

Analytical Study, Numerical Modelling and Experimental Results of the Forces which Act in the Magnetic Liquid Placed between the Poles of an Electromagnet

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Abstract – This paper makes an analytical study, a numerical and an experimental analysis of the magnetic forces which act in magnetic liquids during the experiment called Quincke's effect. Quincke's effect consists in the rise of the magnetic liquid between the poles of an electromagnet. In ferrohydrodynamics theory, the forces acting in the magnetic liquid are not always treated in a uniform manner. The main goal of this paper is to give a consistent macroscopic view, pointing out the importance of the magnetostriction in the force localization understanding. If the absence of the magnetostrictive term in the forces expression leads to an exclusively superficial localization of the forces, the presence of this term leads both to a superficial localization of forces as well as their presence in the magnetic liquid volume. Moreover, in the magnetic liquid rise, the volume forces have a significant contribution than the surface forces, so the magnetostrictive term shall not be neglected. In this analysis, the magnetic liquid will be considered as a linear and nonconducting medium placed in a stationary (or quasistationary) field. In order to support the analytical study, a numerical analysis of the magnetic forces and some qualitative experiments were made. The value of the magnetic field forces which act in the magnetic liquid were established in Matlab using the magnetic field strength established by FEM analysis. Analyzing the numerical results and the magnetic liquid deformation obtained by experiments, the main conclusion consists in the fact that the magnetostrictive term has an important contribution in the localization of the forces, so it shall be taken into consideration.

Cuvinte cheie: lichide magnetice, ferrofluid, forță magnetică de volum, forță magnetică de suprafață, efectul Quincke, modelare cu elemente finite.

Keywords: magnetic liquids, ferrofluid, magnetic volume force, magnetic surface force, Quincke's effect, FEM modelling.

I. INTRODUCTION

The expression of the force density exerted by field on a nonconducting liquid from classical electrodynamics [1, 2], is known as:

$$\mathbf{f}_v = \mathbf{f}_v' + \mathbf{f}_v'' = -\frac{1}{2}H^2\nabla\mu + \frac{1}{2}\nabla\left(H^2\frac{\partial\mu}{\partial\tau}\right), \quad (1)$$

where H is the magnetic field strength, μ the permeability and τ the mass density of the fluid. The term \mathbf{f}_v' is called maxwellian (which was established by J.C. Maxwell), and \mathbf{f}_v'' is the magnetostrictive term (which was established by D. Korteweg and H. Helmholtz).

Studying the effects of the field forces (including the famous experiment Quincke), ferrohydrodynamics systematically neglects the magnetostrictive term [3-5]. The main reason is that the liquid is (basically) considered to be incompressible, so $\mu(\tau) = \text{const.}$ and $\mathbf{f}_v'' = 0$. Another reason that might lead to this simplifying hypothesis is the fact that being a conservative term (expressed by a gradient), the magnetostrictive component \mathbf{f}_v'' does not affect the total force exerted by the field or has a lesser, negligible contribution. Finally, the density expression \mathbf{f}_v which only has the maxwellian term, becomes formally identical with the form obtained by the microscopic forces mediation [6]:

$$\begin{aligned} \mathbf{f}_v &= \mu_0(\mathbf{M} \cdot \nabla)\mathbf{H} = \mu_0 M \nabla H = \\ &= \frac{1}{2}\mu_0 \nabla(MH) - \frac{1}{2}H^2 \nabla\mu, \end{aligned} \quad (2)$$

where M is the magnetization vector and μ_0 is the permeability of vacuum. To obtain the above relation, the following identities have been used: $\mathbf{M} = \chi_m \mathbf{H} = (\mu_r - 1)\mathbf{H}$ (χ_m is the magnetic susceptibility and μ_r is the relative permeability) and

$$(\mathbf{H} \cdot \nabla)\mathbf{H} = \frac{1}{2}\nabla(\mathbf{H} \cdot \mathbf{H}) - \mathbf{H} \times (\nabla \times \mathbf{H}), \quad (3)$$

where $\nabla \times \mathbf{H} = 0$ when there is no electrical conduction.

