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Data-Driven Control of the Second Order Inertial Systems with Astatism

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Abstract - **In order to realize an efficient synthesis of the control algorithm, it is necessary to be known the mathematical model of the control object. This paper deals with the problem of data-driven control of second order astatic inertial systems with, or without time delay, that supposes an experimental identification of the mathematical model in the closed-loop and algorithm for synthesis the PD and PID controllers. The control algorithm was synthesized according to the maximum stability degree method with iterations. The closed-loop identification method and control algorithms of the PD and PID controllers were verified by computer simulation in MATLAB and there are obtained good results in model estimation and in the tuning of the PD and PID controllers.**

Cuvinte cheie: *identificarea în buclă închisă, control bazat pe date, PD şi PID regulator, modelarea matematică, modele de obiecte cu inerţie şi astatism.*

Keywords: *closed-loop identification, data driven control, PD and PID controllers, mathematical modelling, second order inertial systems with astatism.*

I. INTRODUCTION

In the realm of control systems engineering, traditional methods of tuning the proportional-integral-derivative (PID) controllers often rely on mathematical models. The mathematical models present the center of the design and synthesis the controllers, and while these models work well, when the underlying system dynamics are well understood and predictable, but they can give not so good results for the case of complex, uncertain and nonlinear systems [1]. This limitation has led to the emergence of data-driven control, a novel and powerful approach that leverages real-world data to develop control algorithms and optimize system performance in real time of the system operation [2-4].

Data-driven control refers to a control strategy that utilizes data-driven modelling techniques to design control algorithms and this strategy has gained popularity in recent years due to its ability to handle complex and nonlinear systems, where the traditional control methods may give not so good results. It is particularly useful when the underlying physics of the system are not well understood or when the system's behavior changes over time [16].

The applications of data-driven control span across various industries and fields. From the manufacturing processes and robotics to autonomous vehicles, and this strategy has shown promise in enhancing system performance, reducing energy consumption, improving safety, and handling complex environments.

Key features of data-driven control include model identification based on data analysis, adaptive control mechanisms that adjust in real-time the tuning parameters, and the ability to optimize control strategies using historical and real-time data. In this control strategy various identification methods are used to approximate the dynamics of the process with different transfer functions, where the model identification supposes the approximation of the process with mathematical model from observed input/output data [5]. In the control systems the control object model estimation can be realized by the two different approaches: in the open-loop and in the closed-loop. In an open-loop system identification, the control input is applied to the system, but the system's output is not used to influence the control action. Essentially, the system operates without any feedback control. Open-loop identification methods involves analyzing the system's response to different inputs and disturbances and presents some simple assumptions where the process dynamics is approximated by the model of object with inertia first or second order with, or without time delay.

Opposite to the open-loop identification, there are developed over 30 years the closed-loop identification methods, where the control input is adjusted based on the system's output, creating a feedback loop. This allows to the system to regulate itself and maintain desired performance. Closed-loop identification takes into account the interaction between the control action and the system's response. In both open-loop and closed-loop model identification, the goal is to find a mathematical model that accurately predicts how the system responds to inputs and disturbances. These models can subsequently be used for controller design, system analysis, or optimization [6-7].

The purpose of this paper, is to focus attention on the data-driven control of the second order inertial systems with astatism with, or without time delay. This supposes the model identification in the closed-loop and algorithm for synthesis the PD and PID controllers. The controllers were proposed to be synthesized according to the maximal stability degree method with iterations (MSDI), that offers to the system high performance and good robustness.

II. DATA-DRIVEN CONTROL OF THE SECOND ORDER INERTIAL SYSTEM WITH ASTATISM

A. Closed-Loop System Identification of the Second Order Inertial Systems with Astatism

The data-driven control strategy involves deriving control actions directly from observed data and system behavior, rather than relying solely on a pre-existing, fully known mathematical model.

Many industrial processes operate under feedback control. Due to unstable behavior of the control object, or necessary of the safety of operation, experimental data can be acquired only under closed-loop. Closed-loop identification methods have been proposed by the Karl Johan Åström et al. (1970), Gustavsson et al. (1977), Söderström and Stoica (1989), Lennart Ljung et al. (1999), Tore Hägglund et al. (2000), Graham C. Goodwin (2011), Sudhahar et al. (2020) [5-10]. The closed-loop identification methods are oriented to solve the consistency problem of identification, considering the situation that the control object and the disturbance signal are taken into account in the identified model [11-15].

