Synthesis of the PID Algorithm for Control the Thermal Regime in the 3D Printer

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***Abstract*—In this paper it is described the automatic system for formation the small pieces from polymer, based on the principle of thermoplastic extrusion and it is proposed the graph – analytical algorithm for identification the mathematical model of the control object first and second order inertia of the industrial process by the experimental curve of the transient process as response at the step signal applied to the entrance of the system.The procedure of calculation the approximation of the control object is based on the experimental curve of the process for which the output level is determined at the value of 0.632*k* and the respectively time. As control object is used the thermal stabilization regime in the extruder of 3D printer. The obtained results of identification are compared with results obtained in case of identification in MATLAB. For the obtained identified models was synthesized the PID control algorithm by the analytical methods and auto-tuning regime.**

Cuvinte cheie: *imprimarea 3D, identificarea modelului obiectului, modelul matematic algoritmului PID, algoritmii de acordare, auto-acordare.*

Keywords*:* *3D printing, identification of the mathematical model of object, mathematical model PID algorithm, tuning methods, autotuning.*

I. Introduction

In the automation of the various industrial and technological processes it is necessary to be known the mathematical model (MM) that is attached to the process. The MM of the industrial processes can be obtained based on two classes of methods: analytical and experimental methods [5-7].

Analytical methods use physics, mechanics laws etc., which express the phenomena that characterize the evolution of the input-output of a process and, as a result, it is obtained a differential equations system of the energetic equilibrium.

The MM that were obtained using the analytical methods and represent the systems of differential equations of energetic equilibrium, more or less are complex and these models are used with difficulties in process of synthesis the control algorithm.

Experimental methods represent the procedures for determination the characteristic proprieties of the studied process, based on an experiment oriented towards obtaining a non-parametric MM and performed simultaneously with the recording of the process's input-output evolution. In this case, there are applied the input signals and the evolution of the process output is recorded and there are obtained the experimental curves in time or frequency domain and, as a result of the processing of these curves (identification procedure), there are obtained the systemic representations of parametric MM in the form of differential equations, transfer functions, or frequency functions with known coefficients.

These models of knowledge the process proprieties that are relatively simple (reduced order inertia – first order, second with or without time delay) provide facilities to simulate process dynamics and have a wide use in practice in case of automation the diverse processes.

In these cases, it can be supposed that around a some point of process acting, the small variations of the process input will also generate small variations of the process output, which, as a result, presents a linear dynamic model, which can be written in the form of differential equation or transfer function.

At the current stage of development the automation there are a lot of identification methods of the mathematical model of the control object according to the industrial process response, which allow with high accuracy to identify different structures of object models with or without time delay, with high or low order inertia.

For these purposes, there are implemented methods and software packages for processing the experimental data of the industrial process and, as a result, there are obtained the model structures of the control object with different proprieties and low or high order with or without time delay [5-7]. It is wide use for the identification of the mathematical models by the experimental response the MATLAB software package [5-7].

The applying of the existing identification methods for determination the mathematical model of the control object imply the operations with calculations and graphical transformations, which are somewhat complex.

In order to overcome these difficulties in this paper it is proposed a simple method and with a small number of calculation, that permits to determine the model of the control object by the experimental response of the process.

In this paper it is described the designed system for printing the small objects with a maximum volume of 20535 mm^3, where the control of temperature in the extruder is done based on the PID control algorithm, which was tuned by the maximum stability degree method and by the implemented in the controller the auto-tuning regime.

The obtained results are compared with identification in MATLAB and on the real system, the process of stabilization the thermal regime in the extruder of the 3D printer.

II. The Principle of the 3D Objects Printing

3D printers represent the machines that produce 3D physical models based on the digital data, by printing layer-by-layer. It can produced the physical models of objects which are designed by CAD (Computer Aided Design) programs or scanned through 3D scanners.The format of scanned objects or CAD-designed objects is STL (Stereolithography) and for printing it is necessary to transform this format by the software which is called "slicer"that converts the model into a series of fine layers and produces a G-code that contains instructions adaptedto the different 3D printers. This G-code can be used by the various printing software that load the G-code and use it during the printing process [1-3].