Indeed, in the particular case of uniform field (when $MH = \text{const.}$), (2) is reduced to:

$$\mathbf{f}_v = -\frac{1}{2}H^2 \nabla\mu, \quad (4)$$

formally identical with \mathbf{f}_v' from (1). However, it should be remarked that (1) was deduced in the general case of nonuniform field and the possible presence of the electri-

cal conduction. Moreover, as the microscopic dipoles are placed in vacuum, the microscopic models have no way to highlight the magnetostriction of the liquid.

Considering the liquid as homogeneous medium ($\nabla\mu = 0$), the maxwellian term will also be zero, so that even the density f_v is cancelled ($f_v = 0$). Starting from this fact, the forces exerted by the field, are located only in the discontinuity surface between the two media of μ_1 and μ_2 permeability, [1, 2]:

$$f_S = \frac{\mu_1 - \mu_2}{2} \left[\frac{B_n^2}{\mu_1 \mu_2} + H_t^2 \right] \cdot n_{12}, \quad (5)$$

where B_n and H_t are respectively the normal and tangential components of flux density and field strength, and n_{12} is the unit normal vector to the surface directed towards the second medium. That explains the development in literature study of the interfacial instabilities (including the electrical case [7]).

II. QUINCKE'S EFFECT AND ITS CURRENT INTERPRETATION

The Quincke's effect, which carries the person's name who used it for determining the permittivity [8], consists in the raising of a nonconducting liquid placed in an electric field generated within a plan capacitor's plates.

A similar rise could be observed when a magnetic liquid is placed in a magnetic field generated between the poles of an electromagnet, – Fig. 1.

Because the liquid is considered homogeneous, the force exerted by the field is located only in the free surface of A area. Since it is a field surface, $B_n = 0$, $H_t = H$, the expression (5) becomes for $\mu_1 = \mu$ and $\mu_2 = \mu_0$:

$$f_S = \frac{\mu - \mu_0}{2} H^2 n_{12} = \frac{\mu - \mu_0}{2} H^2 n_{12}. \quad (6)$$

Referring to the n_{12} direction, it is asserted that the force f_S exerts traction, resulting the rise h of the liquid.

In the new equilibrium state, the total force $f_S \cdot A$ is equal in magnitude to the weight of the liquid column:

$$\frac{\mu - \mu_0}{2} H^2 A = \tau \cdot g \cdot h \cdot A, \quad (7)$$

where g is the acceleration due to gravity and h is the height of rise. From (7) it results:

$$h = \frac{\mu - \mu_0}{2\tau g} H^2, \quad (8)$$

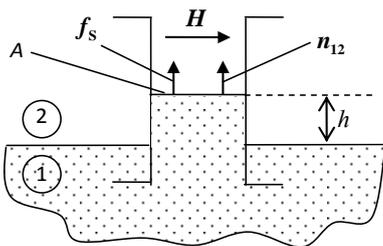


Fig. 1. The Quincke's effect.

Although the deduced relation (8) is well proved by the experience [4, 5], the actual physical mechanism of the effect appears to be totally different. This fact will be proved by taking into account the magnetostrictive term, which cannot be neglected. Actually, the magnetostrictive term has no contribution on the total force generated by the field, but it has a major influence over the spatial distribution of the local forces, [6].

III. THE VOLUME FORCES IN THE PRESENCE OF MAGNETOСТRICTION

Supposing that $\mu = \mu(\tau)$, we can write [2]:

$$\nabla\mu = \frac{\partial\mu}{\partial\tau} \cdot \nabla\tau, \quad (9)$$

and the magnetostrictive term of volume force becomes:

$$f_v'' = \frac{1}{2} \nabla \left(H^2 \frac{\partial\mu}{\partial\tau} \tau \right) = \frac{1}{2} \tau \cdot \nabla \left(H^2 \frac{\partial\mu}{\partial\tau} \right) + \frac{1}{2} H^2 \nabla\mu. \quad (10)$$

In this way, the expression (1) of the force density can be written in the form:

$$f_v = -\frac{1}{2} H^2 \nabla\mu + \frac{1}{2} \tau \cdot \nabla \left(H^2 \frac{\partial\mu}{\partial\tau} \right) + \frac{1}{2} H^2 \nabla\mu = \frac{1}{2} \tau \cdot \nabla \left(H^2 \frac{\partial\mu}{\partial\tau} \right). \quad (11)$$

Due to the fact that the maxwellian term is cancelled by a part of the magnetostrictive term, the relation (11) points out that the magnetostrictive term is much higher than the maxwellian term ($f_v'' > f_v'$). As a consequence, the magnetostrictive term cannot be neglected [1, 2], regardless of the stated reasons.

