Hybrid Systems Modeling Approach with Petri Nets

Mircea – Adrian Drighiciu
University of Craiova, Faculty of Electrical Engineering, Craiova, Romania
adrighiciu@em.ucv.ro

Abstract **- The purpose of this paper is to present the achievement of the Hybrid Petri Nets techniques used for modeling and behavioral analysis of a class of hybrid systems. In our sense, a dynamic hybrid system contains at least two distinct subsystems which interact: a continuous subsystem (linear, or not) and a discrete subsystem with a finite number of states. In this context, after a brief introduction to the basic elements of Hybrid Petri Nets modeling, two hybrid systems, consisting of liquid level control of one and two interconnected tanks were analyzed in different specific scenarios. The analysis starts with the synthesis of a Hybrid Petri Net model for each system, followed by their refinement and validation by simulation of their behavioral properties, in order to achieve and implement a command - control structure (the sequential controller) of the process. For the models synthesis and for their validation, Modified Hybrid Petri Nets were used. Such structure is a extinction of the classical topology of a generalized Hybrid Petri Nets model, achieved by assignation to every continuous transition a firing speed as a algebraic relation whose variables depend of different marking of continuous places, arbitrary or not. The synthesis of the models and their validation was obtained using Visual Object Net++ tool.**

Cuvinte cheie: *sisteme hibride, Rețele Petri, modelare, simulare.*

Keywords: *hybrid systems; Petri Nets; modeling; simulation.*

I. Introduction

A dynamic hybrid system contains at least two distinct subsystems which interact: a continuous subsystem (linear, or not) and a discrete subsystem with a finite number of states. These interactions may lead to the emergence of specific peculiarities in the dynamics of the entire system, especially if the state transition mechanism is based on sequences of external asynchronous events. Thus, switching states can be determined by occurrence a certain type of event and not by an internal time clock, even if time - as an independent variable - appears explicitly in the pattern of the model. For the synthesis of the whole model and then for its quantitative analysis, it is necessary to manipulate specific tools combining - within the same formalism – some elements of classical representations from the theory of continuous systems with others elements specific to the modeling of discrete event drive systems. Such formalism is the one developed around the Petri Nets. Most precisely, it is a particular class of Hybrid Petri Nets, which allows the synthesis of models that incorporate the dynamics of the entire system (process) analyzed, although the paradigms used in the representation of its component subsystems (continuous or discrete) may be different [1], [2], [3], [7], [23], [24], [27], [28].

II. Review on Hybrid Petri Nets

In a discrete Petri Net, the marking of a place may correspond either to the Boolean state of a device (e.g., a supply valve is turned on or off), or to an integer state (e.g., number of parts in a conveyor input buffer). For this type of model, a general analysis method is to compute the set of reachable states and afterwards deduce the different - good or inadequate - properties of the analyzed system. But when a Petri Net marking contains a large number of tokens, the number of reachable states explodes and the method becomes difficult to be applied. This observation allowed researchers community to define and utilize continuous and hybrid Petri Nets in model synthesis [5], [6], [7], [11], [17], [18], [19], [20], [21], [22], [25], [26].

According to this formalism, an Hybrid Petri Net (HPN) is a sextuple HPN = {P, T, Pre, Post, $\mathbf{m}_0$, h} such that: P = {P_1, P_2, …P_n} is a finite, not empty, set of places; T = {T_1, T_2, …, T_m} is a finite, not empty, set of transitions; $P \cap T = \varnothing$ (P and T are disjointed); h, called "hybrid function" indicates for every node whether it is a discrete or a continuous node; Pre : $P \times T \to \mathbf{R}^+$ or $\mathbf{N}^+$, is the input incidence mapping; Post : $P \times T \to \mathbf{R}^+$ or $\mathbf{N}^+$ - the output incidence mapping and $\mathbf{m}_0 : P \to R^+$ or $\mathbf{N}^+$ is the initial marking corresponding to the initial state (Fig.1). Therefore are two parts in a HPN: a discrete part and one continuous, interconnected by means of arcs which links a discrete node (place or transition) to a continuous node (transition or place) [8], [9], [10], [12], [13], [14].

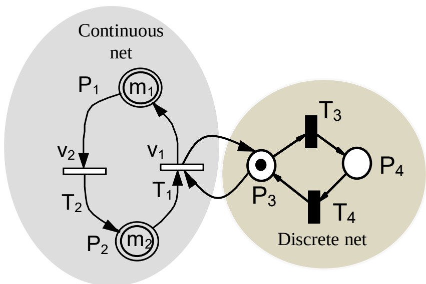
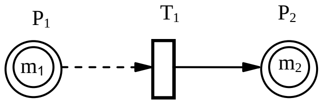


Fig.1. A Hybrid Petri Net structure.

A powerful extinction of the classical structures of HPN consist by the assignation to every continuous transition a firing speed as a function whose variables depending of different marking of continuous places, arbitrary or not [8], [9], [10], [11]. The result was named Modified Hybrid Petri Net (MHPN). Using of continuous functions associated with the execution speeds of the transitions has considerably increased the modeling power of the continuous Petri Nets, the new models possessing a higher abstraction level within a simplified topology, increased modularity and thus a possibility of immediate integration into complex structures, with multiple interactions between subnets of the same category or not. In this way it was possible to translate the mathematical models into an evolved graphic language, proper to the Hybrid Petri Nets formalism, and analyze them with appropriate tools.

