Utilization of Phase Margin for Analyzing Stability of Asynchronous Motors Supplied by Frequency Converters

Monica-Adela Enache, Ion Vlad, Sorin Enache, Mircea Dobriceanu
University of Craiova/Faculty of Electrical Engineering, Craiova, Romania, menache@em.ucv.ro

Abstract— **This paper analyzes a few aspects regarding the dynamic regime operation of the asynchronous motors supplied by variable frequency. The starting point is the conclusion that the stability is a quantitative feature of systems associated to their dynamic behaviour and that has imposed a method for the stability analysis, presented in this paper. The method has as a starting point the mathematical model of the motor written in per unit values. This model has been processed with the help of the Laplace transform. There has been obtained a block scheme with negative reaction which has been used for carrying out a Matlab program for stability analysis. With the help of the program there have been obtained some amplitude-pulsation, phase–pulsation, amplitude-phase characteristics and some hodographs corresponding to a low power asynchronous motor. Comparing the characteristics obtained when some motor parameters have been modified, the conclusions detailed in the final part of the paper have resulted. The notion of phase margin has had a special place for finalizing the analysis. This quantity shows the stability reserve of the machine until the stability limit is reached. The simulations have shown that the phase margin increases when the stator resistance value increases. On the other hand, when the rotor resistance value decreases, the phase margin decreases.**

I. INTRODUCTION

The problem of the stability analysis for driving systems with asynchronous motors is a very present one. This problem is the object of some papers presented in outstanding international conferences [1]-[4] etc. or published in wide spread reviews [5]-[8] etc.

Most methods of analysis that are used are very difficult to be implemented numerically. In addition, these have the drawback that they do not allow to study the inertia moment influence on the stability, a very important aspect, especially in case of low power machines.

In order to eliminate these drawbacks, there has been conceived a new method for studying stability of induction machine supplied by variable frequency.

II. THE MATHEMATICAL MODEL IN PER UNIT VALUES IN OPERATIONAL

The starting point for this analysis is represented by the asynchronous machine equations with representative phasors written in per unit values [9]:

$$
\begin{aligned}
\omega_s^* &= s_{ks}(\underline{\Psi}_s^* - k\underline{\Psi}_r^{/*}) + \frac{d\underline{\Psi}_s^*}{dt^*} + j\omega_s^*\underline{\Psi}_s^* \\
0 &= s_{kr}(\underline{\Psi}_r^{/*} - k\underline{\Psi}_s^*) + \frac{d\underline{\Psi}_r^{/*}}{dt^*} + j(\omega_s^* - \omega^*)\underline{\Psi}_r^{/*} \\
m_r^* &= -\frac{k}{x_{rt}^{/*}}\mathrm{Im}\left[\left(\underline{\Psi}_s^*\right)^* \underline{\Psi}_r^{/*}\right] - h\cdot\frac{d\omega^*}{dt^*}
\end{aligned}
\tag{1}
$$

These equations will be linearized below.

For this, it is considered that the pulsation is modified by saltus with a very small value. This variation will implicitly lead to a voltage modification by saltus, with the same value, so that the ratio of the two quantities is kept constant.

In this hypothesis the system (1) will be modified as follows:

$$
\begin{aligned}
&\omega_s^* + \Delta\omega_s^* = s_{ks}\left[\underline{\Psi}_s^* + \Delta\underline{\Psi}_s^* - k\left(\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*}\right)\right] + \\
&+ \frac{d(\underline{\Psi}_s^* + \Delta\underline{\Psi}_s^*)}{dt^*} + j(\omega_s^* + \Delta\omega_s^*)(\underline{\Psi}_s^* + \Delta\underline{\Psi}_s^*) \\
&0 = s_{kr}\left[\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*} - k\left(\underline{\Psi}_s^* + \Delta\underline{\Psi}_s^*\right)\right] + \\
&+ \frac{d(\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*})}{dt^*} + j(\omega_s^* + \Delta\omega_s^* - \omega^* - \Delta\omega_s^*)(\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*}) \\
&h\cdot\frac{d(\omega^* + \Delta\omega^*)}{dt^*} = - \\
&-\frac{k}{x_{rt}^{/*}}\mathrm{Im}\{[(\underline{\Psi}_s^*)^* + \Delta(\underline{\Psi}_s^*)^*]\cdot(\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*})\}
\end{aligned}
\tag{2}
$$