Considering the magnetic liquid as a homogeneous medium ($\tau \frac{\partial\mu}{\partial\tau} = const.$), (11) becomes:

$$f_v = \frac{1}{2} \tau \frac{\partial\mu}{\partial\tau} \cdot \nabla H^2. \quad (12)$$

Since the magnetic liquids are weak magnetic media, in the domain of applicability of the "Clausius – Mosotti" relation [6]:

$$\frac{\chi}{\mu_r} = C \cdot \tau \quad ; \quad \chi = \mu_r - 1, \quad (13)$$

we have:

$$\tau \frac{\partial\mu}{\partial\tau} = \mu\chi, \quad (14)$$

in the force density expression.

The final force density (12) shows that $f_v \neq 0$ in non-uniform fields (even for homogeneous media), the force is oriented towards the more intensive field areas. In particular case of the Quincke's experiment – Fig. 2, these forces are located around the end effect area, around the lower extremity of the poles, "pushing" the liquid towards the uniform field area (where the field has more intensity).

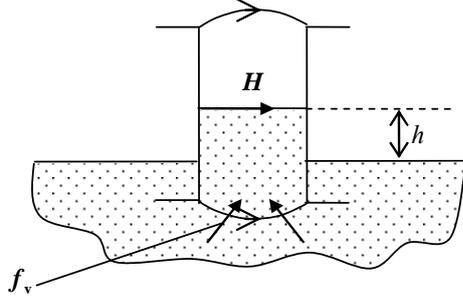


Fig. 2. The volume forces localization.

For a magnetic liquid placed in a magnetic field, these forces are responsible for the excess of the magnetic pressure (with regards to a reference point of zero field):

$$p_{mv} = \frac{1}{2} H^2 \frac{\partial \mu}{\partial \tau} \tau. \quad (15)$$

The expression (15) was well verified by experience in the similar electrical case [9].

IV. THE SURFACE FORCES TAKING INTO ACCOUNT THE MAGNETOSTRICTION

The surface force density along an interface between two different liquids, placed in a magnetic field, is [1, 2]:

$$\begin{aligned} f_S = f_S' + f_S'' = \frac{\mu_1 - \mu_2}{2} \left[\frac{B_n^2}{\mu_1 \mu_2} + H_t^2 \right] \cdot n_{12} + \\ + \frac{1}{2} \left[H_2^2 \tau_2 \frac{\partial \mu_2}{\partial \tau_2} - H_1^2 \tau_1 \frac{\partial \mu_1}{\partial \tau_1} \right] \cdot n_{12} \end{aligned} \quad (16)$$

If the interface between the liquids is a field surface (Fig. 1), $B_n = 0$, $H_t = H$, (16) becomes:

$$f_S = \frac{\mu_1 - \mu_2}{2} H^2 \cdot n_{12} + \frac{1}{2} \left[\tau_2 \frac{\partial \mu_2}{\partial \tau_2} - \tau_1 \frac{\partial \mu_1}{\partial \tau_1} \right] H^2 \cdot n_{12}. \quad (17)$$

Moreover, using Quincke's conditions ($\mu_1 = \mu$, $\mu_2 = \mu_0$), the force density f_S is:

$$f_S = \frac{1}{2} \left[(\mu - \mu_0) - \tau \frac{\partial \mu}{\partial \tau} \right] H^2 n_{12}. \quad (18)$$

which in domain of applicability of "Clausius – Mosotti" relation, becomes:

$$f_S = \frac{1}{2} \left[(\mu - \mu_0) - \mu \chi \right] H^2 n_{12} = -\frac{\chi}{2} (\mu - \mu_0) H^2 n_{12}. \quad (19)$$

As a consequence, on the liquid surface, the forces act as a compression and not as traction, how it resulted in the absence of the magnetostrictive term. – Fig. 3.