In Fig.2, the firing speed of T_1 is represented as a function depending on all making of the model. The transition T_1 is always active, due to the test arc $P_1 - T_1$ (dotted line represented) which does not allow token flow; it can be inactivated only by the empty discrete input places. Hence, using such topology it's easy to achieve various primary subsystems without feedback models: in the dynamics of the model the token quantity of P_1 is not influenced [8], [9], [10].



$v = f(m_1, m_2, ..., m_i, m_{i+1}, ...m_n)$

$m_1(t)$, $m_2(t)$ – quantity of tokens in P_1, P_2
$m_i(t)$, $m_{i+1}(t)$ – quantity of tokens in P_i, P_{i+1}
v – firing speed of continuous transition T_1

Fig. 2. Modified Hybrid Petri Net with test arcs.

For the above HPN, according to the semantics of a continuous Petri Net, the firing speed associated to T_1 continuous transition depends on the markings m_1 and m_2: $v_{T1} = -dm_1/dt = dm_2/dt$

In order to check all of goods properties of the models, algebraic techniques can be used in addition with software tools [4], [5], [6], [15], [16].

III. CASE STUDY

A. *Case of one single tank*

Let us consider an open cylindrical tank of inside circular section S containing liquid (Fig.3). The tank is provided with a ON/OFF elecro-valve V_1 which allows the supply with a constant flow of liquid – q_1. The tank is emptied through a s_c section pipe provided with a ON/OFF elecro-valve V_2. The liquid level in the tank – h – may rise or fall between the two limits - h_{min} and h_{max} – according to the state of the two valves V_1 and V_2 and also depending to the inlet and outlet flow values.

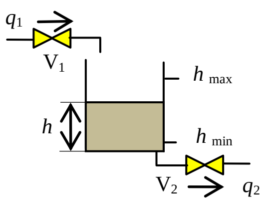


Fig.3. Explanatory to the filling / emptying process.

It is obvious that the variation of the level can be done either commanding V_1 (when V_2 is always open), either commanding V_2 (when V_1 is always open) or by controlling both valves.

Fig. 4 shows the model of this system controlled by both valves, made with HPN, whose elements have the meaning shown in Table I. Obviously, the initial marking of the model correspond to the initial state of the system: V_1 is opened, V_2 is closed and the initial value of the liquid in the tank is shown in the place P_1.

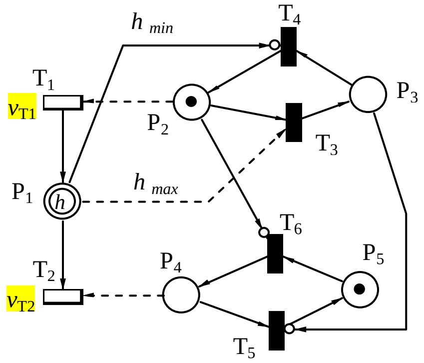


Fig.4. Hybrid Petri Net model of the filling / emptying process.

From the initial state, the level increases and at $h = h_{max}$ the states of the two valves changes: V_1 is turned off and V_2 is turned on. Indeed, the test arc $P_1 - T_3$, whose weight is h_{max} validates the firing of discrete transition T_3, which modifies the marking of P_2 and P_3: $m(P_2) = 0$ and $m(P_3) = 1$. Due of null marking of P_2 place, the inhibitor arc $P_2 - T_6$ allows to T_6 to be fired (turn on V_2) and, as a result $m(P_4) = 1$ and $m(P_5) = 0$. T_2 is now fireable due to the marking of P_4 being fired with a variable speed, depending on the variable output flow q_2 and so on.

The firing speeds v_{T1} and v_{T2}, associated to the T_1 and T_2 continuous transitions are obtained from the flow balance equation, according to the rules of the type of HPN model used [5], [6], [9]:

$$q_1 - q_2 = \frac{dV}{dt} = S\frac{dh}{dt} \qquad (1)$$

where V is the tank volume and h its liquid level.

It results $v_{T_1} = q_1/S$ and $v_{T_2} = q_2/S$.

The output flow can be expressed as: $q_2 = s_c \cdot v_2$. Here s_c is the flow section of the tank outlet which depends on the diameter d_c of the external pipe and v_2 is the instantaneous flow rate of the liquid.

In our case:

$$v_2 \cong \sqrt{2gh} \qquad (2)$$

with: g – gravity acceleration and h – the level of liquid in the tank. Because the value of the S-section is generally much higher than the s_c, we will consider, without further specifications or restrictions, that in the formulas of the above type we can use the equal sign instead of the approximation.

Therefore $q_2 = s_c \cdot \sqrt{2gh} = k\sqrt{h}$, with $k = s_c\sqrt{2g}$

When the level is $h = h_{min}$, V_1 will be turned on and V_2 will be closed. Hence, in the model T_4 and T_5 will be fireables.

