Polynomial Models used in Optimization by Design of Numerical Experiments

Alin-Iulian Dolan
University of Craiova, Electrical Engineering Faculty, Craiova, Romania, adolan@elth.ucv.ro

Abstract **- The paper presents two optimization methods based on the concept of design of experiments: the method by zooms and the method by slidings of designs. Two application variants are presented for each of them. One of the variants only requires access to the values of the objective function at different points in the feasible domain, grouped in designs of experiments, arranged successively, towards the most convenient values. The other variant uses the same technique, but the advance of the designs towards the optimal value is based on the information provided by second-order polynomial models that approximate the objective function on each design, which facilitates the convergence speed of the algorithm. The methods under discussion lend themselves very well to numerical simulations using the finite element method that can provide the values of the objective function at any point in the feasible domain that corresponds to a unique configuration of the simulated device. The paper presents a comparative study of the application of these methods in the two variants, on a 2-D numerical model of an electromagnetic device. Both in the case of the method by zooms and in the case of the method by slidings of designs, the results highlight the simplicity of the first application variant but also the high speed of convergence of the second one, especially for the method by zooms.**

Cuvinte cheie: *optimizare, proiectarea experimentelor, metoda elementului finit bidimensională.*

Keywords: *optimization, design of experiments, two-dimensional finite element method.*

I. INTRODUCTION

Optimization methods based on numerical simulations are very effective tools for improving the performance of electromagnetic devices. Therefore, the technique of design of experiments (DOE) [1]-[2] becomes indispensable in solving these problems, only changing the nature of the experiments from "real" to "numerical" The fact that it is still a topical technique is proven by many scientific works [3]-[12]. The DOE technique can be applied both for the screening of devices for optimization and for the actual optimization through different algorithms.

For example, in [3] the aim was to determine the influential parameters in order to optimize a double-cage induction motor, in [4] critical parameters were subjected to a detailed parametric analysis to select some of them for optimization of switched reluctance motor for ceiling fan design and in [5] through Ansys and Noesis Optimus software based on DOE and response surface methodology is investigated the effect of bump structures and loading conditions on the electromigration properties of solder bumps in Wafer-level chip-scale packaging.

Many works deal with actual optimization problems based on DOE such as [6] which presents the optimization of the operation of a boiler, aiming to reduce the gas-steam ratio to increase its efficiency, or [7] where the robust design by using DOE method and driving cycles is proposed to optimize the brushless direct current motors. Genetic algorithms and DOE were employed in [8] to optimize the stator slot V-shaped PM geometry and the dimension of the rotor bar. The response surface technology combined with DOE, FEM and sequential linear programming is applied in [9] to obtain an optimized linear actuator with permanent magnet for driving a needle in a knitting machine.

Other works focus on multi-objective optimization such of [10] in which DOE is used for adjustment of the limits of design variables to facilitate optimization with finite element method (FEM) of a permanent magnet machine. In [11 the DOE and the Kriging model are used to implement the multi-objective optimization design of the geometric parameters of linear switched reluctance motor with segmental mover and in [12] a combination between DOE and FEM was performed in order to minimize the permanent deflection and internal energy, taking into account three influential parameters.

Some works present the results of the application of different variants of optimization methods based on DOE [13]-[17]. In [13] an optimized solution was proposed for the geometric form of the toroidal modular coil of a superconducting magnetic energy storage device using the method by zooms in combination with FEM. Two geometric parameters were chosen to maximize the magnetic energy stored in a minimal volume of superconducting material. A similar solution was obtained in [14] by applying the method by slidings of designs, also based on DOE. The method by zooms was also applied in [15] to maximize the force developed by a direct current electromagnetic device. The optimization problem considered six parameters to synthesize the optimal geometric shape of the device. In search of a global maximum, in [16] an exhaustive method based on DOE was applied.

The work [17] describes in detail the method by zooms in two application variants: a simpler one that only requires access to the objective function values at different points in the feasible domain, grouped in successive experiment designs, oriented towards the most convenient values and a more complex second one, that, additionally, calculates second-order polynomial models that facilitate the increase of the convergence speed of the algorithm.

This paper adds to the study the method by slidings of designs, allowing comparisons between methods in terms of workload and convergence speed.

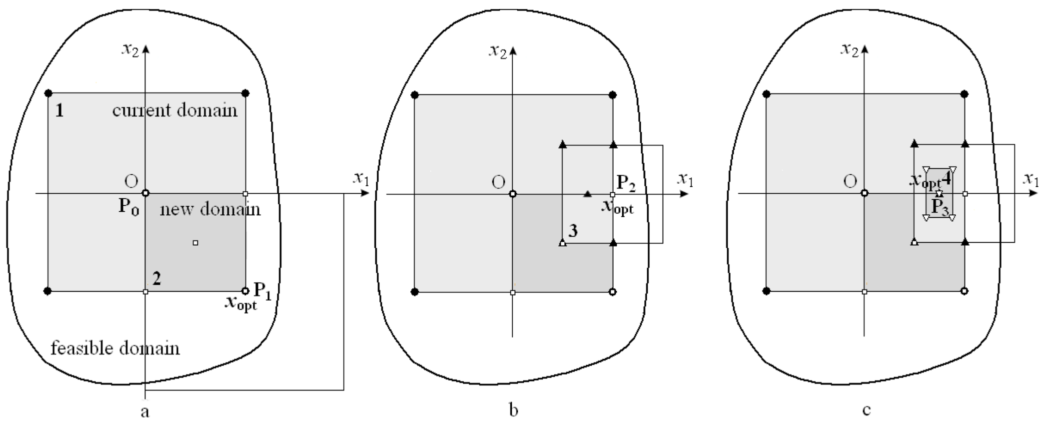

Fig. 1. Graphical illustration of the application of the optimization algorithm by zooms without model calculation [13].