Applying the Laplace transform to the first two equations of the systems (1) and (2), subtracting them member by member and neglecting products of the form $\Delta\cdot\Delta$, it is obtained:

$$
\begin{aligned}
\Delta\omega_s^* &= (s_{ks} + j\omega_s^* + s)\cdot\Delta\underline{\Psi}_s^* - s_{ks}\cdot k\cdot\Delta\underline{\Psi}_r^{/*} + j\cdot\underline{\Psi}_s^*\cdot\Delta\omega_s^* \\
0 &= -s_{kr}\cdot k\cdot\Delta\underline{\Psi}_s^* + (s_{kr} + s)\Delta\underline{\Psi}_r^{/*} + j(\Delta\omega_s^* - \Delta\omega)\underline{\Psi}_r^{/*}
\end{aligned}
\tag{3}
$$

where s is the notation for the operational variable.

It must be said that for simplifying the writing and not creating confusion, both in the previous relations and in the following relations, there have not been indicated the s-dependent quantities ($\Delta\omega_s^*$ (s), $\Delta\omega^*(s)$ etc.) and they are not noted in capitals.

If it is considered that $\Delta\omega_s^*$ is less than 0,1 in the previous relations, the following approximations may be done:

$$j\underline{\Psi}_s^* = 1 \qquad \text{and} \quad j\underline{\Psi}_r^{/*} = k \tag{4}$$

This way, the relations (3) become:

$$\begin{aligned} 0 &= (s_{ks} + j\omega_s^* + s)\Delta\underline{\Psi}_s^* - s_{ks} \cdot k \cdot \Delta\underline{\Psi}_r^{/*} \\ k(\Delta\omega^* - \Delta\omega_s^*) &= -s_{kr} \cdot k \cdot \Delta\underline{\Psi}_s^* + (s_{kr} + s)\Delta\underline{\Psi}_r^{/*} \end{aligned} \tag{5}$$

In the case when $\omega_s^* = 1$ (the motor operates at rated frequency before occurring the saltus), s_{ks} might be neglected relatively to ω_s^* $(s_{ks} = 0{,}1...0{,}15)$.

But if $\omega_s^* = 0{,}1....0{,}2$ the error given when s_{ks} is neglected is important, the stator resistance influencing the dynamic regime.

Further on it will be considered that $R_s \cong 0$, case when in the coordinate system rotating with speed ω_s it may be written:

$$\underline{u}_s^* = j\underline{\Psi}_s^* = 1 \qquad (\Psi_{ds} = 0; \quad \Psi_{qs} = 0) \tag{6}$$

From here there results that when the rotor speed is modified, the stator flux is not modified $(\Delta\underline{\Psi}_s^* = 0)$, even if the flux and currents of the rotor are modified.

In other words, the following computations will not use the first relation of the system (5) anymore, but only the second relation from (5) and the third relation from (2).

This way, the second relation from the system (5) becomes successively:

$$k(\Delta\omega^* - \Delta\omega_s^*) = -s_{kr} \cdot k \cdot \Delta\underline{\Psi}_s^* + (s_{kr} + s)\Delta\underline{\Psi}_r^{/*} \tag{7}$$

Considering the conditions mentioned before it results:

$$-k\Delta\omega_s^* = (s_{kr} + s)\Delta\underline{\Psi}_r^{/*} - k\Delta\omega^* \tag{8}$$