TABLE.I
THE MEANING OF THE HPN ELEMENTS

Element of the HPN	**Meaning**
P_1	Liquid level in the tank; in the initial state the marking of this place correspond to the value of the initial level;
P_2, P_3	Places whose marking indicates the state of the V_1valve: $m(P_2) = 1 - V_1$ is turned on, $m(P_3) = 1 - V_1$ is turned off. Always $m(P_2) + m(P_3) = 1$;
P_4, P_5	Places whose marking indicates the state of the V_2 valve: $m(P_4) = 1 - V_2$ is turned off, $m(P_5) = 1 - V_2$ is turned on. Always $m(P_4) + m(P_5) = 1$;
T_1	Transition whose firing speed is a function, depending on the input flow rate q_1, assumed constantly;
T_2	Transition whose firing speed is a function depending on the output flow q_2;
T_3, T_5	Turn off V_1 and V_2 respectively;
T_4, T_6	Turn on V_1 and V_2 respectively;

For the synthesis and for the validation of the model in various simulation scenarios, starting from different initial states, Visual Object Net++ tool was used (Fig.5) [9], [10], [11].

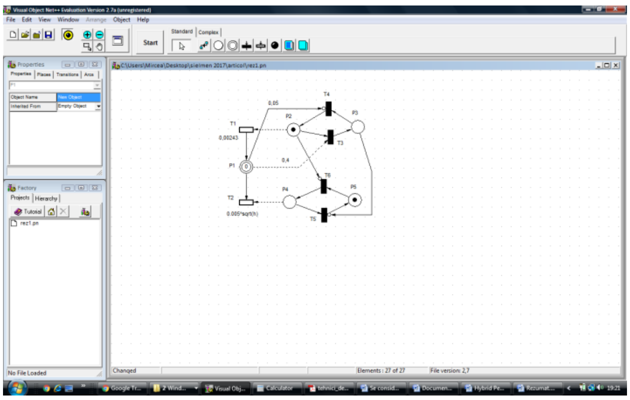


Fig.5. Main view of the Visual Object Net++ tool.

Fig. 6 shows the Hybrid Petri Net model in the initial state and the evolution of the level obtained after simulation, considering an initial value of level in the tank $h_{in} = 0{,}1$ m, D = 0,3 m, $q_1 = 10$ l/min, $d_c = 0{,}01$ m, $h_{min} = 0{,}05$ m and $h_{max} = 0{,}4$ m.

B. *Two interconnected tanks*

Two interconnected tanks plant are considered (Fig. 7). In the initial state, all the three ON/OFF electro-vanes V_1, V_2 and V_3 are closed and the two tanks are empty.

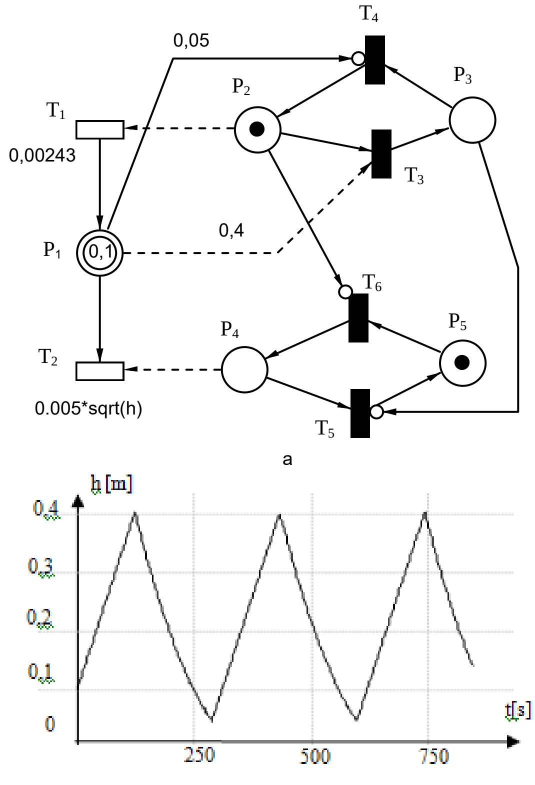


Fig.6. Explanatory at the validation of the model: a) Quantitative model of the process; b) Evolution of the level in a first scenario, obtained by simulation with Visual Object Net++ tool.

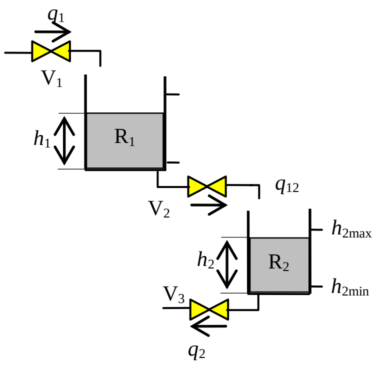


Fig.7. Two interconnected tanks plant.

