Z, \quad (36)$$

the final form of the d.c. component result as

$$Y_{0c}(p) = \frac{Y_M}{p} \cdot ctg(\pi / 2p), \quad (37)$$

whose values fully verify the results obtained with CFT.

In this way, definitively confirmed the existence and the expression of the phenomenon alias of DC for the wave case represented by the voltage rectified, single phase, double alternation.

III. Differential Harmonics Indicators Defining

The experience of harmonics identification and assessment, aligned with the input-output or consumed-useful principles, has emphasized the need to adjust and clarify the ways of defining and calculating the harmonics indicators according to the desired wave type to be obtained, i.e. if it is an a.c. or a d.c. one.

A. Harmonics indicators for a d.c. wave

For a d.c. wave, the direct component is the useful part and all harmonics of the spectrum, including the fundamental, are distorted. Consequently, in this case, the proposals for the harmonics indicators are as follows [4]:

1) The level γ_k of the k order component of the wave, defined as the ratio, expressed as a percentage, between the RMS value Y_{ek} of the *k* order component and the RMS value Y_e of the wave

$$\gamma_k = \frac{Y_{ek}}{Y_e} \cdot 100, \ \%; \quad (38)$$

- **the level of the direct component γ_0** which has the significance of an efficiency of the generation process of the d.c. wave:

$$\gamma_0 = \frac{Y_0}{Y_e} \cdot 100, \%; \quad (39)$$

- the level γ_1 of the first harmonic, having therefore the character of a „fundamental", accordingly the harmonics analysis:

$$\gamma_1 = \frac{Y_{e1}}{Y_e} \cdot 100, \ \%. \quad (40)$$

2) The deforming residue of the d.c. wave must take into account all harmonics, including the „fundamental" because all a.c. components distort the desired d.c. wave, so the correct definition is made by the expression

$$Y_{d\,cc} = \sqrt{\sum_{k=1}^{k_M} Y_{ek}^2}, \quad (41)$$

where k_M is the maximum possible order of harmonics. As stated in [4], for analytically known waves, $k_M \to \infty$.

3) The distortion factor of the d.c. wave that is distorted by the presence of some harmonics, can be defined by reporting the ratio between the d.c. wave deforming residue Y_{dcc} (rel. 41) either to the wave RMS value Y_e

$$\delta_{ce} = \frac{Y_{dcc}}{Y_e} \cdot 100, \ \%, \quad (42)$$

or to the direct component value Y_0, having the significance of the signal useful component:

$$\delta_{c0} = \frac{Y_{dcc}}{Y_0} \cdot 100, \ \%. \quad (43)$$

If the size defined by equation (42) could be called the total distortion factor of the d.c. wave, that defined by relation (43) has rather the significance of a **distortion to direct component** ratio, being further on designated by this term.

Some examples are given further on for the chosen wave in order to give a more complete image over the

family of the harmonics indicators for a d.c. wave:
- direct component level:

$$\gamma_0 = \frac{Y_0}{Y_e} \cdot 100 = \frac{2Y_M / \pi}{Y_M / \sqrt{2}} \cdot 100 = 90{,}03\ \%\ ; \quad (44)$$

- first harmonic level γ_1:

$$\gamma_1 = \frac{Y_{e1}}{Y_e} \cdot 100 = \frac{2\sqrt{2} \cdot Y_M / 3\pi}{Y_M / \sqrt{2}} = 42.44\ \%\ . \quad (45)$$

- deforming residue of the d.c. wave, for $k_M \to \infty$ (rel. 13):

$$Y_{d\,cc} = \sqrt{\sum_{k=1}^{\infty} Y_{ek}^2} = \frac{\sqrt{\pi^2 - 8}}{\pi\sqrt{2}} \cdot Y_M = 0{,}307758 \cdot Y_M\ ; \quad (46)$$

- distortion factor of the d.c. wave:

$$\delta_{ce} = \frac{Y_{dcc}}{Y_e} \cdot 100 = \frac{100\sqrt{\pi^2 - 8}}{\pi} = 43{,}52\ \%\ , \quad (47)$$

- distortion to direct component ratio:

$$\delta_{c0} = \frac{Y_{dcc}}{Y_0} \cdot 100 = 25\sqrt{2\left(\pi^2 - 8\right)} = 48{,}34\ \%\ . \quad (48)$$

B. *Harmonics indicators for an a.c. wave*

For the a.c. wave the signal fundamental is considered the useful and desired part while the direct component as well as all superior harmonics starting from the second order are considered distorted [4].

1) The levels γ_{ke} of the a.c. wave components are defined in the same way as the d.c. wave (rel. 38), as long as reporting the RMS values of the Fourier series components is also made to the wave RMS value Y_e:

$$\gamma_{ke} = \frac{Y_{ek}}{Y_e} \cdot 100\ ,\ \ \%\ . \quad (49)$$

This time, the fundamental level is distinguished as importance among the components levels and is calculated with a relationship similar to (40), expressing the weight of what is useful of all available.

Having in this case an a.c. wave, for which the fundamental has a concrete significance, it is possible to report the RMS values of the other components to the RMS value of fundamental Y_{e1}, instead of the wave RMS value Y_e:

$$\gamma_{k1} = \frac{Y_{ek}}{Y_{e1}} \cdot 100\ ,\ \ \%,\ \ pt.\ k \neq 1\ , \quad (50)$$

variables for which the following denomination is proposed: **components levels versus the fundamental**.