The principle of printing it is the following: the heater it is moving through the *X, Z* axes and the platform is moving through the *Y* axis, depending on the G-code and the filament is melting and it is forming a lot of successive layers of the object.

III. Description of the Designed System

3D printer was designed based on the development board Arduino Mega2560 and a shield Ramps 1.4. The axes of two CD-ROMs and the axis of a Floppy Driver were used as axes for moving the heating element with the print platform.

The model PL15S-020 of the engine was used as actuator element. For the extrusion of the thermoplastic it was used a mechanism with a cogwheel and as actuator element for the extruder was used a stepper motor model Nema17. The block scheme of thedesigned 3D printer is presented in theFig.1.

According to the block scheme from the Fig. 1, the whole process of the printing is controlled by the Arduino Mega2560 microcontroller, which receives instructions in the G-Code format, and control the engines through the A4988 drivers and transmits the commands for moving with certain steps in order to form layer by layer the 3D object and for setting the temperature it is used a mosfet. As a temperature sensor, a thermistor is used, whose resistance varies depending on the temperature.

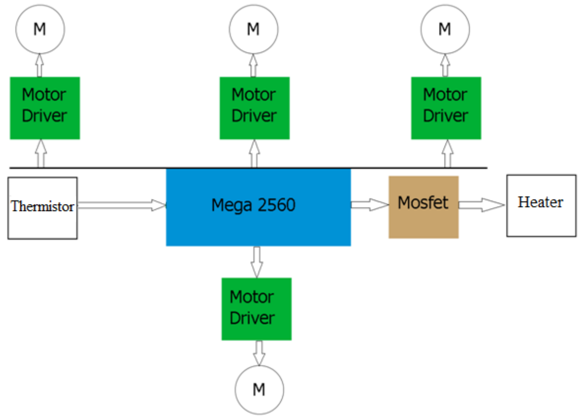

Fig. 1.The block scheme of the designed 3D printer.

IV. The Algorithm for Calculation the Model of the Studied Process

In the discussion it is put the mathematical model of the stabilization process of the thermal regime in the extruder of the 3D printer, the experimental curve of temperature variation is presented in the Fig. 2.

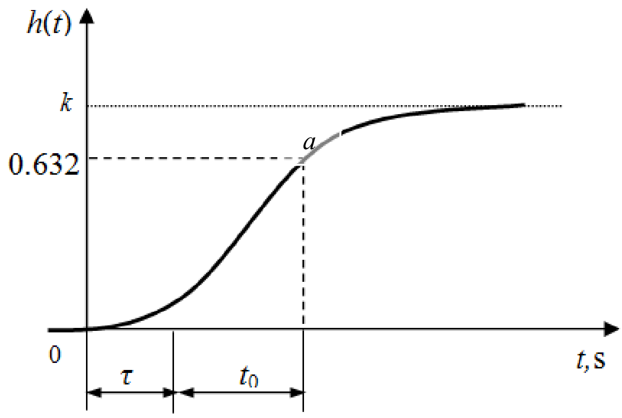

Fig. 2.The response of the industrial process.

The mathematical models of the control object with first and second order inertia and time delay are presented by the transfer functions:

$$H(s) = \frac{ke^{-\tau s}}{Ts+1}, \quad (1)$$

$$H(s) = \frac{ke^{-\tau s}}{(T_1 s+1)(T_2 s+1)} = \frac{ke^{-\tau s}}{a_0 s^2 + a_1 s + a_2}, \quad (2)$$

where k is transfer coefficient, T, T_1, T_2– time constants, τ– time delay, which is determinate from the curve Fig. 2, but the coefficients from (2) are calculated: $a_0=T_1T_2$, $a_1=T_1+T_2$, $a_2=1$.

It is used the following procedure for determination the parameters k, T, T_1, T_2 and τ from relations (1) and (2).

From curve presented in the Fig. 2 it is calculated the numerical values for the model (1) with time delay:

1. The values of the $h(t)$ at the level 0.632k:

$$h(t_2)=0.632k. \quad (3)$$

2. For the level $h(t_2)=0.632k$ it is constructed the parallel line to the abscissa axis up to the intersection with the

curve (point a) and from this point it descends perpendicularly to the intersection with the axis of the abscissa (point t_2).