Applying the Laplace transform to the third equation from (2) it is obtained:

$$h \cdot s \cdot (\omega^* + \Delta\omega^*) = -\frac{k}{x_{rt}^{/*}} \operatorname{Im}(j\underline{\Psi}_r^{/*} + \Delta\underline{\Psi}_r^{/*}) \tag{9}$$

Or, keeping only the terms characterizing the variation:

$$h \cdot s \cdot \Delta\omega^* = -\frac{k}{x_{rt}^{/*}} \cdot \operatorname{Re}(\Delta\Psi_r^{/*}) \tag{10}$$

or equivalently:

$$h \cdot s \cdot \Delta\omega^* + \frac{k}{x_{rt}^{/*}} \cdot \operatorname{Re}(\Delta\Psi_r^{/*}) = 0 \tag{11}$$

Adding here the relation (10), too (where $\Delta\underline{\Psi}_r^{/*} = \Delta\Psi_{dr}^{/*}$) a system having two equations and two unknown quantities ($\Delta\Psi_{dr}^{/*}$ and $\Delta\omega^*$) is obtained:

$$\begin{aligned} -k \cdot \Delta\omega^* + (s_{kr} + s)\Delta\Psi_{dr}^{/*} &= -k \cdot \Delta\omega_s^* \\ h \cdot s \cdot \Delta\omega^* + \frac{k}{x_{rt}^{/*}} \cdot \Delta\Psi_{dr}^{/*} &= 0 \end{aligned} \tag{12}$$

Applying Cramer it is obtained:

$$\Delta\omega^* = \frac{k^2}{x_{rt}^{/*}} \cdot \frac{1}{h} \cdot \frac{1}{s^2 + s_{kr}s + \frac{k^2}{x_{kr}^{/*}} \cdot \frac{1}{h}} \cdot \Delta\omega_s^* \tag{13}$$

respectively:

$$\Delta\Psi_{dr}^{/*} = \frac{1}{s^2 + s_{kr}s + \frac{k^2}{x_{rt}^{/*}} \cdot \frac{1}{h}} \cdot \Delta\omega_s^* \tag{14}$$

If we consider that $\frac{k^2}{x_{rt}^{/*}} = 2M_k^*$, the relations (13) and (14) will get the following form:

$$\begin{aligned} \Delta\omega^* &= \frac{2M_k^*}{h} \cdot \frac{1}{s^2 + s_{kr}s + \frac{2M_k^*}{h}} \cdot \Delta\omega_s^* \\ \Delta\Psi_{dr}^{/*} &= -\frac{ks}{s^2 + s_{kr}s + \frac{2M_k^*}{h}} \cdot \Delta\omega_s^* \end{aligned} \tag{15}$$

Observation

The previous relations are valid only in the case when the stator resistance is neglected. This simplifying hypothesis leads to satisfactory results only in the range $\omega_s^* \in (0{,}5 \div 1)$.

So, it is imposed to analyze the situation when $R_s \neq 0$, but also considering further on that the studied phenomenon is also linearized.

In this situation the third equation of the system (15) might be written as:

$$h\frac{d(\Delta\omega^*)}{dt} = -\frac{k}{x_{st}^*}\mathrm{Im}\left[\left(\underline{\Psi}_s^*\right)\cdot\Delta\underline{\Psi}_r^{/*} + \underline{\Psi}_r^{/*}\cdot\Delta\left(\underline{\Psi}_s^*\right)^*\right] \quad (16)$$

When $\omega_s^* \geq 0{,}1$ it results that we may consider (approximately):

$$\left(\underline{\Psi}_s^*\right)^* = j \qquad \text{şi} \qquad \underline{\Psi}_r^{/*} = -jk \quad (17)$$

In these conditions, applying the Laplace transform to the relation (16), it will be obtained:

$$hs\cdot\Delta\omega^* = -\frac{k}{x_{st}^*}\mathrm{Re}(\Delta\underline{\Psi}_r^{/*} - k\Delta\underline{\Psi}_s^*) \quad (18)$$

or, equivalently:

$$hs\cdot\Delta\omega^* = -\frac{k}{x_{st}^*}(\Delta\underline{\Psi}_{dr}^{/*} - k\Delta\Psi_{ds}^*) \quad (19)$$

$$hs\cdot\Delta\omega^* = -k\Delta i_{dr}^{/*} \quad (20)$$

It is considered that, before modifying the frequency, the motor was operating without load. In this situation, owing to the low frequency of the rotor current, its reactive component may be neglected.