In the author paper [17], it was proposed an algorithm for identification of the astatic system in the closed-loop, where the transfer function that approximates the control object is:

$$H(s) = \frac{k}{s(T_1 s+1)(T_2 s+1)} = \frac{k}{a_0 s^3 + a_1 s^2 + a_2 s}, \quad (1)$$

where T_1, T_2 are time constants; k is transfer coefficient of the system; $a_0 = T_1 T_2$, $a_1 = T_1 + T_2$, $a_2 = 1$.

The identification algorithm in the closed-loop consists from the following steps:

1. Analysis of the process, that involves identifying the key characteristics that define the nature of the system.

2. Implementation of the feedback control system with P controller.

3. Variation of the proportional tuning parameter $k_p > 0$, until the system achieves the limit of stability and further determination the value of critical transfer coefficient k_{cr} and period of oscillations – T_{cr}.

4. Calculation of the natural frequency value according to the relation:

$$\omega_n = \frac{2\pi}{T_{cr}}. \quad (2)$$

5. Calculation of the system transfer coefficient according to the relation:

$$k = \lim_{t\to\infty} \frac{y_{st} - y_{initial}}{u - u_{initial}}, \quad (3)$$

where y_{st} is the steady-state output value, $y_{initial}$ is the initial value of the output response, u – input signal, u_{inital} is the initial value of the input signal.

6. Calculation the parameters' values of the control object (1):

$$\begin{cases} a_0 = \dfrac{1}{\omega_n^2}; \\ a_1 = \dfrac{k_{cr} k}{\omega_n^2}; \\ a_2 = 1. \end{cases} \quad (4)$$

B. Closed-Loop System Identification of the Second Order Inertial Systems with Astatism and Time Delay

For the case of the astatic system with time delay, in the author paper [17], it was proposed an algorithm for system identification in the closed-loop, where the transfer function that approximates the control object is following:

$$H(s) = \frac{k e^{-\tau s}}{s(T_1 s+1)(T_2 s+1)} = \frac{k e^{-\tau s}}{a_0 s^3 + a_1 s^2 + a_2 s}, \quad (5)$$

where T_1, T_2 are time constants; k is transfer coefficient of the system; τ - time delay; $a_0 = T_1 T_2$, $a_1 = T_1 + T_2$, $a_2 = 1$.

The identification algorithm of the astatic system with time delay in the closed-loop is following:

1. Analysis of the process, that involves identifying the key characteristics that define the nature of the system.

2. Implementation of the feedback control system with P controller.

3. Variation of the proportional tuning parameter $k_p > 0$, until the system achieves the limit of stability and further determination the value of critical transfer coefficient k_{cr} and period of oscillations – T_{cr}.

4. From the undamped transient response of the closed loop system determination the value of time delay - τ.

5. Calculation of the natural frequency value according to the relation (2).

6. Calculation of the system transfer coefficient according to the relation (3).

7. Calculation the parameters' values of the control object (5):

$$\begin{cases} a_0 = \dfrac{a_2\omega_n - k_{cr} k \sin \tau\omega_n}{\omega_n^3}; \\ a_1 = \dfrac{k_{cr} k \cos \tau\omega_n}{\omega_n^2}; \\ a_2 = 1. \end{cases} \quad (6)$$

C. Synthesis the PD Controller to the Model of Object with Inertia Second Order and Astatism

It is considered that the control system is presented in the Fig. 1 and it is formed from the controller with transfer function $H_R(s)$ and control object (1).

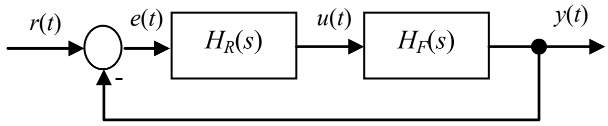

Fig. 1. Structural scheme of the automatic control system.

The PD controller is described by the transfer function [1]:

$$H_R(s) = k_p + k_d s, \quad (7)$$

where the tuning parameters of PD controller - k_p, k_d.

One of the tuning methods of the controllers is maximum stability degree (MSD) criterion [18] and according to this criterion the value of stability degree is calculate by the expression:

$$J = \frac{a_1}{na_0}, \tag{8}$$

where the n is the order of the closed-loop characteristic equation with respectively controller.