A nonlinear and hybrid mathematical model was considered for describing the two tank system:

$$\frac{dh_1}{dt} = \frac{1}{S_1}\left(k_1 \cdot q_1 - k_2 \cdot q_{12}\right) \quad (3)$$

$$\frac{dh_2}{dt} = \frac{1}{S_2}\left(k_3 \cdot q_{12} - k_3 \cdot q_2\right) \quad (4)$$

where h_1 and h_2 are liquid levels in R_1 and R_2 respectively, S_1 and S_2 - the values of the R_1, respectively R_2 inside areas, q_1, q_{12} and q_2 – the flows (Fig.7) and k_i, $i = \overline{1,3}$ a discrete variable, corresponding to the ON/OFF state of the V_i, $i = \overline{1,3}$ electro-valves.

To achieve this scenario, starting from the initial state, the controller must assure the following sequences: turn on V_1, then, after t_1 time units since the valve V_1 was opened the V_2 is also turned on. After t_2 time units from V_2's opening, the V_3 vane is also turned on. If the liquid level in R_2 dropped till h_{2min} amount, V_3 must be turned off while V_2 will remain turned on. If the liquid level in R_2 rises till h_{2max} amount, V_2 vane must be turned off while V_3 will remains turned on.

During this time period, V_1 remains open, filling the tank R_1. If the liquid level in R_1 raises and reaches the maximum amount, h_{1max}, the V_1 vane will be turned off until tank R_1 empties completely ($h_1 = h_{1min}$). At this point, both V_2 and V_3 vanes are automatically turned off regardless of their state. The process is then restarted in the same succession of sequences at a new START command.

The Hybrid Petri Net model of this process is shown in Fig. 8.

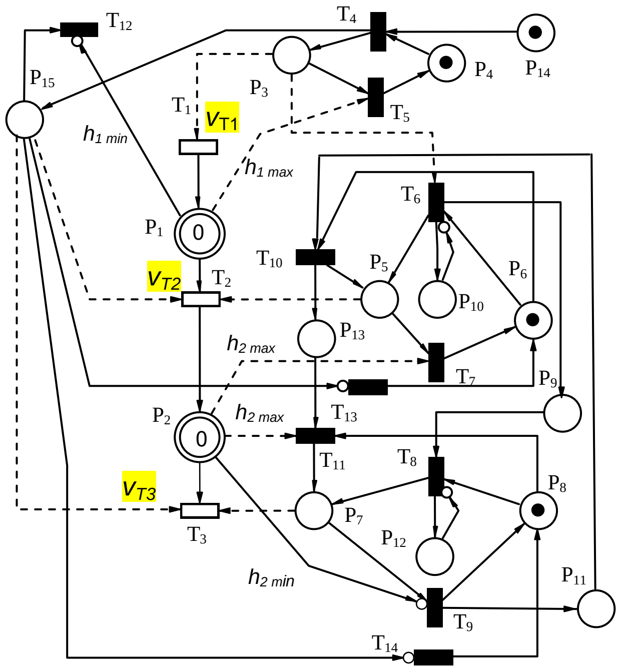


Fig.8. The Hybrid Petri Net model of the process, according to the aforementioned sequences.

The meaning of the elements presented in the HPN model is as shown in Table II.

TABLE.II
THE MEANING OF THE HPN ELEMENTS

Element of the HPN	**Meaning**
P_1, P_2	The liquid level in tank R_1 and R_2 respectively; in the initial state the marking of these places correspond to the initial values of their liquid levels;
P_3, P_5, P_7	Places whose marking indicates the opened state of the V_1, V_2 and V_3 respectively valves: $m(P_3) = 1 - V_1$ is turned on, $m(P_5) = 1 – V_2$ is turned on and $m(P_7) = 1 – V_3$ is turned on;
P_4, P_6, P_8	Places whose marking indicates the closed state of the V_1, V_2 and V_3 respectively valves: $m(P_4) = 1 - V_1$ is turned off, $m(P_6) = 1 – V_2$ is turned off and $m(P_8) = 1 – V_3$ is turned off;
P_9	Place which authorizes the firing of T_8 after t_2 time units from V_2's turning on;
P_{10}, P_{12}	Places which, by the way of connecting with T_6 and T_8 transitions respectively, authorizes their executions one time only, after t_1 respectively t_2 units of time starting from the initial state;
P_{11}	Place whose non-null marking validates the firing of T_{10} (turning on V_2) when the liquid level in R_2 reached the minimum amount h_{2min};
P_{13}	Place whose non-null marking allows the firing of T_{11} (V_3 vane's turning on) when the liquid level in R_2 reached the maximum amount h_{2max};
P_{14}	Place which allows the V_1 vane opening command (START command);
P_{15}	Place whose non-null marking validates the firing of T_2 and T_3 after the level in the tank R_1 has fallen to the minimum value h_{1min}, until the tank R_2 is completely emptied;
T_1	Continuous transition, whose firing speed - v_1 - depends on the value (assumed constant) of the supply flow rate;
T_2	Continuous transition whose variable firing speed - v_2 - is represented as a function depending on the feed rate of the second tank R_2;
T_3	Continuous transition, whose variable firing speed - v_3 - depends on the output flow rate;
T_4	Turn on the V_1 vane after a START command;
T_5	Turning off of V_1 vane when the liquid level in R_1 reached the maximum amount h_{1max};
T_6, T_8	Temporized opening of V_2 and V_3 vane respectively;
T_7, T_9	Turning off of V_2 and V3 vane respectively;
T_{10}	Turning on of V_2 vane when the liquid level from R_2 tank reached minimum amount h_{2min};
T_{11}	Turning on of V_3 vane when the liquid level from R_2 tank reached maximum amount h_{2max};
T_{12}	A target transition which, through its execution, cancels P_{15} place marking and authorizes the incidental commands to turn off the V_2 and V_3 vanes when the liquid level from R_1 reached minimum amount h_{1min};
T_{13}, T_{14}	The incidental command to turn off the V_2 and V_3 vanes when the liquid level from R_1 reached minimum amount h_{1min};

