The Influence of the Output Current Deforming Regime on the Input Current Harmonics for a Three-Phase Rectifier Bridge

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Abstract - **The present paper was created based on the desire of the author to realize a systemic approach of static converters in general and especially of rectifiers. Looked as system from energetic point of view, a rectifier could be represented "as a black box" without internal energetic sources. It is connected to external environment with an electric DC load and a power supply (mono-phase or three-phase). On the other side the electric current represents an ordered movement of electric charge carriers. In order to supply a certain current to the DC load, the electrons must be "transferred" by the static converter from power supply. Thus, a certain wave form of the current at the output can be obtained only based on some wave forms of the currents from the supply phases. The present paper answers to two questions: (1): does the modification of the spectrum of the load current produces the modification of the spectrum of the currents from the supply phases; (2):if yes, how it modifies the spectrum of the current from the supply phase, only in the magnitude of the basic harmonics, or it appears new harmonics? The paper proves the fact that the spectrum of the currents from the supply phases of the three-phased bridge rectifier depends on the harmonic spectrum of the output current. This dependence is materialized only on the change of the components of basic harmonic spectrum, without some visible new harmonics.**

Cuvinte cheie: *redresor trifazat in punte, regim deformant, influența armonicilor, fundamentarea matematică.*

Keywords: *three-phase bridge rectifier, deforming regime, harmonics influence, mathematical foundation.*

I. INTRODUCTION

In the industrial system, most production halls are powered by three-phase voltage systems at low voltage. As a result, industrial machines that require DC supply have a three-phase AC to DC power conversion system. In this paper we consider the three-phased bridge rectifier presented in Fig.1.

A large number of scientific papers published recently have as their study object the emergence of voltage and current harmonics and their limitation through various methods or devices [1-12]. At modern power supply sources equipped with static converters, the existence of these harmonics is inevitable, because these types of high power electronic assemblies are known as deforming elements.

The present article is proposing to show the way how the deforming regime, which affects the current at the output of a three-phased bridge rectifier, influences the current, absorbed by the rectifier from the supply, inducting a non-sinusoidal regime in the connection point of the supply.

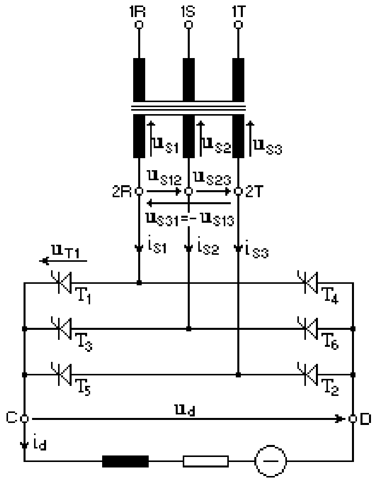

Fig.1. The three-phase bridge rectifier

The analysis of such a rectifier can be achieved by considering that the supply voltage system is symmetrical or asymmetrical. Power supply with asymmetric voltages is not desirable, and the requirements for power supply installations seek to reduce and even eliminate asymmetries. An analysis of the harmonics of the current produced by such a rectifier powered by asymmetrical voltages is presented in [13].

Typically, it can be considered that the rectifier bridge is under a symmetrical voltage regime. In this situation the analysis of the circuit's behaviour can be done in the context of one of the three possible theories (idealized, conventional or real).

II. THE ANALYSIS OF THE POWER SUPPLY PHASE CURRENT OF THE RECTIFIER - IDEALIZED THEORY

The study under idealized theory implies us to consider an infinite DC inductance and zero switching inductances on the supply phases. These presumptions ensure a continuous current through the load and currents formed from rectangular blocks on its supply phases [15].

The harmonic analysis of the currents through the supply phases leads to the following results [14]:

- Zero continuous component

- A fundamental of amplitude $I_1 = 2\sqrt{3} \cdot I_d$, where I_d represents the value of the current at the output of the rectifier.
- A serie of harmonics of order $k = 6\upsilon \pm 1$, with $\upsilon = 1,2,3,4,...$, having amplitudes of $I_k = \frac{1}{k} \cdot I_1$.