Next, utilizing the MSD method with iterations [19], there are calculated expressions for tuning parameters of the PD controller:

$$k_p = \frac{1}{k}(a_0 J^3 - a_1 J^2 + a_2 J) + k_d J, \tag{9}$$

$$k_d = \frac{1}{k}(-3a_0 J^2 + 2a_1 J - a_2), \tag{10}$$

where J is the stability degree of the system.

From the expression (8) is calculated the value of system stability degree

$$J = \frac{a_1}{3a_0},$$

and according to this expression, the analytical expressions for calculation the tuning parameters of the PD controller (9)-(10) can be rewritten in the following way:

$$k_p = \frac{a_1^3}{27ka_0^2}, \tag{11}$$

$$k_d = \frac{a_1^2 - 3a_0 a_2}{3ka_0}. \tag{12}$$

According to the expressions for calculation the values of control object's parameters (4), the expressions (11)-(12) can be rewritten as:

$$k_p = \frac{k_{cr}^3 k^2}{27\omega_n^2}, \tag{13}$$

$$k_d = \frac{k_{cr}^2 k^2 - 3\omega_n^2}{3k\omega_n^2}. \tag{14}$$

D. Synthesis the PID Controller to the Model of Object with Inertia Second Order and Astatism

The control algorithm PID is described by the following transfer function [1]:

$$H_R(s) = k_p + \frac{k_i}{s} + k_d s, \tag{15}$$

where the tuning parameters of PID controller - k_p, k_i, k_d.

The analytical expressions for calculation the tuning parameters of the PID controller were obtained in concordance with MSD method with iterations [19]:

$$k_p = \frac{1}{k}(4a_0 J^3 - 3a_1 J^2 + 2a_2 J) + 2k_d J, \tag{16}$$

$$k_i = \frac{1}{k}(-a_0 J^4 + a_1 J^3 - a_2 J^2) - k_d J^2 + k_p J, \tag{17}$$

$$k_d = \frac{1}{k}\left(-6a_0 J^2 + 3a_1 J - a_2\right), \tag{18}$$

where J is the stability degree of the system.

From the expression (8) it is calculated the value of system stability degree

$$J = \frac{a_1}{4a_0}.$$

And in concordance with the value of stability degree, the analytical expressions (16)-(18) can be rewritten as:

$$k_p = \frac{a_1^3}{16ka_0^2}, \tag{19}$$

$$k_i = \frac{a_1}{256ka_0^3}, \tag{20}$$

$$k_d = \frac{3a_1^2 - 8a_0 a_2}{8ka_0}. \tag{21}$$

According to the expressions for calculation the value of control object's parameters (4), the expressions (19)-(21) can be rewritten as:

$$k_p = \frac{k_{cr}^3 k^2}{16\omega_n^2}, \tag{22}$$

$$k_i = \frac{k_{cr}\omega_n^4}{256}, \tag{23}$$

$$k_d = \frac{3k_{cr}^2 k^2 - 8\omega_n^2}{8k\omega_n^2}. \tag{24}$$

E. Synthesis the PID Controller to the Model of Object with Inertia Second Order, Astatism and Time Delay

The control algorithm PID is described by the transfer function (15) and the control object is described by the transfer function (5). The analytical expressions for calculation the tuning parameters of the PID controller were obtained in concordance with MSD method with iterations [19]:

$$k_p = \frac{e^{-\tau J}}{k}(-\tau a_0 J^4 + (\tau a_1 + 4a_0)J^3 - (a_2\tau + 3a_1)J^2 + 2a_2 J) + 2k_d J, \tag{25}$$

$$k_i = \frac{e^{-\tau J}}{k}(-a_0 J^4 + a_1 J^3 - a_2 J^2) - k_d J^2 + k_p J, \tag{26}$$

$$k_d = \frac{e^{-\tau J}}{2k}(-\tau^2 a_0 J^4 + (\tau^2 a_1 + 8\tau a_0)J^3 - (\tau^2 a_2 + 6a_1\tau + 12a_0)J^2 + (4a_2\tau + 6a_1)J - 2a_2) \tag{27}$$

where J is stability degree of the system.