As in the first case we have analyzed, the continuous transitions T_1, T_2 and T_3 have different firing speeds which depend on the continuous places' markings P_1 and P_2 and show the amount of the liquids' flow rate from both tanks. Therefore, starting from (3) and (4) results: $v_{T_1} = \frac{q_1}{S_1}$, $v_{T_2} = \frac{q_{12}}{S_1} = \frac{k_{f1}\cdot\sqrt{h_1}}{S_1}$ and $v_{T_3} = \frac{q_2}{S_2} = \frac{k_{f2}\cdot\sqrt{h_2}}{S_2}$, where: V_1 and V_2 are the volumes of the two tanks, S_1 and S_2 - the values of the R_1, respectively R_2 inside areas, h_1, h_2 – liquid level in each of the two tank and k_{fi} , i = 1,2 – two flow coefficients, each having its own expression:

$$k_{fi} = s_{ci}\sqrt{2g} \quad \mathrm{i} = 1, 2 \tag{5}$$

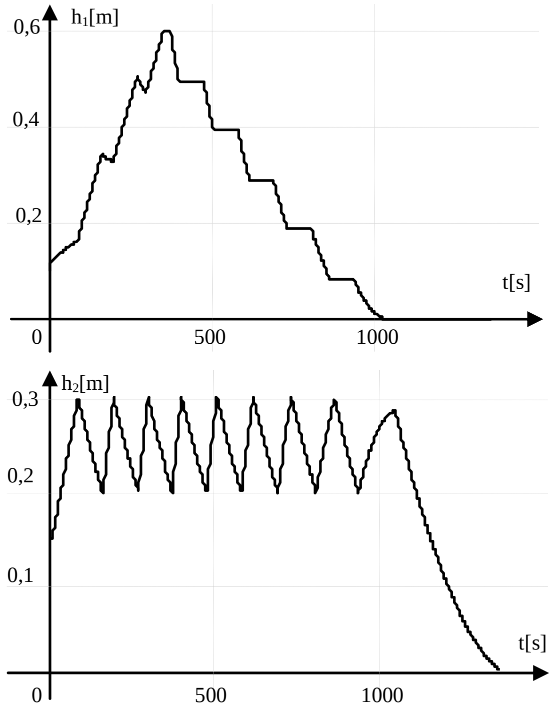
In (5), s_{ci} are the sections of drainage holes from R_1 and R_2 respectively tanks and g is gravitational acceleration.

The dynamic of the model is determined by the fired rules of the HPN transitions, as well as its initial marking: in the initial state the three valves are turned off. The mark from P_{14} authorizes the execution of T_4 transition to which the 'turn on V_1 valve' event is associated to. After its opening (the mark in P_3) the R_1 tank is supplied with the flow rate q_1 of constant value. The test arc P_3- T_1 authorizes the firing of the continuous transition T_1 with the speed v_{T1}. After t_1 time units from V_1's turning on, the T_6 transition is fired (valve V_2 opens). After other t_2 time units from V_2's opening, the V_3 valve also turns on (T_8 transition is fired). By firing both T_6 and T_8 transitions, the new place marking for P_{10} and P_{12} leads to the subsequent disable of T_6 and T_8 transitions. Moreover, the new marking, associated with P_5 and P_7 places, authorizes the continuous firing of T_2 and T_3 transitions with the speeds v_{T2} and v_{T3}. R_1 and R_2 tanks are filled with liquid and the three electro-vanes are opened. If the liquid level from R_2 drops beneath the h_{2min} ($h_2 \leq h_{2min}$) value, the inhibitor arc P_2-T_1, of weight equal to h_{2min} (Fig.8), validates the T_9 transition to which the 'turn off V_3 vane' event is associated to. The marking of P_8 becomes $m(P_8) = 1$ and T_3 transition is disabled (P_7 is not marked anymore). When $h_2 = h_{2max}$, the test arc P_2-P_{11} validates T_{11} transition and determines the V_3 vane's commutation state ($m(P_7) = 1$ and $m(P_8) = 0$). The test arc P_2-P_7 authorizes, in turn, the T_7 transition firing (the V_2 vane is closing).