III. The Analysis of the Power Supply Phase Current of the Rectifier - Conventional Theory

The transition from the analysis under idealized theory to the analysis under conventional theory implies the transition from $L_k = 0$ and $L_d \to \infty$, to $L_k \neq 0$ and $L_d \to \infty$.

This ensures a good filtration and the continuous current at the rectifier output is constant, but the consideration of the switching inductances $L_k \neq 0$ leads to some current pulses of approximately trapezoidal shape occurring on the primary phases of the power supply of the rectifier, due to the existence of the switching periods.

The same harmonics continue to be maintained, but due to the switching phenomenon the amplitude of the harmonic rank k becomes:

$$I_k = k_k \frac{I_1}{k} \tag{1}$$

where k_k is a subunit coefficient.

The switching angle has the expression:

$$\gamma = \mathbf{arccos}\left(\mathbf{cos}\,\alpha - \frac{I_d}{I_{k\,\mathbf{max}}}\right) - \alpha = \mathbf{arccos}(\mathbf{cos}\,\alpha - 2d_x) - \alpha \tag{2}$$

where $d_x = \frac{I_d}{\sqrt{3} \cdot I_{dk}}$ and where $I_{dk} = \frac{\sqrt{2} \cdot U_f}{\omega L_k}$ represents the shortcircuit current of a supply phase of the rectifier.

In fact, consideration of the switching phenomenon implies accepting that the conduction time of a thyristor increases from the value corresponding to the radius angle $\frac{2\pi}{3}$ to the value corresponding to the radius angle $\left(\frac{2\pi}{3} + \gamma\right)$. If the phase origin of the current wave is considered to be oriented on the center of a trapezoidal block, the fundamental amplitude value is calculated as follows:

$$I_{1\mathbf{max}} = \frac{8}{2\pi}\left[\int_0^{\frac{\pi}{3}-\frac{\gamma}{2}} I_d \,\mathbf{cos}\,\omega t \cdot \mathbf{d}\,\omega t + \int_{\frac{\pi}{3}-\frac{\gamma}{2}}^{\frac{\pi}{3}+\frac{\gamma}{2}} I_d\left(\frac{\pi}{3\gamma} + \frac{1}{2} - \frac{\omega t}{\gamma}\right)\mathbf{cos}\,\omega t \cdot \mathbf{d}\,\omega t\right] =$$

$$= \sqrt{2}\,\frac{\sqrt{6} \cdot I_d}{0{,}5 \cdot \gamma \cdot \pi}\,\mathbf{sin}\,0{,}5\gamma \tag{3}$$

In [14] is presented a detailed analysis which has as a result the obtaining of some variation curves of the coefficients k_k that intervene in calculating the amplitude of the harmonics of the phase current. The curves are plotted depending on the parameter $I_{rap} = \frac{I_d}{I_{k\,\mathbf{max}}}$, where $I_{k\,\mathbf{max}}$ is the maximum value of the switching current.

Graphic curves were made for four values of the control angle (0º, 30º, 60º, 90º). The analysis of the six graphs indicates that as the rank of the harmonics increases, it "benefits" of a coefficient k_k with decreasing value when incrementing I_d with respect to the $I_{k\,\mathbf{max}} = const.$, which ultimately leads to the decrease of the weight of the respective harmonics within the wave.

Decreasing is faster with respect to the parameter I_{rap}, if the steering angle is lower. This is a consequence of the fact that at smaller control angles, the switching angle is higher and the shape of the pulse of current is more different from the rectangular shape.

It should be noted that the value of the coefficient k_k is a function of the control angle α, so that for a given value of the parameter I_{rap} we obtain the curves in the Fig.2.

In [14] are presented the curves which describe the variation of the amplitude of harmonics in relation to the fundamental amplitude variation, relative to the control angle α and for different values of the parameter I_{rap}. It is noted that as I_{rap} increases, decreases the weight of the higher harmonics and harmonic 5 (the strongest), does not exceed 20% of the fundamental. By increasing the I_{rap} parameter, both the value of the starting point of the harmonics (the value at the $\alpha = 0°$ command angle) and the maximum they reach are diminished. The charts in Fig.3 provide a synthetic indication of the k_k parameter, according to the α command angle and for various I_{rap} as parameter values.