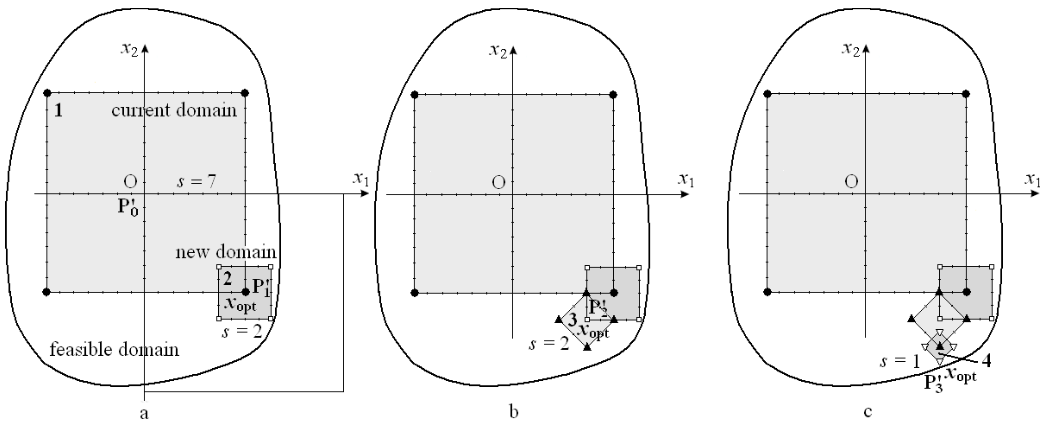

Fig. 2. Graphical illustration of the application of the optimization algorithm by slidings of designs without model calculation [14].

A. Optimization method by zooms

a) Variant without calculation of models

It directly uses the objective function values in the experimental points. Experimental designs centered on points determined by the algorithm (origin points) are used, formed by points arranged diagonally, on one side and on the other of the origin point, at equal distance from it, giving to the corresponding domain a hyper-rectangular shape. For k factors there are 2^k diagonal points relative to the origin. It starts with a design that covers most of the feasible domain. The algorithm of the variant without calculation of models is the following:

1) The first experiment is carried out in a point P_0 (Fig. 1) in the feasible domain (initial point) which is considered the first origin point;

2) The points diagonal to the origin point are established;

3) A number of $N = 2^k$ experiments are performed in the diagonal points (full factorial design);

4) If there is a diagonal point where the response is more convenient than the origin point, this point becomes the new origin point of the next iteration and step 2 is resumed (a "zoom" operation of the same size is made around the found diagonal point);

5) Otherwise, the dimensions of the initial design are reduced with the rates $\tau_k \geq 1$ and step 2 is resumed (usually $\tau_k = 2$, or 2^2, or 2^3 etc.).

The constraints on position can be taken into account when establishing diagonal points. If they are violated by one of the diagonal points, the dimensions of the design are reduced with rates $\tau_k > 1$ and the algorithm is restarted.

During the running of the algorithm it is possible that some points subject to analysis have already been analyzed in previous iterations (recovered points). Their number (N_{rec}) contributes to reducing the required number of experiments (N_{tot}).

b) Variant with calculation of second-order polynomial models

At each iteration, a second-order polynomial model F_{mod} for objective function F is calculated that allows determining the direction of the best values, the position and dimensions of the next design. The matrix equation of a second order model is written [2]:

$$F_{\text{mod}}(\boldsymbol{X}) = b_0 + \boldsymbol{X}^{\mathrm{T}} \cdot \boldsymbol{b} + \boldsymbol{X}^{\mathrm{T}} \cdot \boldsymbol{B} \cdot \boldsymbol{X} \tag{1}$$

$$\boldsymbol{X} = \begin{pmatrix} x_1 \\ \vdots \\ x_k \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} b_1 \\ \vdots \\ b_k \end{pmatrix}, \boldsymbol{B} = \begin{pmatrix} b_{11} & b_{12}/2 & \cdots & b_{1k}/2 \\ b_{12}/2 & b_{22} & \cdots & b_{2k}/2 \\ \vdots & \vdots & \ddots & \vdots \\ b_{1k}/2 & b_{2k}/2 & \cdots & b_{kk} \end{pmatrix} \tag{2}$$

An important simplification of the equation can be obtained if the model is viewed from a particular coordinate system ($S, x_1', x_2', \ldots, x_k'$), rotated and translated relative to the original system ($O, x_1, x_2, \ldots, x_k$) (Fig. 3a), so that only the terms containing x_j^2, $j = 1, \ldots, k$ are kept.

Thus, the canonical analysis is used, which allows the determination of the origin and axes of the new system, as

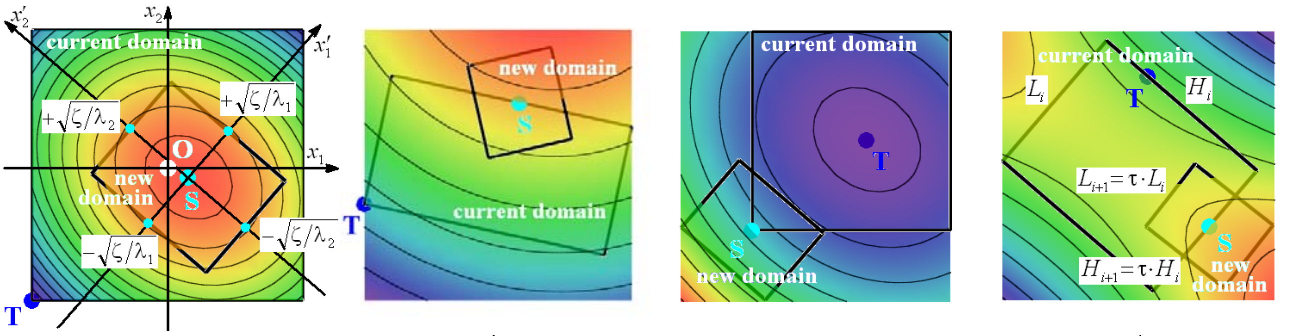

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Fig. 3. Graphical illustration of the application of the optimization algorithm by zooms with calculation of second order polynomial model in case the optimum sought is a maximum [17].