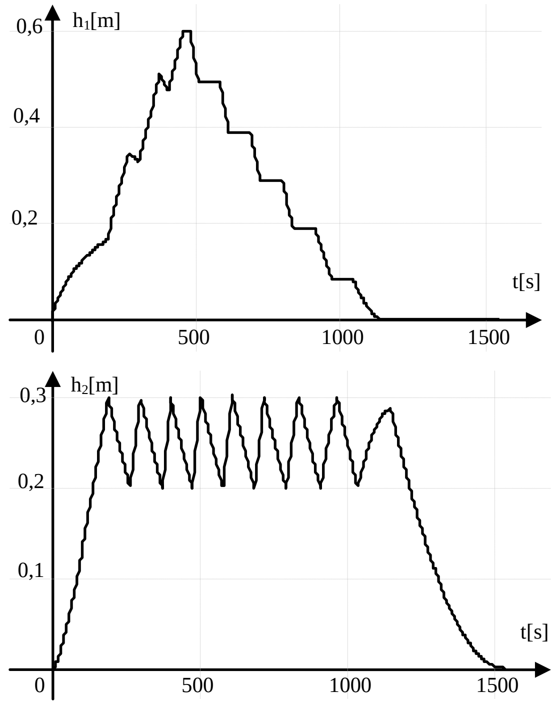
Next, the control of the h_2 liquid level between the limits h_{2min} - h_{2max} is assured through the periodical commutation of V_3 and V_2 vanes states: to $h_2 = h_{2min}$ V_3- turned off, V_2 turned on and for $h_2 = h_{2max}$, V_3- is turned on and V_2- is turned off.

During this time period, R_1 tank is filling with liquid while V_1 vane is turned on. When $h_1 = h_{1max}$, T_5 transition will be fired (V_1 vane will be turned off), the P_3 place marking will disable the ulterior fire up of T_1 continuous transition. From this point on, the liquid level from R_1 starts to drop. When $h_1 = h_{1min}$, the inhibitor arc P_1-T_{12} will allow the firing of T_{12} final transition. Therefore, the null marking of P_{16} place will disable the firing of T_2 and T_3 transition but will authorize the firing T_{13} and T_{14} transitions (V_2 and V_3 vanes are also closed). At a new START command (by placing a new mark in P_{14}) the process will continue in the same manner with the one described above.

The synthesis of the HPN model was done using the Visual Object Net ++ tool [9], [10], [11] and its validation resulted after various simulation scenarios. For example, Fig.9 shows a series of results obtained for different initial conditions, considering the diameters of the two tanks D_1 = 0,3 m and D_2 = 0, 2 m, q_1 = 10 l/min., the diameter of connection pipe d_{c1} = 1 cm, the diameter of the outlet pipe d_{c2} = 0,005 m, h_{1max} = 0,6 m, $h_{2\,max}$ = 0,3 m, $h_{2\,min}$ = 0,2 m, t_1 = 10s, t_2 = 15 s.



a.



b.

Fig.9. Liquid level variation in the two tanks obtained during simulation of the model in Visual Object Net++ tool: a) h_{1in} = 0,1 m and h_{2in} = 0,15 m; b) $h_{1in} = h_{2in}$ = 0 m.

Fig. 9.a shows the liquid level variation in the two tanks for h_{1in} = 0,1 m and h_{2in} = 0,15 m, and Fig. 9.b

shows the same variations considering that the two tanks are completely empties (h_{1in} = 0 m, h_{2in} = 0 m).

Another possibility to interconnect the two tanks is the one presented in Fig. 10. In this case, between the two tanks there is a level variation, R_2's supplying pipe being situated at H height from its base.

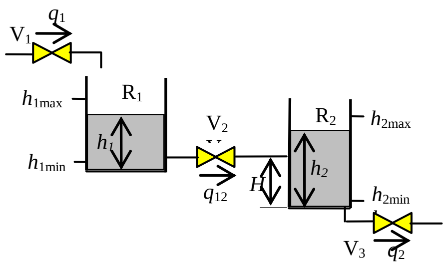


Fig. 10. Explanatory to another case of interconnection of the two tanks.

And in this case also, the mathematical model of the process is a hybrid one, similar with the one described through equations (3) and (4). The difference between this one and the previous one stands in the flow's expression q_{12}, with which tank R_2 is supplied (Fig. 10).

$$q_{12} = \begin{cases} s_{c1}\sqrt{2g}\sqrt{h_1-(h_2-H)} & pentru\ h_2 > H \\ s_{c1}\sqrt{2g}\sqrt{h_1} & pentru\ h_2 \le H \end{cases}, \quad (6)$$

where s_{c1} is the flowing section through the linking pipe, with diameter d_{c1}, between tanks R_1 and R_2. For the flowing rates q_1 and q_2 the following relations have been kept: $q_1 = \text{const.}$, $q_2 = s_{c2}\sqrt{2g}\sqrt{h_2}$.

The HPN process' model is shown in Fig.11. In this case, the Petri net topology has two continuous transitions for the flowing rate of V_2 valve, according to relation (6): $T_{2,1}$ and $T_{2,2}$.

Their firing speeds are as follows: $v_{T2,1} = q_{12}/S_1$, where q_{12} is given by (6) for $h_2 \le H$ and $v_{T2,2} = q_{12}/S_1$, with the value of q_{12} obtained for $h_2 > H$.