IV. The Analysis of the Power Supply Phase Current of the Rectifier - Real Theory

The modification of the form of the rectified current distributed on the charge having the constant of time with finite value $\tau = \frac{L}{R}$ (which places the analysis on the context of the real theory), [15], is made in fact on the transfer of this current through the rectifier from the supply phases. The mathematical simulations made in the context of the conventional theory ($L \to \infty$) and the real theory have sent to the idea that the harmonic spectrum of the current through the phases of the rectifier is modifying very little, only at the amplitude of the harmonics, not in their order [14].

If we compare the harmonic spectrums in the conditions of the two theories for an angle of command $\alpha = 30°$, $\tau = 0{,}002$ s and $I_{rap} = \frac{2\varpi L_K I_d}{\sqrt{6} \cdot U_f} = 0{,}355098$, we can see that the two spectrums are alike, and the values of the harmonics magnitude (in Amps) are presented in Table I.

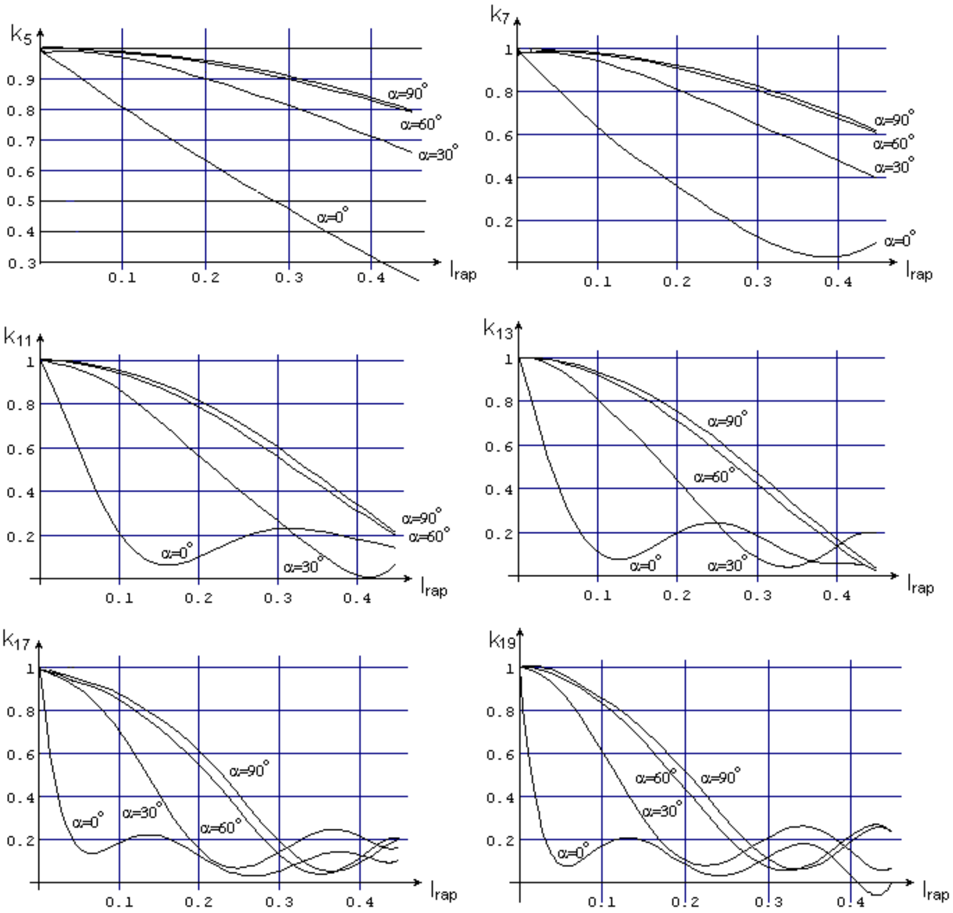

Fig.2. Change of the coefficients k_k based on I_{rap}, for different values of the command angle.