3. It is calculated the value of time delay $t_1=\tau$ for that the output values is null or a value 0.05 from the stationary value of the process. It is calculated the value of the time segment by the relation

$$t_0= T=t_2-t_1=t_2-\tau, \tag{5}$$

during that its level is $h(t_2)$=0,632k.

Its value T from (5) represents the time constant for the process of approximation the mathematical model to model of object with first order inertia starting from the point where the time delay is finished (Fig. 2).

4. It is calculated the values of the time constant for models (1) and (2).

For the model (1):

$$T=t_0. \tag{6}$$

For the model (2) it is calculate the values of the time constant [4,8]:

$$\begin{gathered} T_1/T_2=0.5, \\ T_2=0.64t_0=0.64T, \\ T_1=0.5T_2=0.5\cdot 0.64t_0=0.32t_0=0.32T. \end{gathered} \tag{7}$$

By the relations (7) are calculated the coefficients a_0, a_1, a_2:

$$\begin{gathered} a_0=T_1T_2=0.32\cdot 0.64T^2=0.2048T^2, \\ a_1=T_1+T_2=0.32T+0.64T=0.96T, \\ a_2=1. \end{gathered} \tag{8}$$

As a result of the calculations according to the proposed algorithm the mathematical models (1) and (2) are given by transfer function:

$$H(s)=\frac{ke^{-\tau s}}{Ts+1}=\frac{ke^{-\tau s}}{t_0 s+1}, \tag{9}$$

$$H(s)=\frac{ke^{-\tau s}}{0.2048T^2s^2+0.96Ts+1}=\frac{ke^{-\tau s}}{0.2048t_0^2s^2+0.96t_0s+1}.$$

The model parameters (9) - k, and T or t_0 are calculated by the experimental curve.

These approximation models of control object are used to synthesize the control algorithms. If it is known the model of object with second order inertia and it is necessary to obtain the model of object with first order inertia it is used the (7) relations and it is obtained the relation for calculation the time constant:

$$T=t_0=\frac{T_1}{0.32}=\frac{T_2}{0.64}, \tag{10}$$

but the parameter k is the same from the model of object with second order inertia.

In case when is known the model of control object with first order inertia with or without time delay, it can be calculated the model object with second order inertia applying the (7) - (9) relations with parameter k, τ, which are the same as in the case of model object with inertia first order.

V. Identification the Mathematical Model of the Control Object

In the 3D printer the extruder is the core of the printer and represents the place where the plastic is pulled, melted and pushed. It is very important to keep the temperature in the extruder with high precision. For this reason, it is used the PID controller. For optimal tuning of the controller it is necessary to know the mathematical model of the control process.

To determinate the mathematical model of the thermal process in the extruder it was obtained the experimental curve of the temperature variation in the extruder, where the reference temperature was settled at the 218 C (12V). The experimental curve was registered by the Simplify 3D software and it is presented in the Fig. 3.

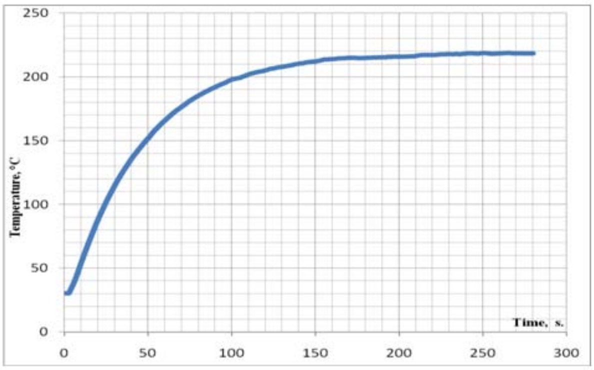

Fig. 3.The experimental curve.