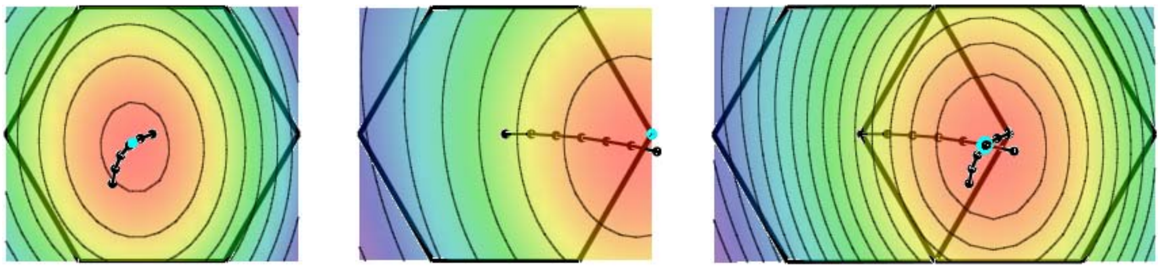

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Fig. 4. The optimal paths and the optimal point.

well as the new form of the model, using the diagonal matrix $\boldsymbol{\Lambda}$ of the eigenvalues of $\boldsymbol{B}$ [2]:

$$\boldsymbol{B}\cdot\boldsymbol{M}=\boldsymbol{M}\cdot\Lambda \tag{3}$$

where $\boldsymbol{M}$ = matrix of eigenvectors of $\boldsymbol{B}$.

The origin S($x_{1\mathrm{S}}$, …, $x_{k\mathrm{S}}$) of the new system represents the optimal point (minimum or maximum) of the second order model restricted by the dimensions of the current domain.

This point can be found inside the current domain or on its border. Fig. 3 illustrates four possible cases for a two-dimensional domain in which the sought optimum is a maximum:

a) the model function has the same curvature as the objective function and the maximum of the model function is located inside the current domain (Fig. 3a).

b) the model function has the same curvature as the objective function but the maximum of the model function is located outside the current domain (Fig. 3b);

c) the model function has the opposite curvature of the objective function, admitting a minimum inside the current domain (Fig. 3c);

d) the model function has inflection point inside the current domain (Fig. 3d).

In case *a*, point S is identified with the maximum of the model function, being located inside the current domain. In cases *b*, *c*, *d*, point S coincides with the maximum of the model function restricted by its border, being located either on one of the sides (*b*, *d*) or in a corner of the current domain (c).

The axes of the new system are expressed as:

$$\boldsymbol{X}'=\boldsymbol{M}^{\mathrm{T}}\cdot\left(\boldsymbol{X}-\boldsymbol{X}_{\mathrm{S}}\right) \tag{4}$$

The equation of the model in the new system becomes:

$$F'_{\mathrm{mod}}(\boldsymbol{X}')=F_{\mathrm{mod}}(\mathrm{S})+\boldsymbol{X}'^{\mathrm{T}}\cdot\Lambda\cdot\boldsymbol{X}' \tag{5}$$

$$\boldsymbol{X}'=\begin{pmatrix}x'_1\\ \vdots\\ x'_k\end{pmatrix},\quad \Lambda=\begin{pmatrix}\lambda_1 & \cdots & 0\\ \vdots & \ddots & \vdots\\ 0 & \cdots & \lambda_k\end{pmatrix} \tag{6}$$

The next iteration of the algorithm will use a design of experiments with the sides oriented according to the axes of the new system.

The zoom effect is obtained by successively reducing the dimensions of the domains from one iteration to another with different percentages, depending on the signs of the eigenvalues λ_j. The reduction rate depends on the topology and on the variations of the objective function. Hyper-rectangular designs with three levels per factor ($p = 3^k$) are usually used.

It is denoted by T(x_{T},y_{T}) = the minimum point of the model, S(x_{S},y_{S}) = the optimal point of the model, pr = the percentage reduction of the surface of the current domain to obtain the new domain ($pr < 1$), τ = the reduction rate of each dimension of the current domain, ζ = the reduction percentage of the difference between the maximum and the minimum of the model and ρ = the coefficient of reduction of the difference between the maximum and the minimum of the model.

For the two-dimensional case, if all the eigenvalues λ_j have the same sign (Fig. 3a,b,c), starting from iteration i, the percentage pr is calculated first, then ζ, then the dimensions of the domain at iteration i+1, denoted by L_{i+1}, H_{i+1}. Only in the first iteration, a value $pr < 1$ is first chosen and then the coefficients ρ and ζ are calculated:

$$pr=\frac{\rho\cdot 4\cdot\left|F_{\mathrm{mod}}(x_{\mathrm{T}},y_{\mathrm{T}})-F_{\mathrm{mod}}(x_{\mathrm{S}},y_{\mathrm{S}})\right|}{\sqrt{\lambda_1\cdot\lambda_2}\cdot L_i\cdot H_i} \tag{7}$$

$$\rho = \frac{\sqrt{\lambda_1 \cdot \lambda_2} \cdot pr \cdot L_i \cdot H_i}{4 \cdot \left| F_{\text{mod}}(x_T, y_T) - F_{\text{mod}}(x_S, y_S) \right|} \quad (8)$$

$$\zeta = \rho \cdot \left(F_{\text{mod}}(x_T, y_T) - F_{\text{mod}}(x_S, y_S) \right) \quad (9)$$

$$L_{i+1} = 2 \cdot \sqrt{\frac{\zeta}{\lambda_1}}, \quad H_{i+1} = 2 \cdot \sqrt{\frac{\zeta}{\lambda_2}} \quad (10)$$

Thus, the new domain will be a hyper-rectangle that will circumscribe an ellipsoid with k axes and dimensions $\sqrt{\zeta/\lambda_j}$, j = 1, …, k and which will have $2 \cdot k$ vertices with coordinates $\left[\pm\sqrt{\zeta/\lambda_1}, \pm\sqrt{\zeta/\lambda_2}, \ldots, \pm\sqrt{\zeta/\lambda_k}\right]$.