In Quincke's experiment, the forces (19) induce in liquid an additional pressure:

$$p_{ms} = \frac{1}{2} \left[(\mu - \mu_0) - \tau \frac{\partial \mu}{\partial \tau} \right] H^2. \quad (20)$$

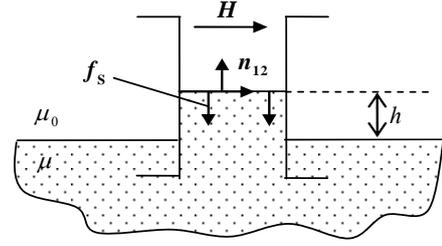


Fig. 3. The compression surface force.

V. THE ACTUAL PHYSICAL MECHANISM OF THE QUINCKE'S EFFECT

The resulting magnetic pressure due to the simultaneous action of volume and surface forces (Fig. 4) is obtained using (15) and (20):

$$\begin{aligned} p_m = p_{mv} + p_{ms} = \frac{1}{2} H^2 \frac{\partial \mu}{\partial \tau} \tau + \frac{1}{2} \left[(\mu - \mu_0) - \tau \frac{\partial \mu}{\partial \tau} \right] H^2 = \\ = \frac{\mu - \mu_0}{2} H^2 \end{aligned} \quad (21)$$

It could be remarked that the magnetostrictive term is lost in the final relation [10]. Since the pressure p_m generates the rise h of the fluid, the new equilibrium state occurs when the total force will be balanced by the liquid weight:

$$p_m A = \tau g A h. \quad (22)$$

From (21) and (22), we have:

$$h = \frac{p_m}{\tau g} = \frac{\mu - \mu_0}{2 \tau g} H^2, \quad (23)$$

which is identical with (8).

Comparing the volume and surface forces influence over the pressure p_m , it could be proved that, for weak magnetic fluids, $p_{mv} > p_{ms}$. Considering (14) as well, it results:

$$\begin{aligned} p_{mv} - p_{ms} = \frac{1}{2} H^2 \frac{\partial \mu}{\partial \tau} \tau - \frac{1}{2} \left[(\mu - \mu_0) - \tau \frac{\partial \mu}{\partial \tau} \right] H^2 = \\ = \frac{\chi (2\mu - \mu_0)}{2} H^2 > 0 \end{aligned} \quad (24)$$

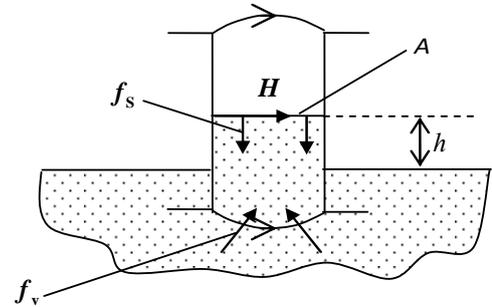


Fig. 4. The actual spatial localization of the forces.

VI. NUMERICAL ANALYSIS USING FEM METHOD

The conclusions from the analytical study can be proved by the results obtained from numerical analysis.

In order to compute the values of the magnetic forces which act in ferrofluid in the Quincke's effect case, the magnetic field strength was established by numerical modelling using the 3D-FEM program Opera 13 of Vector Fields [11]. The analysis program used to solve this problem was TOSCA algorithm which can analyze magnetostatic fields from defined current sources. This algorithm is based on vector potential formulation.

The values of the magnetic field strength were established inside a sample of ferrofluid, having a parallelepipedal shape $2.8 \times 4.5 \times 5.3 \text{ cm}$. This sample was placed between the cylindrical poles of a Weiss electromagnet (with the radius of the poles $r = 5 \text{ cm}$), with the air gap having the length $\delta = 4.5 \text{ cm}$ and the coils being powered by a DC current of $I = 10.05 \text{ A}$. The considered model is a scale reproduction of it. The Weiss electromagnet provides a stationary and quasiuniform field in its air gap. Because we had wanted to point out the effect of deformation of the ferrofluid in the presence and in the absence of the end effect in magnetic fluid, we made the FEM analysis for three positions of the ferrofluid sample between the magnetic poles:

- The ferrofluid sample had placed such that its base was in the nonuniform field
- The entire ferrofluid sample was placed in the uniform field area
- The ferrofluid sample had placed such that its free surface was in the nonuniform field

The magnetic fluid is considered linear with relative magnetic permeability $\mu_r = 2.4$.