It was proposed to approximate the control object with five types of the mathematical models:

1. Model of object with first order inertia and time delay

$$H(s)=\frac{ke^{-\tau s}}{Ts+1}=\frac{1.0035e^{-2.847s}}{44.2696s+1}. \tag{11}$$

2. Model of object with second order inertia and time delay

$$H(s)=\frac{ke^{-\tau s}}{(T_1s+1)(T_2s+1)}=\frac{1.0041e^{-0.002s}}{(37.87s+1)(38.55s+1)}. \tag{12}$$

3. Model of object with second order inertia

$$H(s)=\frac{k}{(T_1s+1)(T_2s+1)}=\frac{1.0031}{(44.003s+1)(2.2272s+1)} \tag{13}$$

4. Model of object with third order inertia

$$\begin{gathered} H(s)=\frac{k}{(T_1s+1)(T_2s+1)(T_3s+1)}= \\ =\frac{1.0033}{(44.1315s+1)(2.0042s+1)(1.9689s+1)}. \end{gathered} \tag{14}$$

5. Based on the proposed algorithm in the chapter IV consider that the time constant of the model object with first order inertia is T=44.2696 from the model (11), when the calculations are

$$T_2=0.64\cdot 44.2696=28.3325,$$

T_1=0.5·28.3325=14.1663.

In this case the obtained transfer function is

$$H(s)=\frac{ke^{-\tau s}}{(T_2 s+1)(T_1 s+1)}=\frac{1.0035e^{-2.847s}}{(28.3325s+1)(14.1663s+1)}. \quad (15)$$

In the transfer functions (11), (12), (13), (14) and (15) are used the following notations: k is the transfer coefficient; T_1, T_2, T_3 - time constants; τ – time delay.

The mathematical models of the control object were obtained using the module Process Models from System Identification Toolbox from MATLAB and there are obtained transfer functions (11)-(14). The (15) transfer function was obtained based on the method described in the chapter IV.

In the Fig. 4 there are presented the comparison of the identified models, where there are used the following notations: 1 – transient process of the model object (11); 2 – transient process of the model object (12); 3 – transient process of the model object (13); 4 – transient process of the model object (14); 5 – transient process of the model object (15); 6 – the original experimental curve.

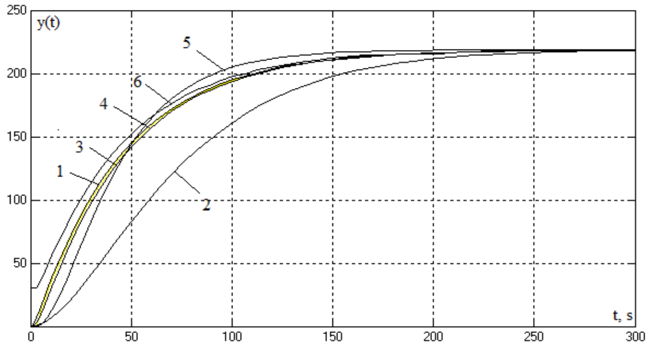

Fig. 4. Comparison of the identified transient processes.

VI. Computer Simulation

It is proposed to synthesize the standard control algorithm PID to the identified model of objects (11)-(15), which is described by the following transfer function:

$$H_{PID}(s)=k_p+\frac{k_i}{s}+k_d s, \quad (16)$$

where k_p, k_i, k_d represent the tuning parameters of the respectively controller [9 - 11].

As tuning methods, it was proposed to use maximum stability degree method with iterations [12-15] and analytical algorithms for synthesis of controller by the maximum stability degree criterion [16, 17]. According to the maximum stability degree method with iterations [12-15], there were obtained the dependencies $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$ for each case of model objects (11)-(15), where J is the maximum stability degree and obtained dependencies are presented in the figures 5-9. Based on the obtained curves were chosen the values sets J - k_p, k_i, k_d of the tuning parameters of the PID controller and these sets are presented in the Table I, where the numbering of the sets corresponds to the models of object (11)-(15).

TABLE I.
The Values of the PID Controller's Parameters

No. curves	J	k_p	k_i	k_d
1	0.17	7.5251	0.3859	20.704
2	0.029	2.673	0.035	50.407
3	0.3	25.382	2.6379	41.843
4	0.26	10.836	0.7611	21.032
5	0.09	2.6854	0.072	23.524

Based on the obtained results it was done the computer simulation of the control system, using the Matlab Simulink and the obtained transient processes are presented in the Fig. 10.