These coordinates are valid in the system formed by the main axes Sx_1', Sx_2', …, Sx_k' associated with the second order model. They can be expressed in the original axis system (O,x_1,x_2,…,x_k) using the matrix relation:

$$\boldsymbol{X} = \boldsymbol{X}_S + \boldsymbol{M} \cdot \boldsymbol{X}' \quad (11)$$

If the eigenvalues λ_j have different signs, the coefficient τ will be calculated first using the percentage pr from the previous iteration, then L_{i+1}, H_{i+1}:

$$\tau = \sqrt{pr} \quad (12)$$

$$L_{i+1} = \tau \cdot L_i, \quad H_{i+1} = \tau \cdot H_i, \quad L_{i+1} \cdot H_{i+1} = pr \cdot L_i \cdot H_i \quad (13)$$

The algorithm of the variant with calculation of second order polynomial model is the following:

1) An origin point $P_0 = (x_1, \ldots, x_k)^T$ is defined in the feasible domain;

2) A design of N experiments centered in P_0 is made, with $N > p$ = the number of coefficients of the second-order polynomial (in 2-D it is rectangular domain, $N = 4^2 > p = 6$);

3) The associated second-order polynomial model is calculated;

4) Determine the optimal point S of the model on the current domain (the center of the new coordinate system);

5) Determine the opposite of the optimal point T of the model on the current domain (the minimum point, if the optimum is a maximum or the maximum point, if the optimum is a minimum);

6) The eigenvalues λ_j of matrix $\boldsymbol{B}$ and the axes of the new system are determined;

7) If all the eigenvalues λ_j have the same sign, first calculate the percentage pr, then calculate ζ (in the first iteration a value $pr < 1$ is chosen, then ρ and ζ are calculated). Otherwise, the coefficient τ is first calculated using the percentage pr from the previous iteration;

8) Calculate L_{i+1}, H_{i+1};

9) The coordinates of the vertices of the new domain are determined;

10) A design of N experiments centered in S and limited by the vertices calculated in step 9 is made, with $N > p$ = the number of coefficients of the second-order polynomial (in 2-D it is rectangular domain, $N = 3^2 > p =$ = 6) and the step 3 is repeated.

As advantages, the method provides a modeling of the objective function over the entire feasible domain and models the neighborhood of the optimal point more and more finely. Instead, the optimum found is a local one, the algorithm is computationally expensive and quickly generates designs with at least one point outside the feasible domain, due to the rotation of the domains. From an economic point of view, there are minimal chances of recovering points.

B. Optimization method by slidings of designs

a) Variant without calculation of models

It directly uses the objective function values in the experimental points. The algorithm uses only discrete factors; in the case of continuous factors, they are discretized into a finite set of values (N_{vk} = total number of values of factor k). A k-dimensional discretization network (grid) is defined, preferably regular, determined by some of the N_{vk} values of the k factors, which will be the support of the optimization algorithm, having great influence on the precision of the results. The structure and dimensions of the experimental designs used are established, in correlation with the dimensions of the feasible domain and of the discretization network.

Two types of experimental designs centered in network nodes determined by the algorithm (origin points) are used: diagonal designs formed by nodes of the network diagonally arranged, on one side and the other of the origin point, equidistant from it, and axial designs formed by nodes of the network axially arranged, on one side and on the other of the origin point, equidistant from it. For k factors there are 2^k diagonal points and 2^k axial points relative to the origin point.

The size of a design is defined by the parameter (step) s being the number of grid nodes traversed in a given direction (diagonal or axial), from the origin point to the points of the design ($s \geq 1$). Its value is initially established in correlation with the N_{vk} numbers. It starts with designs of relatively small dimensions compared to the dimensions of the feasible domain. The algorithm of the variant without model calculation is the following:

1) The first experiment is carried out in a node P_0 from the feasible domain (initial point) which is considered the first origin point (Fig. 2);

2) The points diagonal to the point of origin are established;

3) $N = 2^k$ experiments are performed in the diagonal points (full factorial design);

4) If there is a diagonal point where the answer is more convenient than the origin point, that point becomes the new origin point of the next iteration and step 2 is resumed (the current origin point slides towards the diagonal point);

5) Otherwise, the axial points of the origin point are established;

6) $N = 2^k$ experiments are performed in the axial points (full factorial design);

7) If there is an axial point where the answer is more convenient than the origin point, that point becomes the new origin point of the next iteration and step 2 is resumed (the current origin point slides towards the axial point);

8) Otherwise, the current value of step *s* is decremented by 1, if this is possible, and step 2 is resumed.

All new designs derive from the previous ones by slidings in diagonal or axial direction with different values of step *s*.

The constraints on position can be taken into account when establishing diagonal or axial points. In case they are violated by one of the diagonal or axial points, the current value of step *s* is decremented by 1, if this is possible and the algorithm restarts. The step *s* allows controlling the speed of the algorithm, which is the speed of slidings the designs in the feasible domain in search of the optimal point. A step that is too big can increase the speed but it can reduce the probability of reaching the optimal point, so a compromise is necessary depending on the situation.

b) Variant with calculation of second-order polynomial models

As in the case of the method by zooms, at each iteration a second-order polynomial model is calculated that allows determining the direction of the best values, the position and dimensions of the next designs (1), (2).

The slidings direction for the next iteration will be towards one of the design vertices / side centers, closest to the optimal (stationary) point of the model, on the current design. The designs used must comprise a number of experiments at least equal to the number of coefficients b_k, i.e. $N \geq k + 1$ experiments. The structure and dimensions of the used experimental designs are established in correlation with the dimensions of the feasible domain.