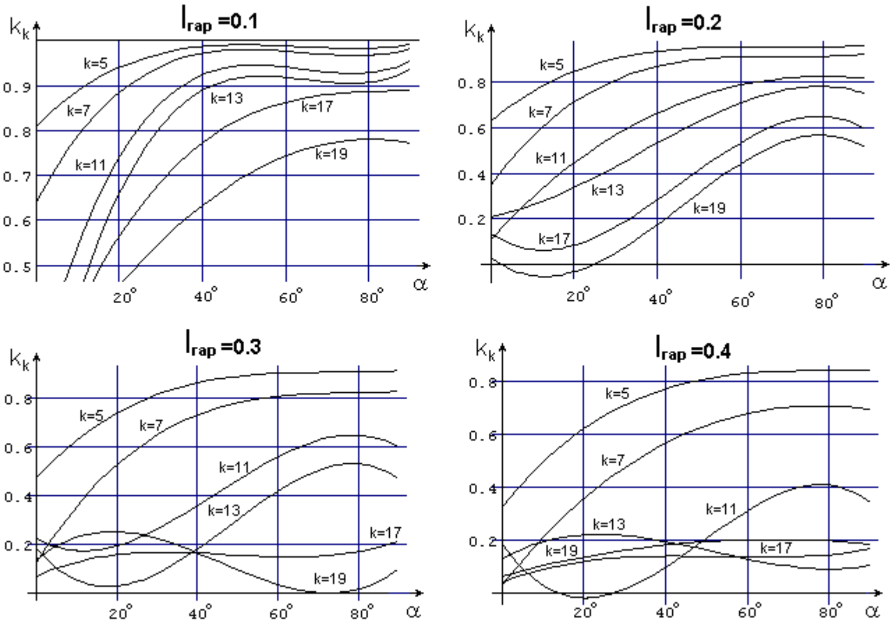

Fig.3. The influence of the command angle on the parameter k_k .

TABLE I.
THE VALUE OF THE HARMONICS

	Conventionally theory	Really theory
harm.1	19.2955	19.3156
harm.5	2.9368	3.0476
harm.7	1.5187	1.4250
harm.11	0.2080	0.0292
harm.13	0.0786	0.1269
harm.17	0.2498	0.2291
harm.19	0.2078	0.1692

A. Mathematical Foundation

The explanation of the way how the deformation of the current through the charge influents the current taken by the rectifier from the supply is based on the next mathematical idea:

We consider two periodical functions, with the same period, which satisfy the conditions of Diriclet and can be developed in Fourier complex series:

$$g_1(t)=g_1(t+kT)=\sum_{-\infty}^{+\infty}\underline{C}_{1n}\cdot e^{jn\omega t} \quad (4)$$

and

$$g_2(t)=g_2(t+kT)=\sum_{-\infty}^{+\infty}\underline{C}_{2n}\cdot e^{jn\omega t} \quad (5)$$

where

$$\underline{C}_{1n}=\frac{1}{T}\int_0^T g_1(t)\cdot e^{-jn\omega t}\cdot \mathbf{d}t$$

$$\underline{C}_{2n}=\frac{1}{T}\int_0^T g_2(t)\cdot e^{-jn\omega t}\cdot \mathbf{d}t$$

The function $f(t)=g_1(t)+g_2(t)=f(t+kT)$ can be developed like:

$$f(t)=\sum_{-\infty}^{+\infty}\underline{C}_{fn}\cdot e^{jn\omega t} \quad (6)$$

where

$$\underline{C}_{fn}=\frac{1}{T}\int_0^T f(t)\cdot e^{-jn\omega t}\cdot \mathbf{d}t=\frac{1}{T}\int_0^T[g_1(t)+g_2(t)]\cdot e^{-jn\omega t}\cdot \mathbf{d}t= =\frac{1}{T}\int_0^T g_1(t)\cdot e^{-jn\omega t}\cdot \mathbf{d}t+\frac{1}{T}\int_0^T g_2(t)\cdot e^{-jn\omega t}\cdot \mathbf{d}t=\underline{C}_{1n}+\underline{C}_{2n} \quad (7)$$

From the above we can say that it can be replaced the calculus of the developing in Fourier series of the composed function $f(t)$, with the operation of summing the harmonics of the two functions and the algebraically summing of the coefficients on each common harmonic. In case of the rectifier considered (in supply phases), over the harmonic spectrum of the trapezoidal wave we superpose the harmonic spectrum due to the rectified current.