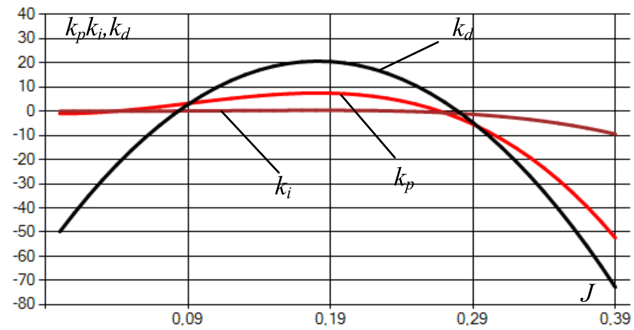

Fig. 5. Dependencies $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$ in case of tuning the PID controller to the model object with first order inertia and time delay.

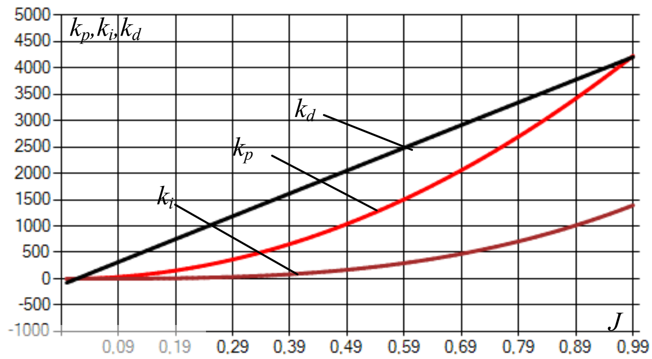

Fig. 6. Dependencies $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$ in case of tuning the PID controller to the model object with second order inertia and time delay.

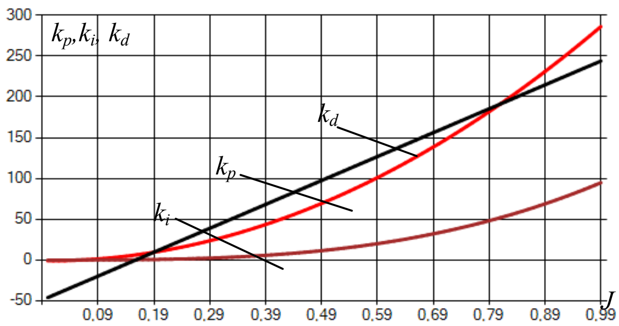

Fig. 7. Dependencies $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$ in case of tuning PID controller to the model object with second order inertia.

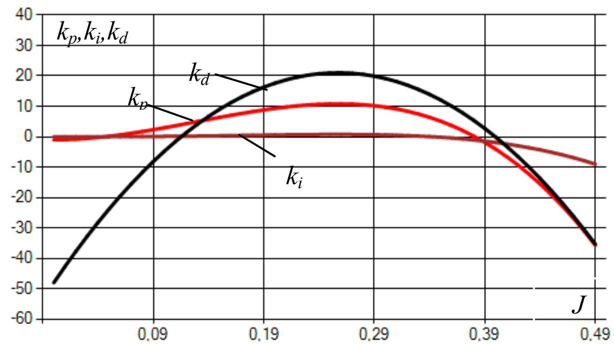

Fig. 8. Dependencies $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$ in case of tuning the PID controller to the model object with third order inertia.

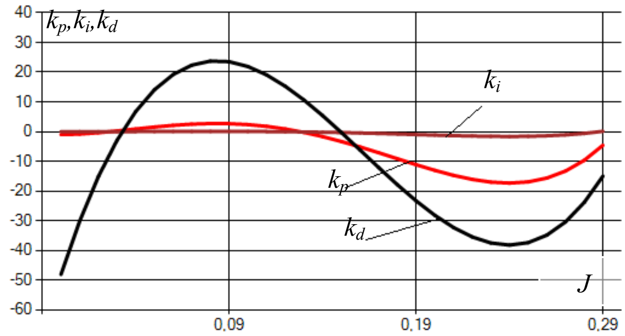

Fig. 9. Dependencies $k_p=f(J)$, $k_i=f(J)$, $k_d=f(J)$ in case of tuning the PID controller to the model object with second order inertia and time delay.