The sliding strategy (Fig. 5) establishes that if D-1 type Doehlert design is used, after determining the optimal (stationary) point S of the model on the current design, the position of the new design is chosen, of the same size, having as center the vertex closest to point S. In this way, 4 points are recovered from one iteration to another (economy design).

If at one of the iterations the optimal point of the model S is obtained inside the current design, then the optimal path is determined, representing the ridge line of the model (Fig. 4a). Another iteration is performed and a new optimal path is calculated (Fig. 4b). The intersection of the two optimal paths can provide an acceptable solution to the optimization problem (Fig. 4c).

The algorithm of the variant with model calculation is the following:

1) An origin point $P_0 = (x_1, \ldots, x_k)^T$ is defined in the feasible domain;

2) A design of *N* experiments centered in P_0 is made, with $N > p$ = the number of coefficients of the second-order polynomial (type D-1 Doehlert design, $N = 7 > p = = 6$);

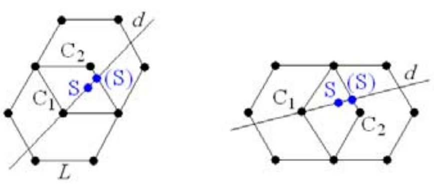

Fig. 5. Sliding strategy for two parameters [2].

3) The associated second-order polynomial model is calculated;

4) Determine the optimal (stationary) point S of the model on the current design;

5) Set the center of the new design at the vertex closest to point S;

6) If the optimum point is not inside the current design, repeat step 3 for the new design;

7) If the optimal point is inside the current design, the optimal path on the current design is determined;

8) If two optimal paths have not yet been determined, step 3 is repeated for the new design;

9) If two optimal paths have already been determined, the solution of the optimization problem is obtained by the intersection of the two optimal paths.

As convergence criteria [13] can be used the error specific to the method that compares the maximum difference between the values of the objective function *F* at the last iteration against the same value if all previous iterations are taken into account:

$$\varepsilon\ [\%] = \frac{f_{\max} - f_{\min}}{F_{\max} - F_{\min}} \cdot 100 \leq \varepsilon_{\max}\ [\%] \qquad (14)$$

or the relative error of the last objective function value $F^{(n)}$ compared to the previous iteration with a different value $F^{(m)}$:

$$\varepsilon_F\ [\%] = \frac{F^{(n)} - F^{(m)}}{F^{(m)}} \cdot 100 \leq \varepsilon_{F\max}\ [\%] \qquad (15)$$

II. Case Study

A. The object of study

Next, the application of the optimization method by zooms is presented to improve the performance of the direct current electromagnetic device analyzed in [18] (Fig. 6). For a comparative study, the simple variant of the method, which does not require calculation of models, was applied first, followed by the application of the variant that uses polynomial models of the second-order. Both variants combine the results of a large number of numerical simulations to find the maximum of the objective function represented by the force developed by the device at the air-gap δ = 1 mm. The numerical model of the device was obtained by 2-D simulation using the FEMM program [19] coupled with the LUA language [20]. The force value was determined in the post-processing stage using the Maxwell Stress Tensor method. The geometrical parameters are given in Table I and the geometrical shape is shown in Fig. 6. The coil is supplied with voltage $U = 115$ V, has resistance $R_b = 2300\ \Omega$ and number of turns $w = 11500$.

TABLE I.
GEOMETRICAL PARAMETERS OF DC DEVICE [18]

h (mm)	52.50	L_1 (mm)	6.35	δ (mm)	1.00
h_1 (mm)	7.90	L_2 (mm)	6.35	h_b (mm)	31.20
h_2 (mm)	7.90	L_{a1} (mm)	13.00	L_b (mm)	7.50
H (mm)	65.70	g (mm)	19.80		
L (mm)	50.90	$S_b = L_b \cdot h_b$ (mm^2)	234.00		

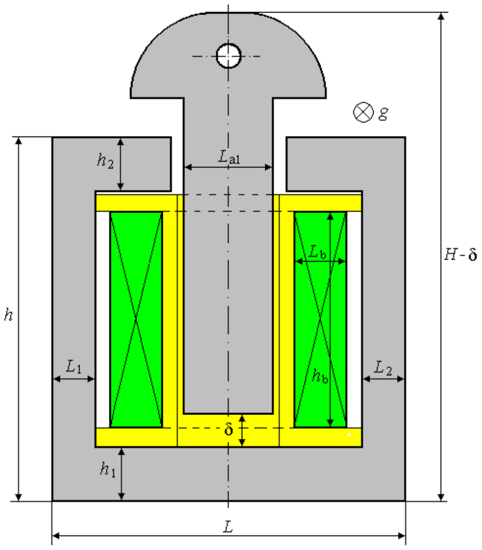

Fig. 6. Geometrical shape of the electromagnetic device [18].

B. *Formulating and solving the optimization problem*

The optimization problem [17] consists in maximizing the force developed by the device at the air-gap δ = 1 mm by varying two geometrical parameters: the ratio between the height and thickness of the coil (k_b) and the ratio between the thicknesses of the core yokes (k_{my}):

$$k_b = \frac{h_b}{L_b} \in [2 \div 6], \quad k_{my} = \frac{h_2}{h_1} \in [0.5 \div 2] \tag{16}$$

The optimization is non-linear and subject to four equality constraints which maintain the overall dimensions of the device (core width L = ct., core height h = ct., total device height H = ct.) and coil section ($S_b = h_b \cdot L_b$ = ct.). The optimization problem is described mathematically as follows:

$$\mathrm{P}: \begin{cases} \max F(k_b, k_{my}) \\ k_{b\min} \le k_b \le k_{b\max} \\ k_{my\min} \le k_{my} \le k_{my\max} \\ g_h(k_b, k_{my}) - h = 0 \\ g_L(k_b) - L = 0 \\ g_H(k_b, k_{my}) - H = 0 \\ g_{Sb}(k_b) - S_b = 0 \end{cases} \tag{17}$$

where the functions g_h, g_L, g_H, g_{Sb} depend on parameters k_b, k_{my} and on other geometric parameters not detailed here.