Thus, it may be written:

$$\Delta\underline{i}_r^{/*} = \Delta i_{dr}^{/*} + j\Delta i_{qr}^{/*} \cong \Delta i_{dr}^{/*} = \frac{\Delta\underline{\Psi}_r^{/*} - k\Delta\underline{\Psi}_s^*}{dx_s^*} \quad (21)$$

It is considered that $\Delta i_{qr}^{/*} = 0$ and, using the notation:

$$\varepsilon = (1-k^2)s_{ks} = \frac{r_s^*}{x_s^*} = \frac{r_s^*}{x_s^*} = \frac{r_s^*}{x_r^{/*}} \quad (22)$$

it is obtained:

$$\Delta i_{dr}^{/*} = \frac{s + j\omega_s^* + \varepsilon}{s^2 + (s_{ks} + s_{kr} + j\omega_s^*)s + s_{kr}(\varepsilon + j\omega_s^*)}\cdot k(\Delta\omega^* - \Delta\omega_s^*) \quad (23)$$

If $\Delta i_{dr}^{/*}$ given by (23) is replaced in the relation (20) we will have:

$$hs\cdot\Delta\omega^* = 2M_k(\Delta\omega_s^* - \Delta\omega)\cdot \cdot\frac{s + j\omega_s^* + \varepsilon}{s^2 + (s_{ks} + s_{kr} + j\omega_s^*)s + s_{kr}(\varepsilon + j\omega_s^*)} \quad (24)$$

This relation, if the fact that $s_{ks}\cdot\varepsilon << \frac{2M_k}{h}$ is taken into account, becomes:

$$\Delta\omega^* = \frac{s + j\omega_s^* + \varepsilon}{s^3 + (s_{ks} + s_{kr} + j\omega_s^*)s^2 + \left(\frac{2M_k^*}{h} + j\omega_s^* s_{ks}\right)s + (\varepsilon + j\omega_s^*)\cdot\frac{2M_k^*}{h}}\cdot \cdot\frac{2M_k^*}{h}\Delta\omega_s^* \quad (25)$$

III. PRINCIPLE OF THE COMPUTATION METHOD

Further on, the starting point for studying the stability of the asynchronous motors operation, will be the following relations obtained before:

$$hs\cdot\Delta\omega^* = -k\Delta i_{dr}^{/*}$$

$$\Delta i_{dr}^{/*} = \frac{s + j\omega_s^* + \varepsilon}{s^2 + (s_{ks} + s_{kr} + j\omega_s^*)s + s_{kr}(\varepsilon + j\omega_s^*)}\cdot k(\Delta\omega^* - \Delta\omega_s^*)$$

The first relation becomes successively:

$$\Delta\omega^* = -\frac{k}{hs}\cdot\Delta i_{dr}^{/*} \Leftrightarrow \Delta\omega^* = G_1(s)\cdot\Delta i_{dr}^{/*} \quad (26)$$

with

$$G_1(s) = -\frac{k}{hs} \quad (27)$$

Similarly the second relation is processed:

$$\Delta i_{dr}^{/*} = G_2(s)\cdot(\Delta\omega_s^* - \Delta\omega^*) \quad (28)$$

where

$$G_2(s) = \frac{s + j\omega_s^* + \varepsilon}{s^2 + (s_{ks} + s_{kr} + j\omega_s^*)s + s_{kr}(\varepsilon + j\omega_s^*)}\cdot k \quad (29)$$

Using (26) and (28) the following configuration may be depicted.