Due to the model symmetry, just a quarter of the geometry was used in numerical analysis. The model mesh has quadratic elements having approximately 20000 nodes (Fig. 5).

In the domains of interest of the model (the ferromagnetic core, the air gap and the magnetic liquid), the magnetic vector potential A fulfils a Laplace equation $\nabla^2 A = 0$, and the magnetic field fulfils $\text{div} \mathbf{B} = 0$, $\text{curl} \mathbf{H} = 0$ and $\mathbf{B} = \mu \mathbf{H}$ in ferromagnetic core, $\mathbf{B} = \mu_0 \mathbf{H}$ in the air and $\mathbf{B} = \mu_f \mathbf{H}$ in magnetic liquid. In the sources domains, the magnetic vector potential A fulfils a Poisson equation $\nabla^2 A = -\mu \cdot \mathbf{J}$.

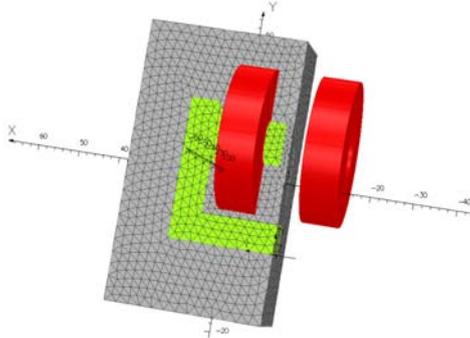


Fig. 5. The mesh of the model.

All analyzes have the following boundary conditions: the boundary passing through the center of the air gap being parallel with the poles surfaces is normal magnetic boundary $(\nabla \times \mathbf{A}) \times \mathbf{n} = 0$, and all the rest are tangential magnetic boundaries $\mathbf{A} \times \mathbf{n} = 0$.

The values of the magnetic field strength established by modelling (Fig. 6a, b,c) were used to compute the values of the surface and volume forces, for all three positions of the ferrofluid sample between the magnetic poles.