The algorithms of the two variants of the two methods were initialized with the point corresponding to the center of the feasible domain (P_0), close to the point describing the initial geometry (P_{in}) (Table I, Fig. 7).

The evolutions of the optimization algorithms are graphically illustrated in Figs. 7-8 and Figs. 11-12 for the variants without model calculation, respectively, Figs. 9-10 and Figs. 13-14 for the variants with second-order polynomial models. The calculated values are written in Tables II-V.

TABLE II.
EVOLUTION OF THE OPTIMIZATION ALGORITHM BY ZOOMS WITHOUT CALCULATION OF MODELS [17]

Iterations	N_{tot}	N_{rec}	k_b	k_{my}	F	ε (%)	$ε_F$ (%)	L_b (mm)	h_b (mm)	h_1 (mm)	h_2 (mm)	L_1, L_2 (mm)	L_{a1} (mm)
0	1	-	4.000	1.250000000	22.939	-	-	7.65	30.59	7.29	9.11	6.35	12.70
1	5	1	3.000	1.625000000	24.380	100.00	6.28	8.83	26.50	7.81	12.69	5.76	11.52
2	5	2	2.000	2.000000000	24.496	46.50	0.48	10.82	21.63	8.46	16.91	4.77	9.53
3	5	2	3.000	2.000000000	24.818	20.15	1.32	8.83	26.50	6.84	13.67	5.76	11.52
4	5	1	2.500	2.000000000	24.924	17.20	0.43	9.67	24.19	7.60	15.21	5.34	10.68
5	5	3	2.500	1.953125000	24.878	12.73	-0.18	9.67	24.19	7.73	15.09	5.34	10.68
6	5	1	2.750	1.976562500	24.898	4.31	0.08	9.22	25.37	7.27	14.37	5.56	11.13
7	5	4	2.500	2.000000000	24.924	4.00	0.11	9.67	24.19	7.6	15.21	5.34	10.68
8	5	2	2.500	1.988281250	24.909	3.51	-0.06	9.67	24.19	7.63	15.18	5.34	10.68
9	5	1	2.625	1.994140625	24.915	2.63	0.02	9.44	24.78	7.42	14.80	5.45	10.91
10	5	4	**2.500**	**2.000000000**	**24.924**	**1.28**	**0.04**	**9.67**	**24.19**	**7.60**	**15.21**	**5.34**	**10.68**
TOTAL	**51**	**21**											

TABLE III.
EVOLUTION OF THE OPTIMIZATION ALGORITHM BY ZOOMS WITH SECOND-ORDER MODELS [17]

Iterations	N_{tot}	N_{rec}	k_b	k_{my}	F	ε (%)	$ε_F$ (%)	L_b (mm)	h_b (mm)	h_1 (mm)	h_2 (mm)	L_1, L_2 (mm)	L_{a1} (mm)
0	1	-	4.000000	1.250000	22.939	-	-	7.65	30.59	7.29	9.11	6.35	12.70
1	16	0	3.000000	1.625000	24.380	100.00	6.28	8.83	26.50	7.81	12.69	5.76	11.52
2	9	1	2.475191	1.856124	24.753	29.97	1.53	9.72	24.07	8.03	14.90	5.31	10.63
3	9	1	2.510789	1.959299	24.867	8.55	0.46	9.65	24.24	7.69	15.07	5.35	10.70
4	9	1	**2.503167**	**1.993066**	**24.914**	**2.78**	**0.19**	**9.67**	**24.20**	**7.62**	**15.18**	**5.34**	**10.68**
TOTAL	**44**	**3**											

TABLE IV.
EVOLUTION OF THE OPTIMIZATION ALGORITHM BY SLIDINGS OF DESIGNS WITHOUT CALCULATION OF MODELS

Iterations	N_{tot}	N_{rec}	k_b	k_{my}	F	ε (%)	$ε_F$ (%)	L_b (mm)	h_b (mm)	h_1 (mm)	h_2 (mm)	L_1, L_2 (mm)	L_{a1} (mm)
0	1	-	4.000	1.25000	22.939	-	-	7.65	30.59	7.292	9.114	6.351	12.701
1	5	1	3.750	1.34375	23.380	100.00	1.92	7.90	29.62	7.414	9.963	6.225	12.451
2	5	2	3.500	1.43750	23.768	66.40	1.66	8.18	28.62	7.541	10.841	6.087	12.173
3	5	2	3.250	1.53125	24.104	45.70	1.41	8.49	27.58	7.673	11.750	5.932	11.865
4	5	2	3.000	1.62500	24.380	32.86	1.15	8.83	26.50	7.811	12.693	5.759	11.518
5	5	2	2.750	1.71875	24.589	23.43	0.86	9.22	25.37	7.957	13.676	5.563	11.126
6	5	2	2.500	1.81250	24.707	14.97	0.48	9.67	24.19	8.111	14.702	5.338	10.675
7	5	2	2.750	1.90625	24.818	16.13	0.45	9.22	25.37	7.444	14.189	5.563	11.126
8	5	3	**2.500**	**2.00000**	**24.924**	**11.67**	**0.43**	**9.67**	**24.19**	**7.604**	**15.209**	**5.338**	**10.675**
TOTAL	**41**	**16**											