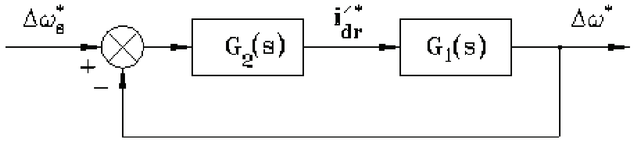


Fig. 1. Block scheme of the machine in the situation we mentioned.

This scheme with negative reaction may be used for analyzing the stability for different concrete cases.

The validity of this method has been demonstrated in [10].

IV. STABILITY CHARACTERSITICS

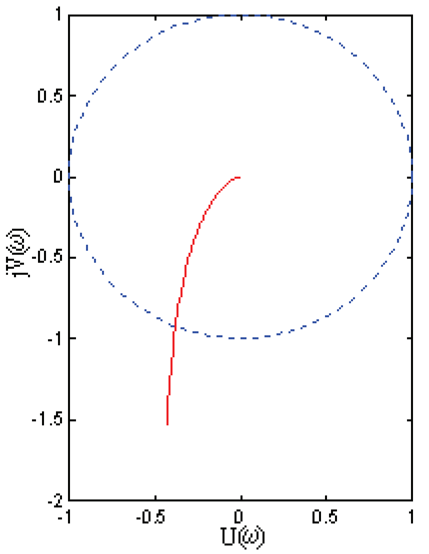
According to the principle detailed before a Matlab program [11] has been conceived for plotting the following characteristics [12]:

- hodograph;
- amplitude-pulsation characteristic;
- phase-pulsation characteristic;
- amplitude-phase characteristic.

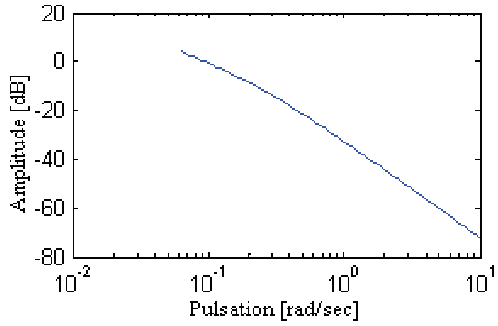
For exemplification these characteristics are presented for a motor having the following data:

R_s=5,5 Ω; R'_r=4,5 Ω; L_s=0,63 H; L'_r=0,62 H; L_{sh}=0,59 H; J=0,006 kg m^2; p=2; f_{1N}=50 Hz; I_{1N}=3,8 A; U_{1N}=220 V (Fig. 2).

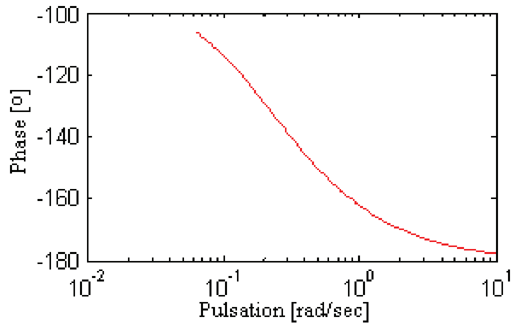
In this case a phase margin of 67,7 degrees has been obtained.



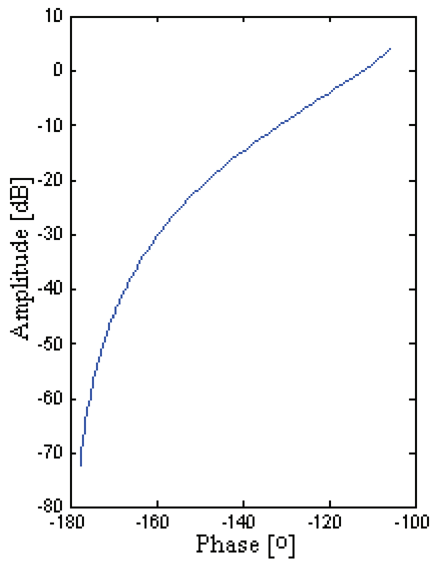
a) Hodograph.



b) Amplitude-pulsation characteristic.