It is noticed that the two ways of defining, according to relations (49) and (50) lead to very different values, the harmonics level being preferably defined by relation (49), by reporting the harmonics RMS values to the wave RMS value. Given that sometimes the fundamental may have reduced values in comparison with the wave RMS value, calculating the harmonics level with relation (50) would be completely meaningless.

2) The deforming residue is firstly considered in the known form, defined [4], but which must also take into account the direct component calculus so that for the **complete deforming residue of the a.c. wave** the following relationship is proposed:

$$Y_{dca} = \sqrt{Y_0^2 + \sum_{k=2}^{k_M} Y_{ek}^2}\ , \quad (51)$$

where k_M is the harmonics maximum order, in accordance with the considerations presented in equation [4].

3) The distortion factor of an a.c. wave is defined by the ratio expressed as percentage of the complete deforming residue of the a.c. wave and the wave RMS value Y_e.

TABLE I.
HARMONICS INDICATORS DEFINITION IN ACCORDANCE WITH THE WAVE TYPE

Harmonics indicator		**Wave type**	
		d.c. wave	a.c. wave
Component level	generally	$\gamma_k = \frac{Y_{ek}}{Y_e} \cdot 100\ ,\ \ \%$	
	for the direct component	$\gamma_0 = \frac{Y_0}{Y_e} \cdot 100\ ,\ \%$	
	for the first harmonic	$\gamma_1 = \frac{Y_{e1}}{Y_e} \cdot 100\ ,\ \ \%$	
Deforming residue	generally	$Y_{def} = \sqrt{\sum_{k=2}^{k_M} Y_{ek}^2}$	
	completely	$Y_{d\,cc} = \sqrt{\sum_{k=1}^{k_M} Y_{ek}^2}$	$Y_{dca} = \sqrt{Y_0^2 + \sum_{k=2}^{k_M} Y_{ek}^2}$
Distortion factor		$\delta_{ce} = \frac{Y_{dcc}}{Y_e} \cdot 100\ ,\ \%$	$\delta_{ae} = \frac{Y_{dca}}{Y_e} \cdot 100\ ,\ \ \%$
Distortion to useful component ratio		Distortion to direct component ratio: $\delta_{c0} = \frac{Y_{dcc}}{Y_0} \cdot 100\ ,$	Distortion to fundamental ratio: $\delta_{a1} = \frac{Y_{dca}}{Y_{e1}} \cdot 10$

$$\delta_{ae} = \frac{Y_{dca}}{Y_e} \cdot 100 \text{ , \% ;} \qquad (52)$$

in the specialized literature, instead of the wave RMS value, the rated value Y_n is used sometimes, especially if the variable Y designates a voltage.

As in the d.c. waves case, for the a.c. waves we can proceed as well to report the complete deforming residue of the a.c. wave to the fundamental RMS value, accordingly the relation

$$\delta_{a1} = \frac{Y_{dca}}{Y_{e1}} \cdot 100 \text{ , \% ;} \qquad (53)$$

which has rather the significance of a **distortion to fundamental ratio**, being denominated further on with this term. When the fundamental RMS value Y_{e1} has a lower weight in the wave RMS value Y_e, this ratio can reach values rather alarming than informative.

Finally, the above mentioned variables are exemplified for different meanings and values of the double alternations, single-phase, rectified voltage, considered now as an a.c. wave:

- a.c. wave deforming residue, for $k_M \to \infty$ (rel. 51):

$$Y_{dca} = \sqrt{Y_0^2 + \sum_{k=2}^{\infty} Y_{ek}^2} = \frac{\sqrt{18\pi^2 - 32}}{6\pi} \cdot Y_M = 0{,}640263\, Y_M \text{ ;} \qquad (54)$$

- a.c. wave distortion factor:

$$\delta_{ae} = \frac{Y_{dca}}{Y_e} \cdot 100 = \frac{100\sqrt{9\pi^2 - 16}}{3\pi} = 90{,}55 \text{ \% ,} \qquad (55)$$

- distortion to fundamental ratio:

$$\delta_{a1} = \frac{Y_{dca}}{Y_{e1}} \cdot 100 = 25\sqrt{9\pi^2 - 16} = 213{,}3 \text{ \% .} \qquad (56)$$

To summarize, in a comparative manner, the harmonics indicators, defined in accordance with the wave type, Table I was conceived to emphasize not less than seven major indicators. The first four indicators, namely the components levels (general, direct and first harmonic) and the deforming residue are common, because they are specific to the Fourier analysis.

IV. Conclusions

In addition to alias phenomenon there is a redefinition of a multiple ($2p$) rank harmonics, which - using DFT - appear as continuous components, algebraically summarizing with the real continuous component.

Reducing errors in Fourier analysis of real signals requires working with a larger number of samples over a period and, if possible, analyzed signal filtering with a low pass filter, which does not alter the harmonic values which require being determined.

Finally, it is necessary to refute the assumption that the number of samples per period increases the accuracy of harmonics identification. In fact, the number of samples over a period primarily determines the identifiable harmonic number and thereby - **the accuracy of periodic function identification.** Secondly, the large number of samples over a period of reduces translatable harmonics and, if their amplitudes decrease with increasing order, the magnitude of alias phenomenon is diminished. In the absence of the alias phenomenon, the number of samples ($2p$) per period does not influence accuracy of determining the Fourier coefficients for identifiable harmonics.

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