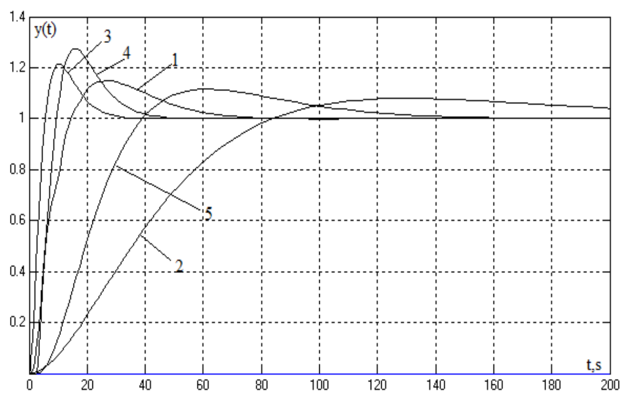

Fig. 10. The transient processes of control system.

Analyzing the obtained transient processes from Fig. 10, it can be observed that the best performance was obtained for the case of tuning PID controller to the model object with inertia first order and time delay (11).

The obtained values of the tuning parameters of the PID controller were settled and verified on the installation, the set point was settled equal with 190 □C and steady-state error is considered equal with $\varepsilon=\pm 5\% y_{st}$. The obtained experimental curves of the temperature variation in the extruder are presented in the figures 11-15.

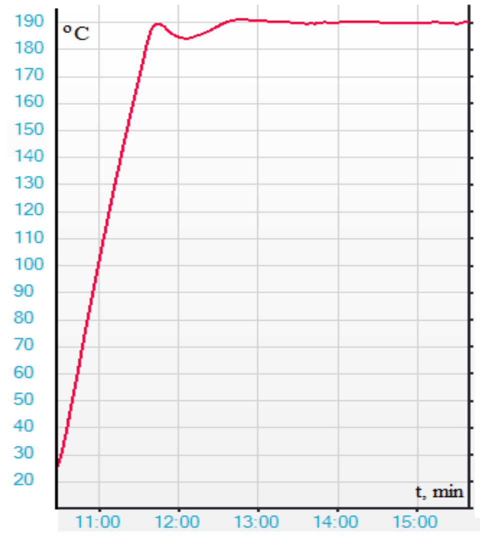

Fig. 11. The transient experimental process of the control system with PID controller, $k_p=7.5251; k_i=0.3859; k_d=20.704$, for the model (11).

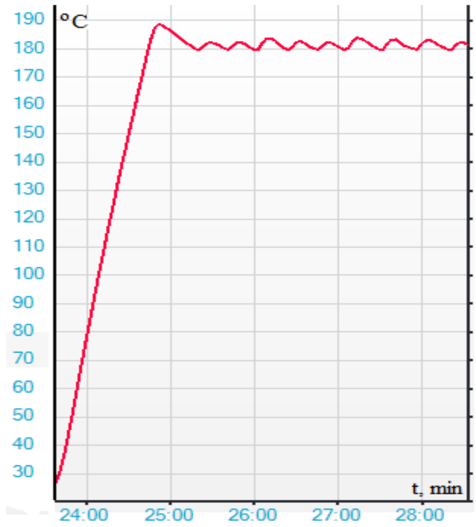

Fig. 12. The transient experimental processes of the control system with PID controller, $k_p= 2.673; k_i= 0.035; k_d= 50.407$, for the model (12).

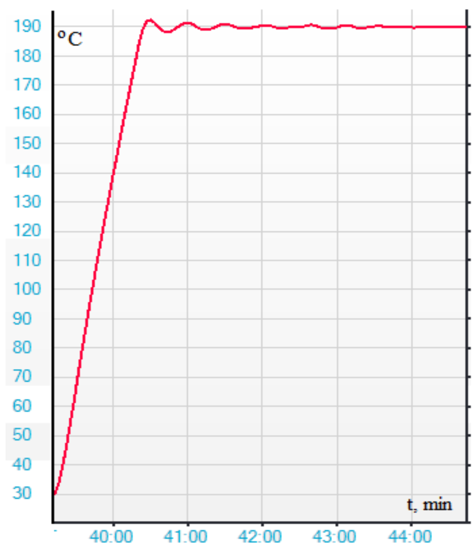

Fig. 13. The transient experimental processes of the control system with PID controller, $k_p= 25.382; k_i= 2.6379; k_d= 41.843$, for the model (13).