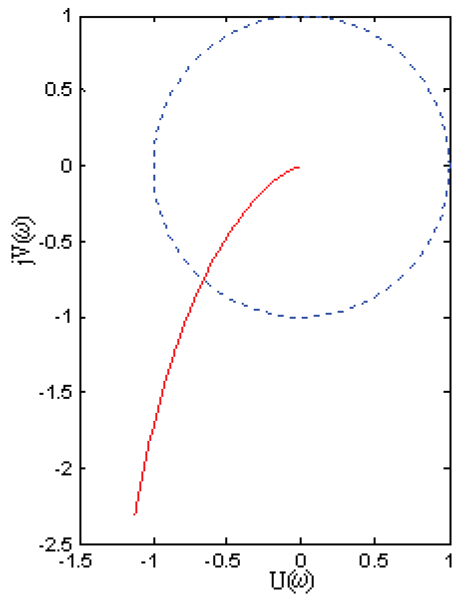


c) Phase-pusation characteristic.

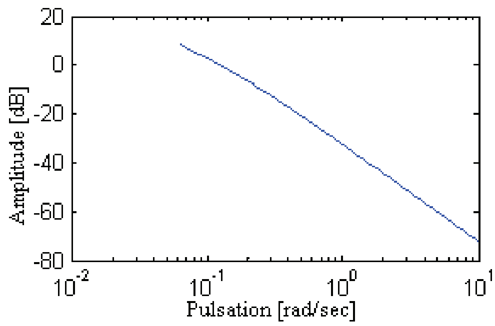


d) Amplitude-phase characteristic.

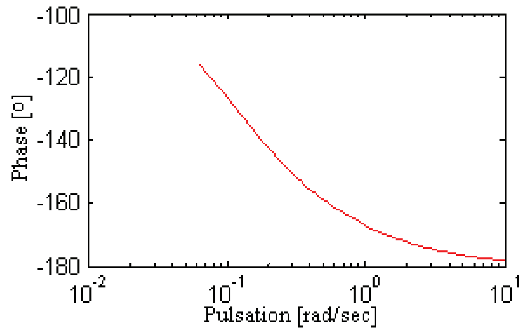
Fig. 2. Stability characteristics for the motor having the real parameters (R_s=5,5 Ω; R'_r=4,5 Ω).



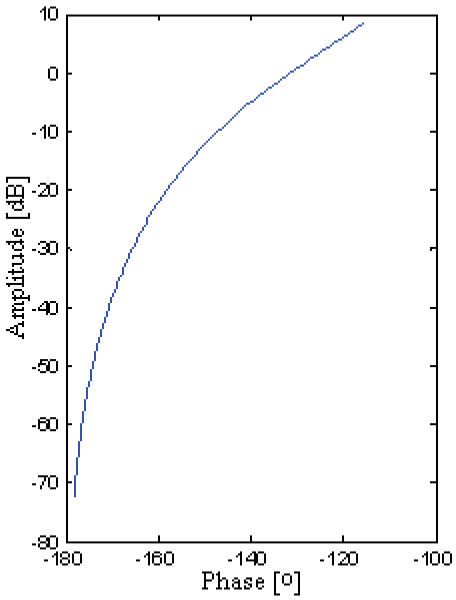
a) Hodograph.



b) Amplitude-pulsation characteristic.