TABLE V.
EVOLUTION OF THE OPTIMIZATION ALGORITHM BY SLIDINGS OF DESIGNS WITH SECOND-ORDER MODELS

Iterations	N_{tot}	N_{rec}	k_b	k_{my}	F	ε (%)	$ε_F$ (%)	L_b (mm)	h_b (mm)	h_1 (mm)	h_2 (mm)	L_1, L_2 (mm)	L_{a1} (mm)
0	1	-	4.000	1.250	22.939	-	-	7.65	30.59	7.292	9.114	6.351	12.701
1	7	4	3.800	1.380	23.668	100.00	3.18	7.85	29.82	7.219	9.962	6.251	12.503
2	7	4	3.600	1.510	23.740	49.26	0.31	8.06	29.02	7.162	10.814	6.144	12.288
3	7	4	3.400	1.640	23.371	50.23	-1.56	8.30	28.21	7.12	11.674	6.027	12.054
4	7	4	3.194	1.766	24.445	28.13	4.60	8.56	27.34	7.108	12.552	5.896	11.791
5	7	4	3.000	1.900	24.767	23.80	1.32	8.83	26.50	7.072	13.433	5.759	11.518
6	7	4	2.800	2.000	24.920	16.27	0.62	9.14	25.60	7.065	14.338	5.604	11.208
7	**7**	**4**	**2.664**	**2.000**	**24.954**	**10.63**	**0.14**	**9.37**	**24.97**	**7.272**	**14.759**	**5.489**	**10.978**
TOTAL	**50**	**22**											

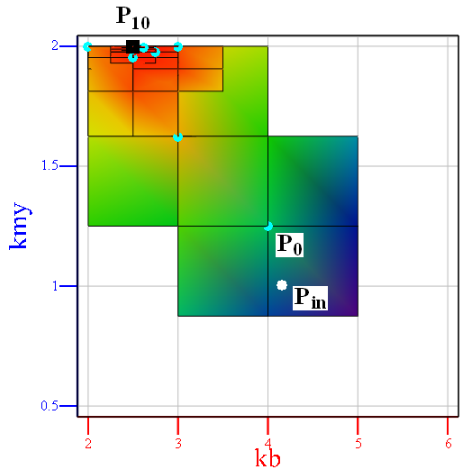

Fig. 7. Two-dimensional representation of the optimization algorithm of the method by zooms without calculation of models [17].

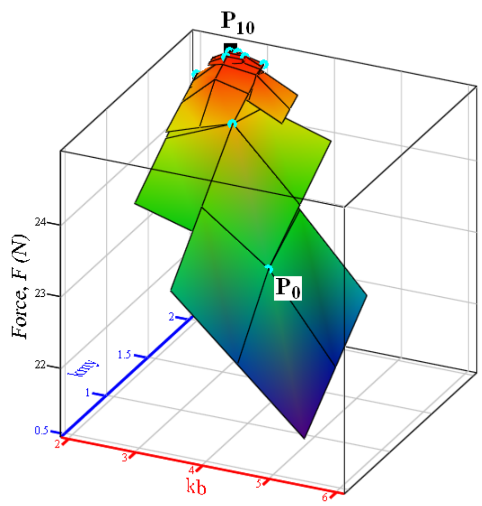

Fig. 8. Three-dimensional representation of the optimization algorithm of the method by zooms without calculation of models [17].

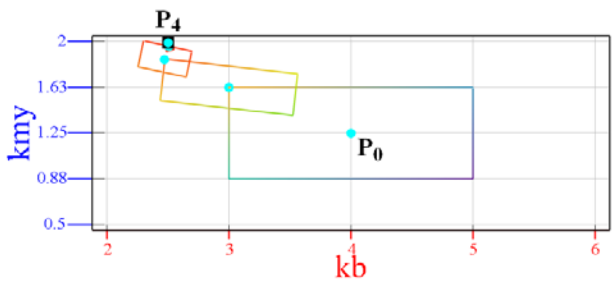

Fig. 9. Two-dimensional representation of the optimization algorithm of the method by zooms with second-order polynomial models [17].

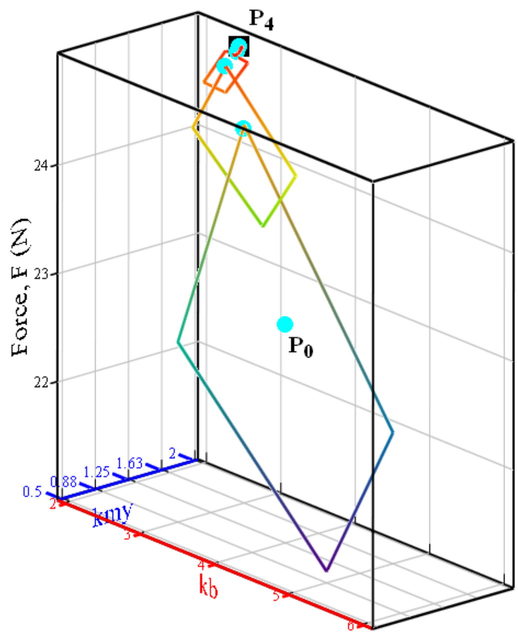

Fig. 10. Three-dimensional representation of the optimization algorithm of the method by zooms with second-order polyn. models [17].

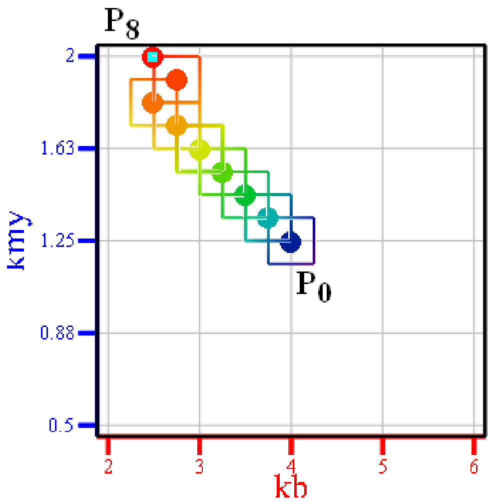

Fig. 11. Two-dimensional representation of the optimization algorithm of the method by slidings of designs without calculation of models.