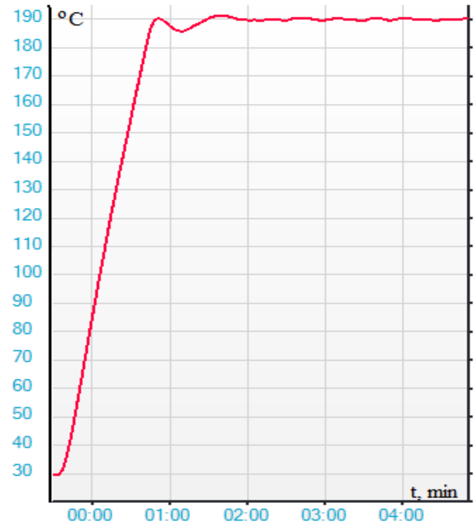

Fig. 14. The transient experimental processes of the control system with PID controller, k_p=10.836;k_i=0.7611;k_d=21.032, for the model (14).

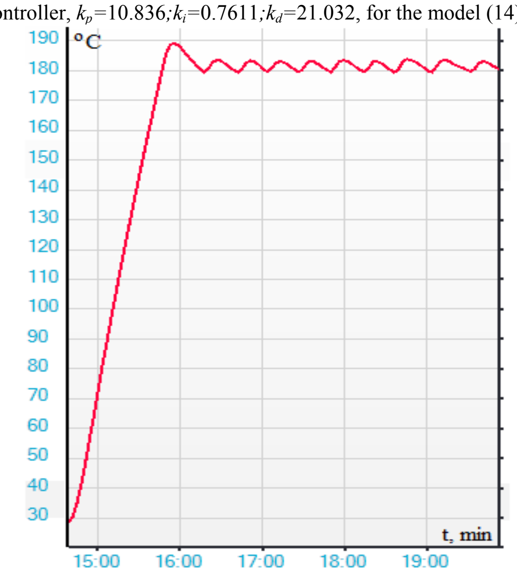

Fig. 15. The transient experimental processes of the control system with PID controller, k_p=2.6854;k_i=0.072;k_d=23.524, for the model (15).

From figures 11-15 it can be observed that the best performance was obtained for the case of tuning PID controller by the maximum stability degree method with iterations to the model object with third order inertia without time delay (14). In this case it was obtained the overshoot equal with σ = 1.5%, settling time T_s =105 seconds.

The worst results were obtained for the case of tuning PID controller by the maximum stability degree method with iterations to the model object with second order inertia and time delay (12), in this case it was obtained the oscillatory process with overshootequal with σ = 8%.

Next, it was proposed to tune the PID controller by the analytical algorithm by the maximum stability degree criterion [16, 17] to the model object with third order inertia (14) and it was obtained the following tuning values: J=0.36, k_p=31.3962; k_i=2.9154; k_d=87.0241, and to the model object with second order inertia and time delay (15) and it was obtained the following tuning values: J=0.135, k_p=14.23; k_i=0.671; k_d=81.58. The experimental curve of the thermal process in the extruder in case of tuning the PID controller by the analytical algorithms is presented in the Fig. 16 and Fig 17.

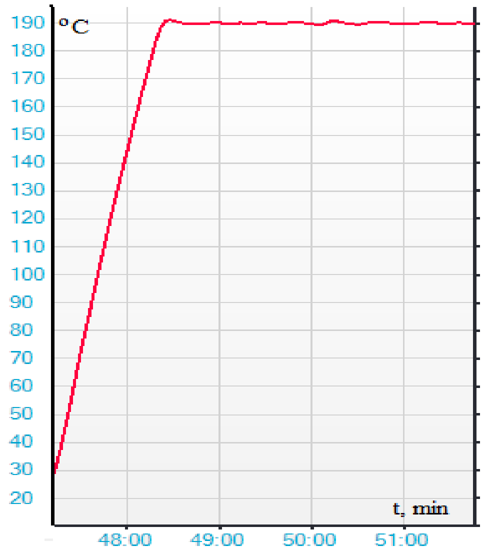

Fig. 16. The transient experimental processes of the control system with PID controller tuned by the analitycal algorithms, k_p=31.3962; k_i=2.9154; k_d=87.0241