F. Algorithm for Data Driven Control

According to the procedure of experimental identification in the closed-loop of the astatic system with inertia second order and analytical expressions for calculation the tuning parameters, the algorithm for data-driven control of the astatic systems with inertia is following:

1. P control algorithm establishment.
2. Achievement the undamped transient response by the variation the proportional tuning parameter $k_p > 0$.
3. Data extraction from step response as T_{cr}, k_{cr}, k, τ and calculation the value of natural frequency ω_n.
4. Model object chosen and parameter estimation by the (4) or (6) expressions in dependency of time delay presence.
5. Tuning the PD controller by the expressions (9)-(10), or PID controller by the expression (18)-(20), or (25)-(27).

III. Application and Computer Simulation

A. Closed-Loop System Identification and Control of the Second Order Inertial Systems with Astatism

To verify the proposed algorithm, it is considered that the control object is described by the following transfer function:

$$H(s) = \frac{2}{s(2s+1)(3s+1)} = \frac{2}{6s^3 + 5s^2 + s}. \quad (28)$$

Next, the control system with P controller was simulated and k_p tuning parameter was varied until it was achieved the system response presented in Fig. 2.

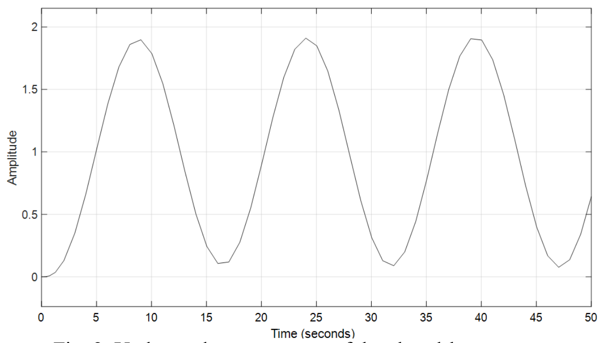

Fig. 2. Undamped step response of the closed-loop system.

There are obtained the following parameters:

$$k_{cr} = 0.42,\ T_{cr} = 15.314 \text{ s.},\ \omega_n = 0.4101.$$

Based on the expressions (4), there are calculated the transfer function of the control object:

$$H(s) = \frac{2}{5.9459s^3 + 4.99s^2 + s}. \quad (29)$$

In the Fig. 3, it is presented the comparation between original step response of the open loop system described by the transfer function (28) – curve 1 and identified transfer function (29) – curve 2.

From Fig. 3, it can be observed that the algorithm for identification in the closed loop offers good estimation of the mathematical model.

Next, to the identified transfer function (29), it is proposed to tune the PD controller according to the (13)-(14) expressions. In the Table I, there are presented the calculated values of the tuning parameters and it was done the comparison with MSDI method, parametrical optimization (PO) from MATLAB and genetic algorithm (GA).

In case of genetic algorithm, the fitness function was designed based integral square error (GA1) and integrated time absolute error (GA2).

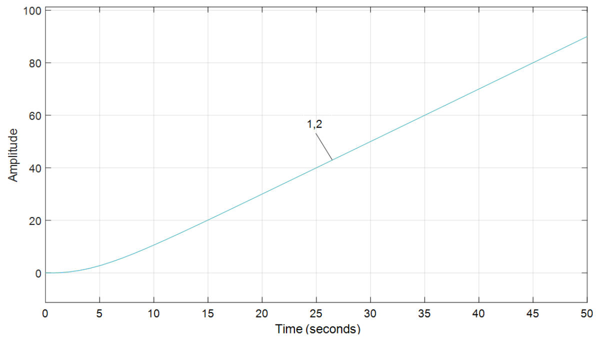

Fig. 3. Comparation of the system step responses in the open loop.

TABLE I.
System Performance and Tuning Parameters of the PD Controller

No	Method	J	k_p	k_d	t_s	t_r	σ
1	Proposed Method of Tuning		0.065	0.199	22.2	22.2	-
2	MSDI	0.29	0.064	0.193	22.7	22.7	
3	MSDI	0.36	0.049	0.147	32.1	32.1	
4	PO		0.206	0.886	14.3	3.1	7.5
5	GA1		0.302	35.01	13.1	0.33	69
6	GA2		3.008	26.84	14.6	0.54	70

In Fig. 4, it is presented the step responses of the automatic control system with PD controller.