B. The Influence of a Harmonic of the Rectified Current over the Supply Phases Current

For load current through three-phase bridge rectifier we take the harmonics in order $k=6\nu$, $\nu=1,2,3,4,...$ [16]:

$$i_k=\sqrt{2}\cdot I_k\,\mathbf{cos}(k\omega t) \quad (8)$$

Although the periods of variation of the current in phase are function of the command angle α and of the switching angle, γ, we can consider with a good approximation that the wave is symmetrical compared with the point $\alpha+\frac{\gamma}{2}-\frac{\pi}{6}$, the symmetry being an even type, this means $f(t)=f(T-t)$. In consequence, the wave will discompose only in harmonics in cosine that will complete the wave of phase current from the idealized theory [14]. Because, at the translation with $\frac{T}{4}=\frac{\pi}{2}$ the function becomes odd, and the developing in Fourier series will contain only the harmonics in odd order in cosine:

$$f(t)=\sum_{p=0}^{\infty}M_{2p+1}\cdot\mathbf{cos}(2p+1)\omega t \quad (9)$$

where

$$M_{2p+1}=\frac{8}{T}\int_0^{\frac{T}{4}}f(t)\cdot\mathbf{cos}(2p+1)\omega t\cdot\mathbf{d}\,\omega t= =\frac{4}{\pi}\int_{\alpha+\gamma}^{\alpha+\frac{\gamma}{2}+\frac{\pi}{3}}\sqrt{2}\cdot I_k\cdot\mathbf{cos}\,k\omega t\cdot\mathbf{cos}(2p+1)\omega t\cdot\mathbf{d}\,\omega t \quad (10)$$

Results

$$M_{2p+1}=\frac{2\sqrt{2}I_k}{\pi}\left[\frac{1}{k+2p+1}\left[\mathbf{sin}(k+2p+1)\left(\alpha+\frac{\gamma}{2}+\frac{\pi}{3}\right)-\right.\right. -\mathbf{sin}(k+2p+1)(\alpha+\gamma)\Big]+ +\frac{1}{k-2p-1}\left[\mathbf{sin}(k-2p-1)\left(\alpha+\frac{\gamma}{2}+\frac{\pi}{3}\right)-\right. \left.\left.-\mathbf{sin}(k-2p-1)(\alpha+\gamma)\right]\right] \quad (11)$$

In Fig.4 are presented, reported at unit, the influences that the 6-th order harmonics of the rectified current have on the harmonics of the current from first phase of the rectifier power supply. To obtain the absolute value of the harmonics influence, the value from the graphic must to be multiplied with the amplitude of the 6-th order harmonics of the output current for the specified command angle and I_{rap} respectively.

The five graphics show a wavy variation for the harmonics components, when α and I_{rap} are changing.

Considering the fact that the variation of the parameter $I_{rap}=\frac{I_d}{I_{k\,\mathbf{max}}}$ for a certain command angle is possible only if we consider the charge variable, the graphics presented in Fig.4 are quite difficult to use by the user of the rectifier. At constant load ($\tau = const.$) and at a certain command angle, corresponds a certain pare ($I_{d\alpha}, \gamma$) and at $L_k = const.$, results o certain value for I_{rap} .