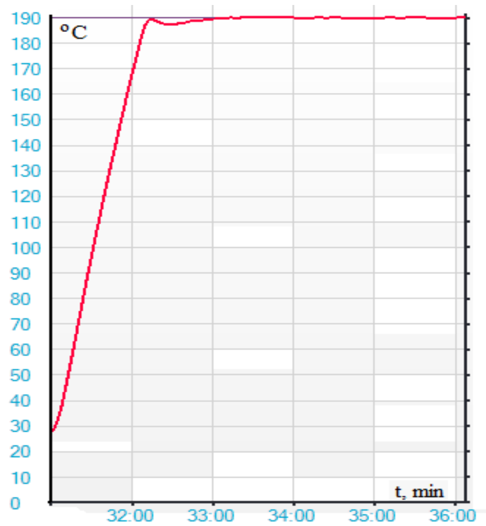

Fig. 17. The transient experimental processes of the control system with PID controller tuned by the analitycal algorithms, k_p=14.23; k_i=0.671; k_d=81.58.

Comparing the obtained experimental results in case of tuning PID controller by the maximum stability degree method with iterations and by the analytical algorithm by the maximum stability degree criterion, it was observed that the best performance was obtained for the case of tuning PID controller by the analytical algorithm. In this case it was obtained the overshoot equal with σ=1%, settling timeT_s = 90 seconds.

In the controller it is implemented the auto-tuning regime and by this procedure it was obtained the following values of the tuning parameters: k_p=30.43; k_i=3.27; k_d=70.83. The experimental curve of the temperature variation in the extruder for the case of tuning the PID controller by the auto tuning regime it is presented in the Fig. 18.

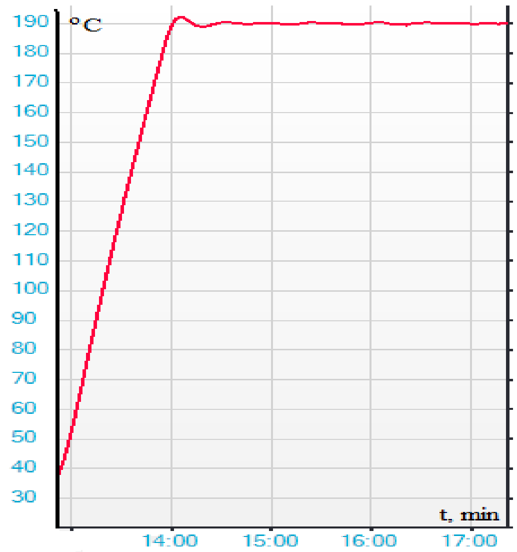

Fig. 18. The transient experimental processes of the control system.

In case of using the auto-tuning regime it was obtained the transient experimental curve of thermal process in the extruder with overshoot equal σ = 1.5%, settling time T_s =115 seconds.

VII. CONCLUSIONS

The 3D printer represents the device that produces 3D physical models and the core of the 3D printer represents the extruder. It is very important to keep the temperature in the extruder with high precision and in the settled range, because on the temperature variation in the extruder depends the quality of the printed objects. In this paper was identified the mathematical model of the thermal process in the extruder and in the result it was determinate five types of model objects (11)-(15). To the identified model of objects it was tuned the PID controller by the maximum stability degree method with iterations, analytical algorithmby the maximum stability degree criterion and auto-tuning regime.

Analyzing the obtained results it can be done the following conclusions:

1. Analyzing the computer simulation results of tuning the PID controller and experimental results, it was observed that results obtained in the computer simulation is different from the experimental results. In case of computer simulation the best result was obtained for the case of tuning PID controller to the model object with inertia first order and time delay (11), but in experimental implementation the best result was obtained for the case of tuning PID controller to the model object third order inertia (14).

2. Comparing the results after tuning the PID controller, the best performance were obtained for the case of tuning PID controller by the analytical algorithm (Fig. 16, Fig. 17), but the tuning method with iterations and auto-tuning regime (Fig. 18) also gave the satisfactory results.

3. From figures 11-18 can be observed that the steady state error was obtained more then $\varepsilon=\pm 5\% y_{st}$ in case of tuning PID controller to the model objects (12) and (15) by the maximum stability degree method with iterations, in other case of tuning PID controller the steady-state error is equal with $\varepsilon = \pm 5\% y_{st}$.

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