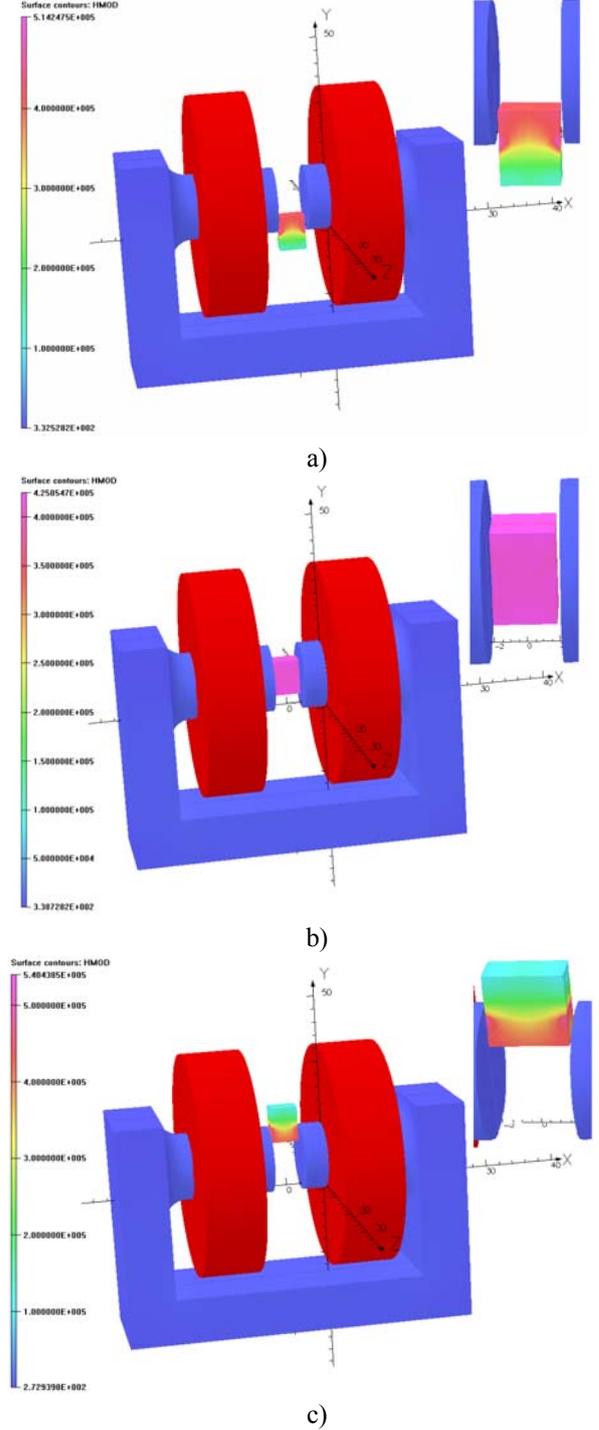
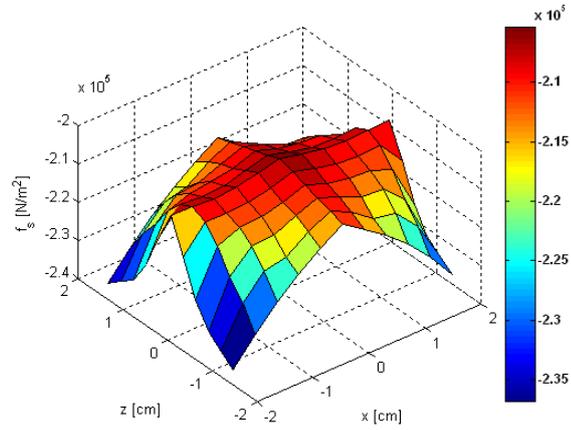


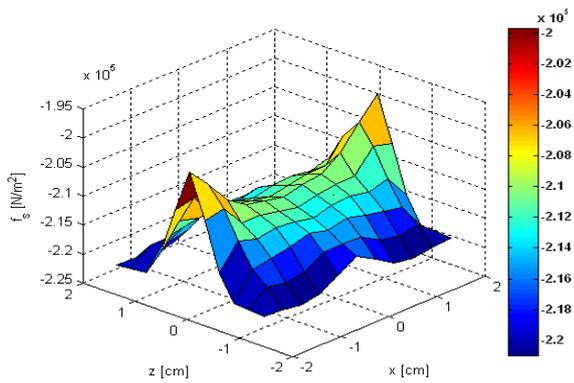
Fig. 6. The magnetic field strength distribution (complete model and detail) for: a. the ferrofluid sample having the base placed in the nonuniform field; b. the ferrofluid sample placed in the uniform field; c. the ferrofluid sample having the free surface placed in the nonuniform field.

The surface forces were computed using the magnetic field strength in the magnetic liquid free surface. The volume forces were established in two parallel planes with the poles surface passing through the coordinates $x = 0\text{cm}$ and $x = 1.8\text{cm}$ and in two perpendicular planes on the poles surface passing through the coordinates $z = 0\text{cm}$ and $z = 1.2\text{cm}$. The values of both forces were computed in Matlab with (15) and (19).

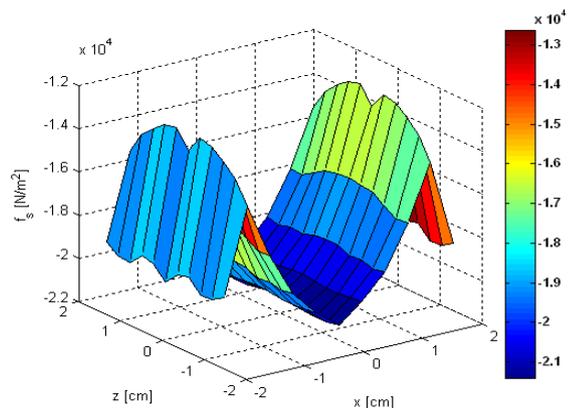
All results are presented in graphical forms. Fig. 7.a, b, c shows the surface force distribution:



a)



b)