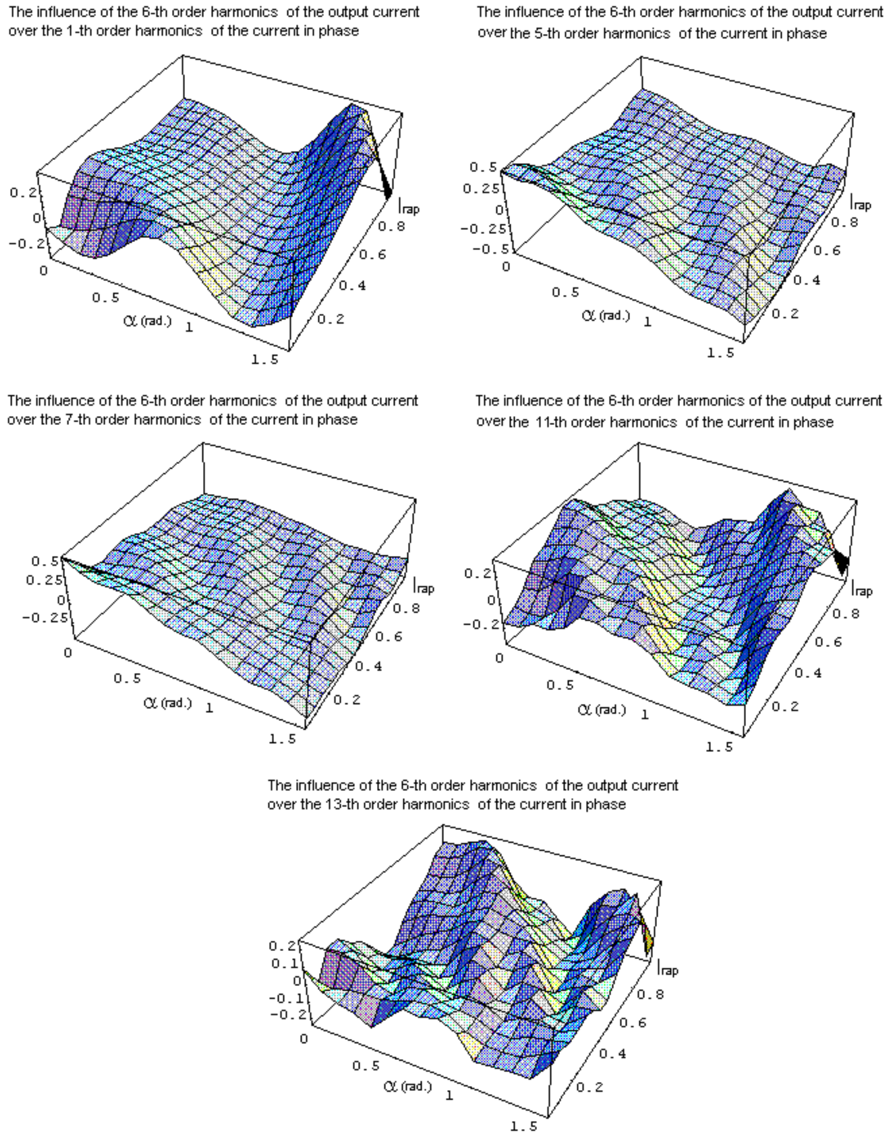

Fig.4. The influence of the harmonics 6 of the current rectified over the current from charging phases.

In [14] is presented a punctual database which was made through mathematical simulation, where for each command angle $\alpha = 0°,15°,30°,45°,60°,75°,89°$, was calculated ($I_{d\alpha}, \gamma$) and I_{rap} . Based on the corresponding values of the harmonics of the rectified current was calculated at the end the values of the coefficients of the harmonics transferred in the wave of the current of the 1-th phase power supply of the rectifier. The database had been calculated for two time constants of the charge $\tau = 0{,}0005s$ and $\tau = 0{,}0025s$. Then are calculated the algebraically sums of the harmonic's coefficients, and the total influence of the harmonics 6,12,18 of the rectified current over the harmonics of the current from phase of power supply.

Even if the harmonics in order $3(2p+1)$ have coefficients different from zero, their last value can be negligee, because the principal wave does not contain these harmonics.

In Fig.5 is presented in a compared way the influence of the 6-th order harmonics and the influence of the sum of the harmonics 6, 12 and 18 of the rectified current over the harmonics of the phases supply current, for the two values of time constant τ . The similar way of variation indicates the fact that the influence of the harmonics in order 6 is much more powerful.