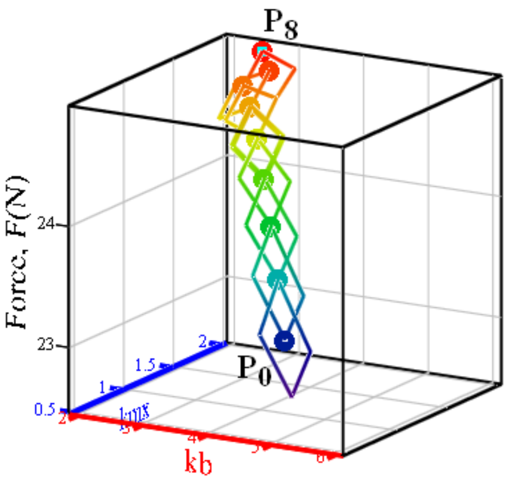

Fig. 12. Three-dimensional representation of the optimization algorithm of the method by slidings of designs without calculation of models.

Since in the last iteration the optimal point was obtained on the border of the domain, the determination of the optimal paths would not bring a great gain because their intersection would take place outside the feasible domain. Therefore, the algorithm stops at this point.

To facilitate the comparison between the methods and their application variants, Table VI compares the results derived from Tables II-V: the number of iterations (N_{it}), the number of experiments (simulations) actually performed ($N = N_{tot} - N_{rec}$)) and the force increase (ΔF).

It can be seen that similar results were obtained after a comparable number of actually performed numerical simulations. Although the second variant of the method requires additional complex calculations, it converges much faster.

TABLE VI.
COMPARISON BETWEEN APPLIED METHODS

Method	Variant	N_{it}	N	ΔF (%)
Zooms	Without models	10	30	8.66
	With 2-D polynomial models	4	41	8.61
Slidings of designs	Without models	8	25	8.66
	With 2-D polynomial models	7	28	8.78

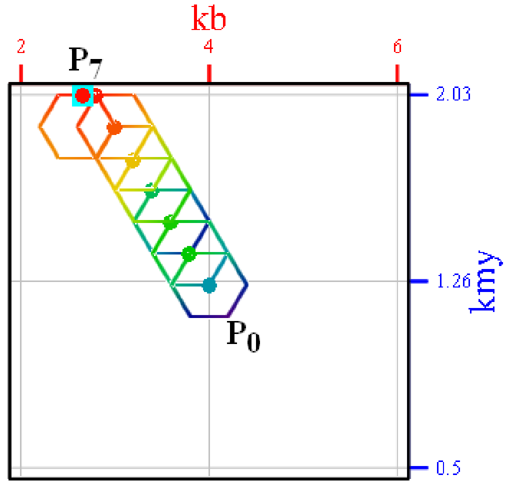

Fig. 13. Two-dimensional representation of the optimization alg. of the method by slidings of designs with second-order polyn. models.

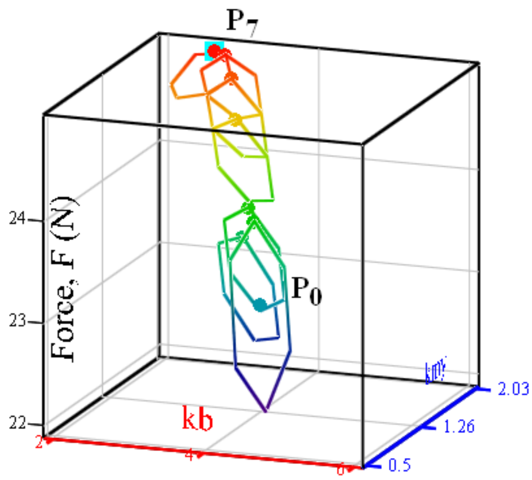

Fig. 14. Three-dimensional representation of the optimization algorithm of the method by slidings of designs with second-order polynomial models.

In Fig. 15 shows the optimal solution obtained by applying both, the method by zooms and the method by slidings of designs, in the variants without and with calculation of models (about the same solution) with the distribution of the magnetic flux density at the air-gap δ = 1 mm, obtained in FEMM, as a planar solution.

III. CONCLUSIONS

The paper presents two variants of two optimization method based on DOE and FEM, ones that only require access to the objective function values at certain points of the feasible domain and others that, in addition, calculates second order polynomial models that approximate the objective function on subdomains. The application on a 2-D numerical model of an electromagnetic device allowed a comparative study of them.

The results highlighted simplicity of the application of the first variant and the speed of convergence of the second for both methods, obtaining similar results with a number of iterations reduced to less than half, but at the price of increasing the number of simulations.

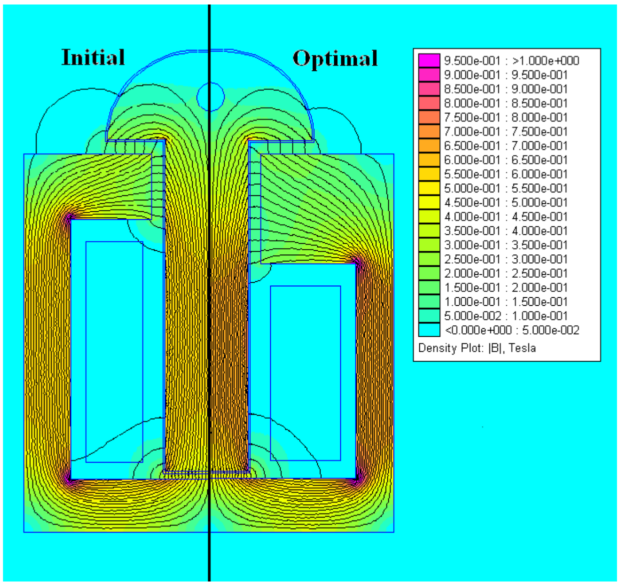

Fig. 15. The optimal solution obtained by the method by zooms (both variants) with the magnetic flux density distribution (δ = 1mm, planar FEMM solution) [17].

As disadvantages, the presented optimization methods determine a local optimum and require a numerical model of the device under optimization. The variants with model calculation are based on a mathematical apparatus of high complexity.

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