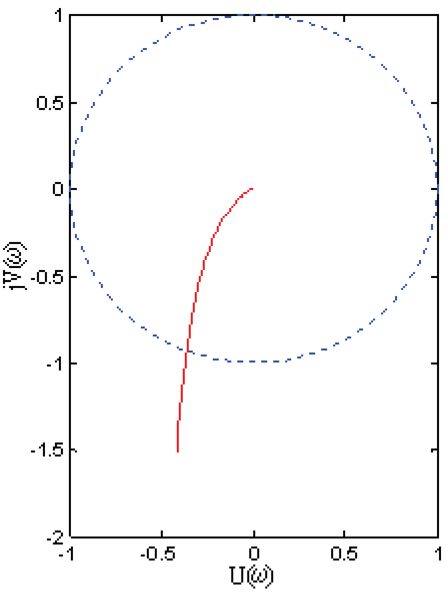


c) Phase-pulsation characteristic.

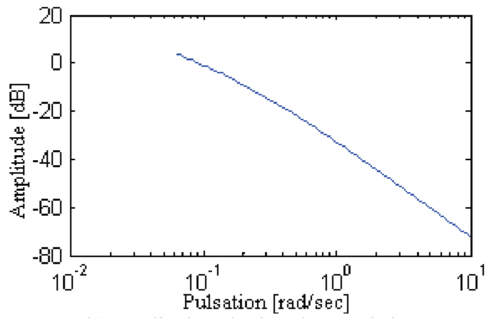


d) Amplitude-phase characteristic.

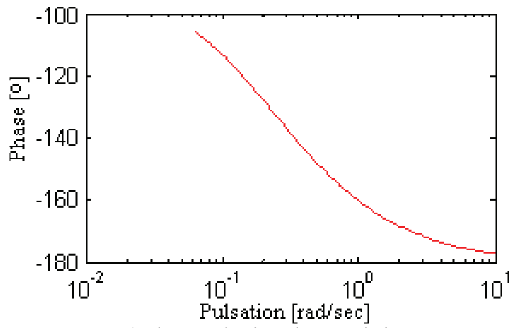
Fig. 3. Stability characteristics for R'_r=2,5 Ω.



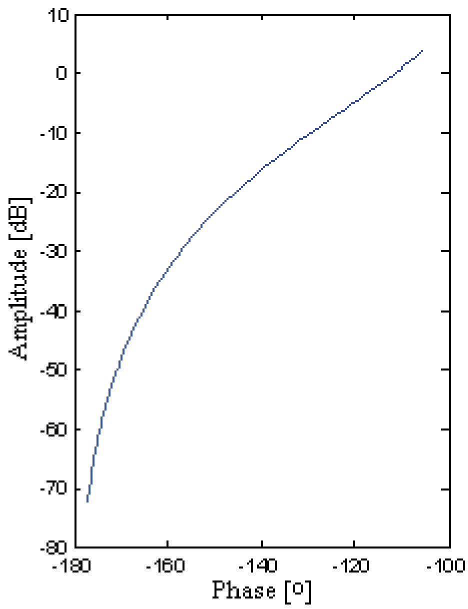
a) Hodograph



b) Amplitude-pulsation characteristic.



c) Phase-pulsation characteristic.



d) Amplitude-phase characteristic.

Fig. 4. Stability characteristics for R_s=7,5 Ω.

Further on the value of the rotor winding resistance was modified (it has been decreased to the value R'_r=2,5 Ω).

The graphic results obtained by running the Matlab program are presented in figure 3.

For this case a phase margin of 48,7 degrees has been obtained.

Then the stator resistance has been increased to the value R_s=7,5 Ω (Fig. 4). The corresponding phase margin has been of 68,5 degrees.

V. Conclusions

As it can be noticed, the method proposed here is easy to be implemented and it provides fast quantitative and qualitative information about the stability of asynchronous motors supplied by variable frequency.

This information concerns the influences of the motor parameters on the stability, in the case of this paper, the effects of the modification of the two windings resistances.

Thus, it is noticed that:

- the comparison between the figures 2 and 3 shows that the stability is strongly influenced by the rotor winding resistance (a high decrease of the phase margin when the rotor resistance decreases);

- the comparison between the figures 2 and 4 shows that the stator resistance has a minor influence on stability (the phase margin has a little increase when the stator resistance increases).

Acknowledgment

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