The meaning of the HPN model's elements from Fig.11 it's, mostly similar to the one described in TABLE II. To illustrate the model dynamics, the same working scenario has been kept, the one that led to the corresponding firing sequences from Fig.8. Hence, from the initial state, V_1 valve is turned on, then, after t_1 time units from V_1's opening, the V_2 vane will be is opened too. After t_2 time units from V_2's opening, the V_3 valve also turns on. Both R_1 and R_2 tanks are filled with liquid and the three electro-valves are opened. If the liquid level from R_2 drops beneath the h_{2min} value, V_3 will be closed. The level in R_2 tank increases and at $h_2 = h_{2max}$, the state of V_3 valve changes again (V_3 is turned on). Hence, the control of the h_2 liquid level between the limits h_{2min} - h_{2max} is assured through the periodical commutation of V_3 and V_2 vanes states: to $h_2 = h_{2min}$ V_3-turns off, V_2-turns on and for $h_2 = h_{2max}$, V_3- will be turned on, while V_2- will be turned off.

During this time period, R_1 tank is filled with liquid while V_1 vane is turned on. When h_1=h_{1max} , V_1 vane commutes and from this point on, the level in R_1 starts to drop. When h_1=h_{1min}, V_2 and V_3 valves will be also turned off and so on.

As in the above cases, to synthesize the model, the Visual Object Net ++ tool has been used, validation of the model and the analysis of its good behavioral properties being done by simulation, in various initial conditions and in various scenarios, assuming that the process proceeds without failures.

Starting from the above model, Fig.12 a. and b. shows the results achieved by simulation of the HPN model in Visual Object Net++, considering the diameters of the

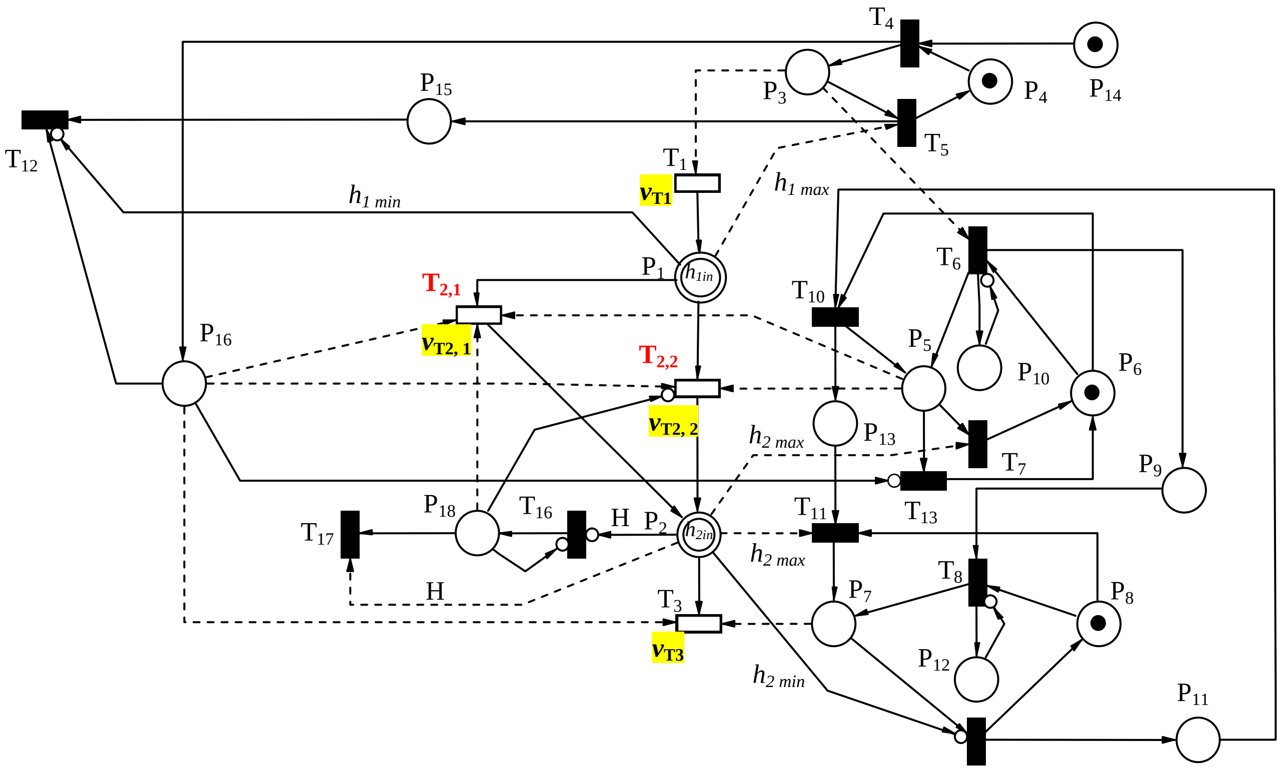


Fig.11. The Hybrid Petri Net model of the process, according to the above connexion of tanks.

two tanks D_1 = 0,3 m and D_2 = 0, 2 m, q_1 = 10 l/min., the diameter of connection pipe d_{c1} = 0,01 m, the diameter of the outlet pipe d_{c2} = 0,05 m and t_1 = 10s, t_2 = 15 s.