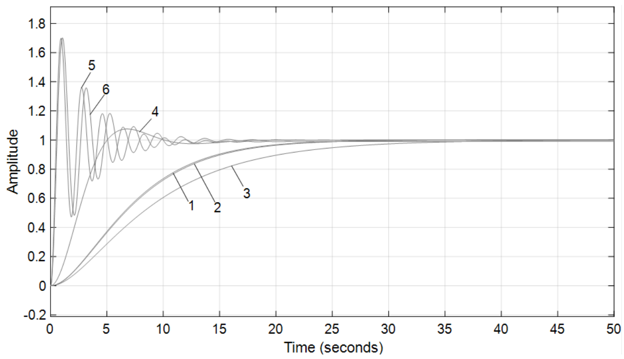

Fig. 4. Step response of the automatic control system with PD controller tuned by: 1 – proposed method of tuning; 2 – MSDI method; 3 – MSDI; 4 – parametrical optimization from MATLAB; 5 – GA1 (fitness function ISE); 6 – GA2 (fitness function ITAE).

In case of using the genetic algorithm, there were obtained oscillated transient responses with high overshoot more than 20%.

Next, it was proposed to be tuned the PID controller to the identified model of object (29) based on the analytical expressions (22)-(24). The obtained results are presented in the Table II and there was done the comparison with MSDI method, parametrical optimization from MATLAB and genetic algorithm, where fitness function was designed based integral square error (GA1) and integrated time absolute error (GA2). Tuning parameters of the PID controller and system performance are presented in the Table II.

TABLE II.
SYSTEM PERFORMANCE AND TUNING PARAMETERS OF THE PID CONTROLLER

No	Tuning Method	J	k_p	k_i	k_d	t_s	t_r	σ
1	Proposed Method of Tuning		0.1	4.640 5e-05	0.28	19.7	11.5	2.8
2	MSDI	0.2	0.10	0.005	0.27	43.1	8.2	26
3	MSDI	0.28	0.07	0.001	0.21	122	13.5	8.4
4	PO		0.2	0.006	0.92	45	4	10
5	GA1		0.25	0.014	18.17	18	0.48	57
6	GA2		5.89	0.008	67.62	11.76	0.32	83

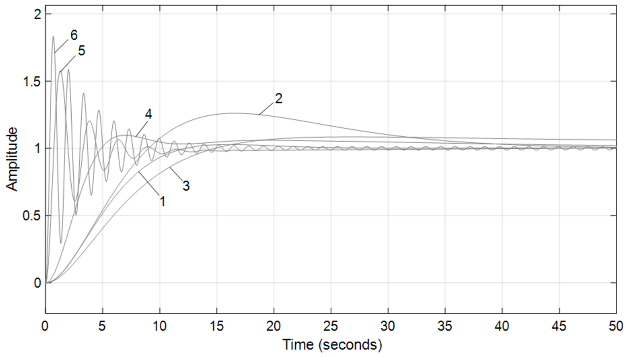

Fig. 5. Step response of the automatic control system with PID controller tuned by: 1 – proposed method of tuning; 2 – MSDI method; 3 – MSDI method; 4 – parametrical optimization from MATLAB; 5 – GA1 (fitness function ISE); 6 – GA2 (fitness function ITAE).

B. Closed-Loop System Identification and Control of the Second Order Inertial Systems with Astatism and Time Delay

It is given the control object described by the following transfer function:

$$H(s)=\frac{3e^{-10s}}{s(5s+1)(2s+1)}=\frac{3e^{-10s}}{10s^3+7s^2+s}. \tag{30}$$

According to the identification algorithm, the control system with P controller was simulated and k_p tuning parameter was varied until it was achieved the system response presented in Fig. 6.

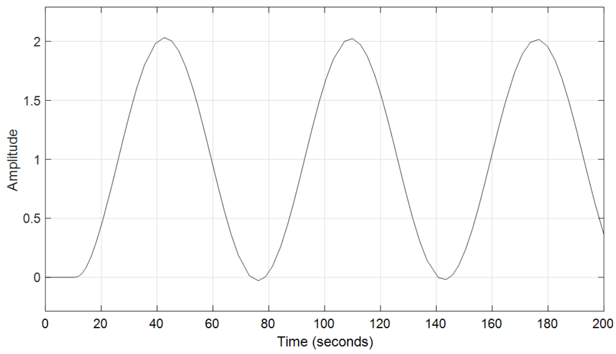

Fig. 6. Undamped step response of the closed-loop system.