Because the graphics had been made after some linear approximations, it shows a few electrical degrees zone for the values of the command angle, where the curves pass through zero. If we draw directly the variation of the harmonic coefficients for a certain value of the I_{rap} parameter, we obtain a common point of intersection of the curves with the Ox axe. (For example, for $I_{rap} = 0{,}355098$ all the harmonic influences become zero at $\alpha = 57{,}2958$ electrical degrees).

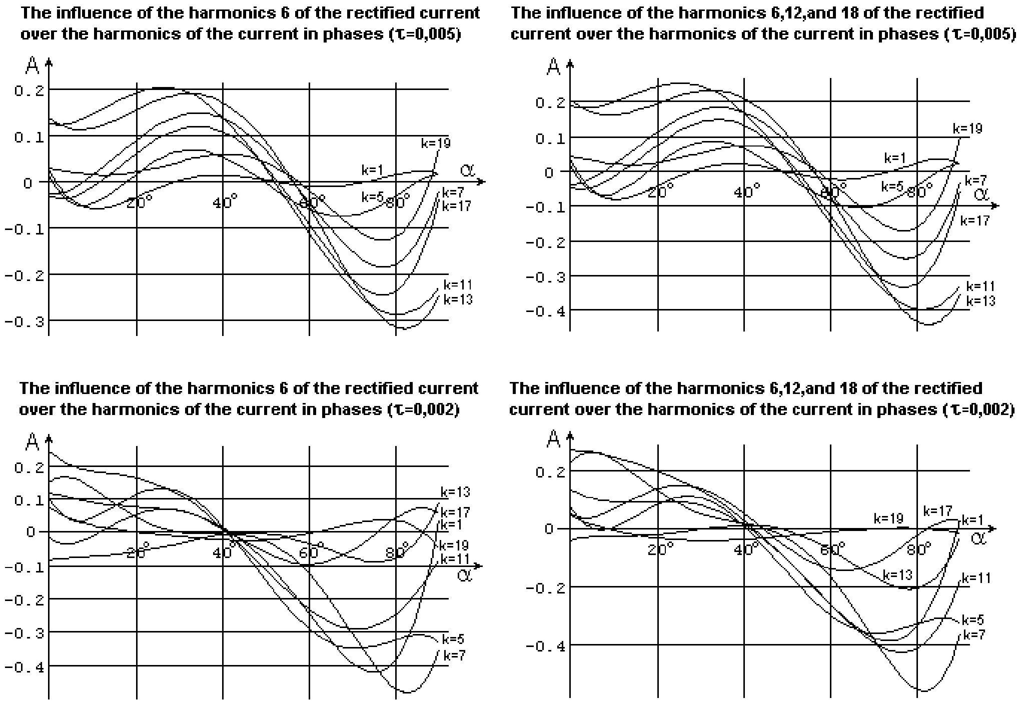

Fig.5. The influence of the harmonics of the rectified current over the harmonics of the current in charging phase.

If we consider the idealized theory ($\gamma = 0$), for a command angle $\alpha = 0°$ we obtain:

$$M_{2p+1} = \frac{2\sqrt{2}I_k}{\pi}\left[\frac{1}{k+2p+1}\left[\mathbf{sin}(k+2p+1)\left(\frac{\pi}{3}\right)\right] + \frac{1}{k-2p-1}\left[\mathbf{sin}(k-2p-1)\left(\frac{\pi}{3}\right)\right]\right] = \frac{2\sqrt{2}I_k}{\pi}\left[\frac{1}{k+2p+1}\left[\mathbf{sin}\left(2\nu\pi + (2p+1)\frac{\pi}{3}\right)\right] + \frac{1}{k-2p-1}\left[\mathbf{sin}\left(2\nu\pi - (2p+1)\frac{\pi}{3}\right)\right]\right] \quad (12)$$

and finally:

$$M_{2p+1} = -\frac{2\sqrt{2}I_k}{\pi}\cdot\frac{2(2p+1)}{k^2-(2p+1)^2}\cdot\mathbf{sin}(2p+1)\frac{\pi}{3} \quad (13)$$