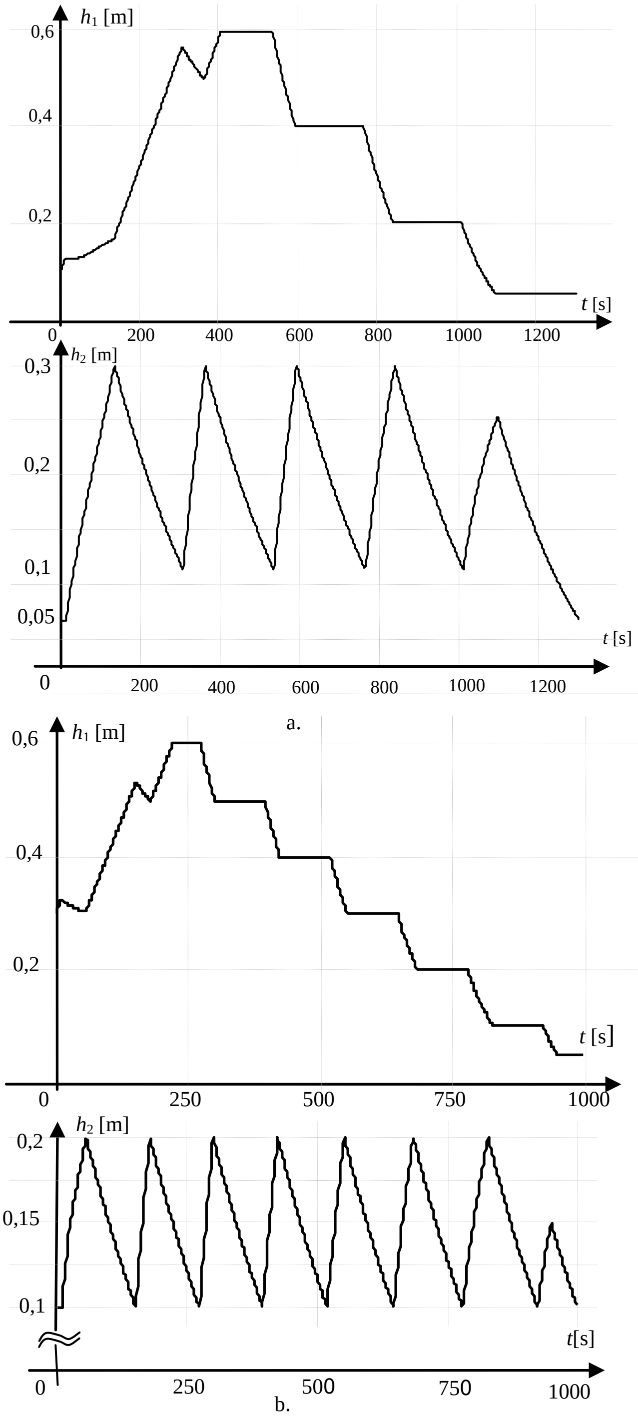


Fig.12. Liquid level variation in the two tanks obtained during simulation of the model in Visual Object Net++ tool for:
a) h_{1in} = 0,1 m; h_{1min} = 0,1 m; h_{1max} = 0,6 m, h_{2in} = 0,05 m, $h_{2\,max}$ = 0,3 m, $h_{2\,min}$= 0,1 m, H = 0,2 m; b) h_{1in} = 0,3 m; h_{1min} = 0,05 m; h_{1max} = 0,6 m, h_{2in} = 0,1 m, $h_{2\,max}$ = 0,2 m, $h_{2\,min}$= 0,1 m, H = 0,15 m;

IV. CONCLUSIONS

In the paper, two hybrid systems were analyzed using Hybrid Petri Nets model techniques: a liquid level control process in a tank and then, starting from this basic structure, a two interconnected tanks plant. Continuous subsystems have been represented as Continuous Petri Nets with variable speed of transitions, depending on the marking of continuous places and the discrete part (controller) was modelled using discrete Petri Nets. The accuracy and correctness of the models was verified by simulation with Visual Object Ne++ tool, comparing the results for different simulation scenarios with similar ones, obtained through other technical tools (Matlab – Simulink, Modelica etc.), or with laboratory experimental tests.

HPN paradigm remains a powerful tool for analyzing hybrid systems, even if for complex processes the model's achievement remains a laborious activity. In this case, a possible approach is based on Petri Nets Objects, which gives to the models a greater flexibility and an increased modularity.

One of the important advantages of the using this concept is the ability to describe a larger system by the decomposition into interacting objects. Then, using the properties of the synthesized objects, finding an improved version of the model can be made easier. The object-oriented concept unites the advantages of the modules and hierarchies and adds powerful new concepts and skills like reuse, encapsulation and information hiding.

Starting from the achieved models, the next step consists in the synthesis of the sequential controller for the whole process.

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