Based on the step response presented in the Figure 6, there are obtained:

$$k_{cr}=0.0355,\ T_{cr}=67.418\text{ s., } \tau=10\text{ s., } \omega_n=0.0932.$$

Based on the expressions (6), there are calculated the transfer function of the control object:

$$H(s)=\frac{3e^{-10s}}{9.51s^3+7.31s^2+s}. \tag{31}$$

The comparation between original step response of the open-loop system described by the transfer function (30) (curve 1) and identified transfer function (31) (curve 2) is presented in the Figure 7.

From Fig. 7, it can be concluded that the identification algorithm in the closed-loop offers so good model estimation.

Next, to the identified transfer function (31), it was proposed to be tuned PID controller according to the (25)-(27) expressions.

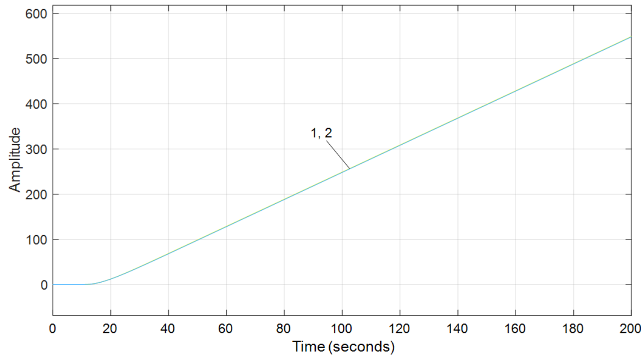

Fig. 7. Comparation of the system step responses in the open loop.

The obtained results are presented in the Table III and there was done the comparison with parametrical optimization (PO) from MATLAB. The genetic algorithm did not give the satisfactory results for the case of tuning the PID controller.

TABLE III.
SYSTEM PERFORMANCE AND TUNING PARAMETERS OF THE PID CONTROLLER

No	Tuning Method	J	k_p	k_i	k_d	t_s	t_r	σ
1	MSDI	0.08	0.016	0.00026	0.09	141	15	39
2	MSDI	0.1	0.015	0.0002	0.08	176	17	32
3	PO		0.016	7.9136e-05	0.11	351	17	12

In the Figure 8, it is presented the computer simulation of the control system with PID controller tuned by the MSDI method – curve 1 and curve 2, parametrical optimization – curve 3.

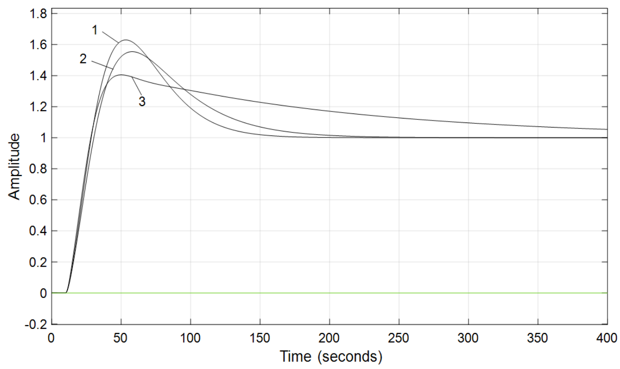

Fig. 8. Step response of the automatic control system with PID controller tuned by: 1 – MSDI method; 2 – MSDI method; 3 – parametrical optimization from MATLAB.

IV. CONCLUSIONS

Traditional control methods and algorithms often depend on accurate mathematical models to design controllers, data-driven approaches leverage the availability of data to achieve control objectives. This paper introduced a new algorithm for data-driven system identification and control of the second order inertial systems with astatism and with, or without time delay. The algorithm's foundation lies in a stepwise approach, commencing with system identification, that is performed within a closed-loop, leveraging the undamped step response of the system and based on the parameters, that are obtained from the undamped step response, there are calculated the mathematical model that approximates the inertial system with astatism.

Next according to data-driven control algorithm, there are proposed simple analytical expressions for calculation the tuning parameters of the PD and PID controllers. The expressions for tuning the PD and PID controllers were obtained based on the MSD method with iterations. The closed-loop identification and synthesis methods was verified by computer simulation and the obtained results were compared with parametrical optimization from MATLAB and genetic algorithm. The proposed procedure of data driven control of second order inertial systems with astatism can be implemented as auto-tuning method of the PD or PID controller and offers to the control system so good performance and high robustness.

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