For $2p+1 = 3(2r+1)$ (multiple of 3) the harmonic coefficients are annulling, because:

$$2p+1 = 3(2r+1) \Rightarrow \mathbf{sin}(2p+1)\frac{\pi}{3} = \mathbf{sin}(2r+1)\pi = 0 \quad (14)$$

For the others coefficients:

$$2p+1 = 3(2r+1)\pm 2 \Rightarrow \mathbf{sin}(2p+1)\frac{\pi}{3} = \mp\frac{\sqrt{3}}{2} \quad (15)$$

and we obtain:

$$M_{3(2r+1)\pm 2} = \pm\frac{2\sqrt{3}\cdot[3(2r+1)\pm 2]}{\pi\left[k^2-[3(2r+1)\pm 2]^2\right]}\cdot\left(\sqrt{2}\cdot I_k\right) \quad (16)$$

The use of the formula from upstairs permits the calculus of the dependence of the harmonics coefficients 1,5,7,11,13,17,19,23,25 of the current in phase compared with the harmonics 6,12,18,24 of the rectified current (presented in Table II):

TABLE II.
THE DEPENDENCE OF THE HARMONICS COEFFICIENTS

	harm.6	harm.12	harm.18	harm.24
M1	-0,031	-0,008	-0,003	-0,002
M5	0,501	0,046	0,018	0,01
M7	0,594	-0,081	-0,028	-0,015
M11	-0,143	0,527	0,059	0,027
M13	0,108	0,573	-0,092	-0,035
M17	-0,074	-0,129	0,536	0,065
M19	0,064	0,097	0,566	-0,097
M23	-0,051	-0,066	-0,124	0,540
M25	0,047	0,057	0,092	0,563

The ripple of the rectified current and the appearance of the harmonics sent to the influence of the harmonic content into a part of the alternating current of the rectifier. The spectrum himself doesn't change in a big way with the appearance or the removal of some harmonics, because he contains harmonics in order $k = 5,7,11,13,17,19,...$ if we still take in count the ripple rectified current phenomena the amplitude of these harmonics is modifying in a big way, like in Fig.6. It has to be mentioned that this graphic is made in conditions when the switching angle γ had been considered zero [15].

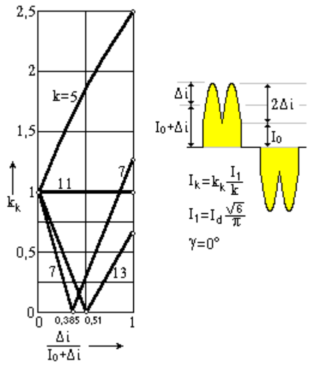

Fig.6. The modify of the harmonics depending on current ripple.

From the graphic we can see the growth of the 5-th order harmonic, the oscillation of the harmonics 7 and 13 and the constancy of the 11-th order harmonic. The 17-th order harmonic and the 23-th order harmonic remain practically uninfluenced just like the harmonics 11, harmonics 19 is becoming zero for a 0,62 ripple of rectified current and the harmonics 25 becoming zero for a 0,67 ripple of rectified current. The growth of the curve of the rectified current is obtained when we modify the constant of time of the load.

V. CONCLUSION

This article is proposing to show the way how the deforming regime on the output, influents the current, absorbed by the rectifier from the supply, inducting in the point of the supply a non-sinusoidal regime. The appearance of the ripple on the rectified current (continuous theoretically) and of the harmonics on c.c. side sent to the influence of the harmonic content into a part of the alternating current of the rectifier. The spectrum himself doesn't change in a big way with the appearance or the removal of some harmonics, because he contains harmonics in order $k = 5,7,11,13,17,19,...$ But the ripple rectified current phenomena take modifying in big way the harmonic's amplitude of supplying current.

The paper presents a series of theoretical considerations and several graphs that allow an estimation of the influence of the harmonics of the current at the output on the spectrum of the currents on the supply phases. Knowing these influences can be useful in harmonic filtering.

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