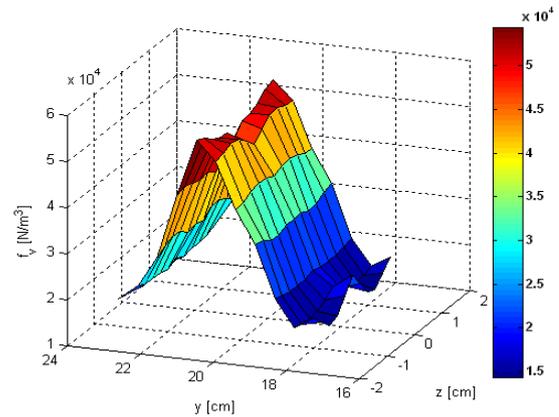


c)

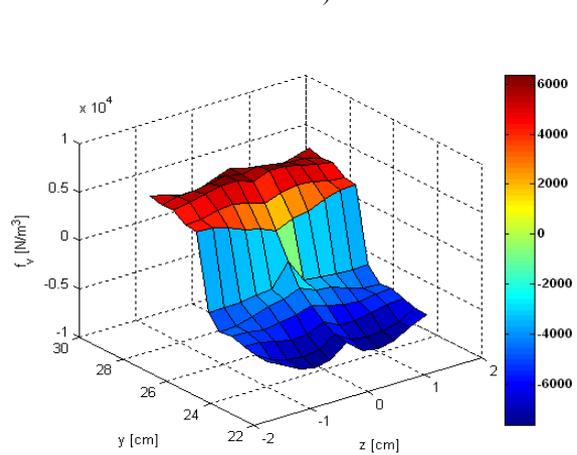
Analyzing qualitatively these results it is observed that the surface forces are negative so they have an opposite orientation to n_{12} , exerting a compression against the magnetic liquid. This compression is more intensive at poles extremity (Fig.7a)) when the ferrofluid is placed with the base in the nonuniform field. In the case with the ferrofluid placed in the uniform field (Fig.7, b)), the surface forces exert a quasiuniform compression in the point of the free surface of the magnetic liquid. In the last case (Fig. 7c)), the compression is more intensive in the central points of the free surface, but the values of forces are the lowest of them all, due to the lower values of the magnetic field strength. The compression due to the surface forces leads to the ferrofluid surface deformation. This deformation is more pronounced in the case a, due to the high value of the magnetic field strength in the free surface points.

The volume force distribution in both parallel planes with the poles surface is shown in Fig. 8a, b, c for $x = 0\text{cm}$ and Fig. 9a, b, c for $x = 1.8\text{cm}$.

Analyzing qualitatively these results, it is observed that they have positive and negative values. Due to the fact that they are expressed by a gradient (∇H^2), the sign gives the clue about their orientation related to the surface forces orientation.

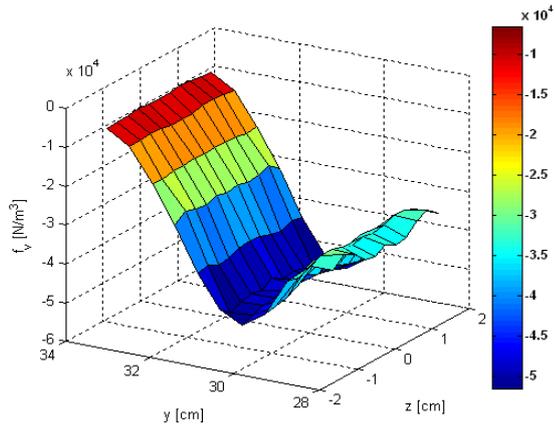


a)

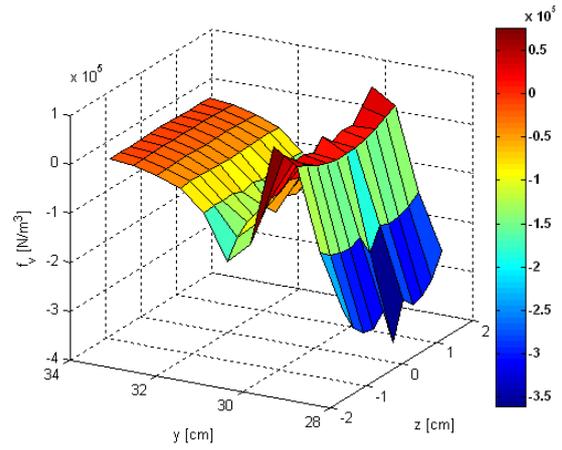


b)

Fig. 7. The specific surface force distribution for: a. the ferrofluid sample having the base placed in the nonuniform field; b. the ferrofluid sample placed in the uniform field; c. the ferrofluid sample having the free surface placed in the nonuniform field.



c)

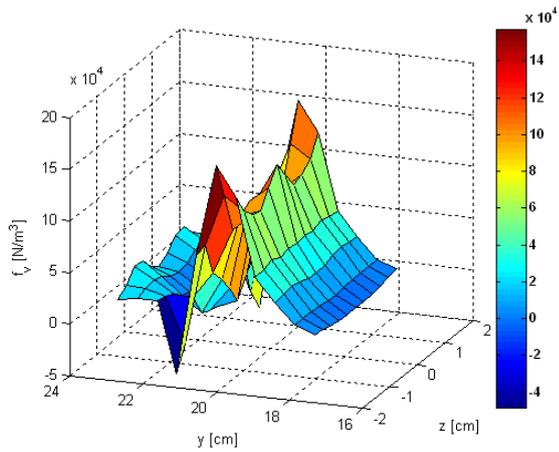


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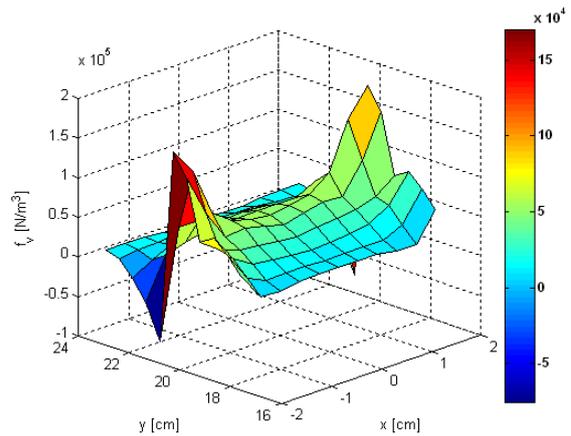
Fig. 8. The specific volume force distribution in a parallel plane with the poles surface passing through $x = 0\text{cm}$, for: a. the ferrofluid sample having the base placed in the nonuniform field; b. the ferrofluid sample placed in the uniform field; c. the ferrofluid sample having the free surface placed in the nonuniform field.

Fig. 9. The specific volume force distribution in a parallel plane with the poles surface passing through $x = 1.8\text{cm}$, for: a. the ferrofluid sample having the base placed in the nonuniform field; b. the ferrofluid sample placed in the uniform field; c. the ferrofluid sample having the free surface placed in the nonuniform field.

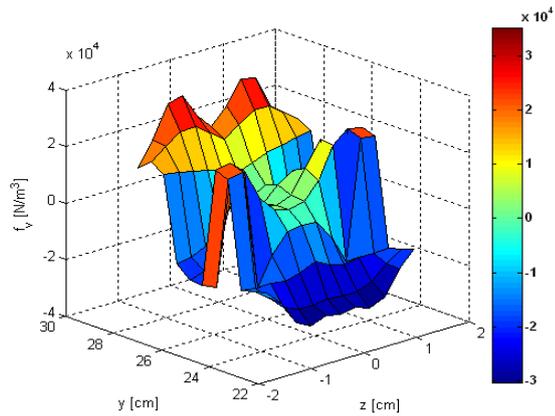
The volume force distribution in both perpendicular planes with the poles surface is shown in Fig. 10a, b, c for $z = 0\text{cm}$ and Fig. 11a, b, c for $z = 1.2\text{cm}$.



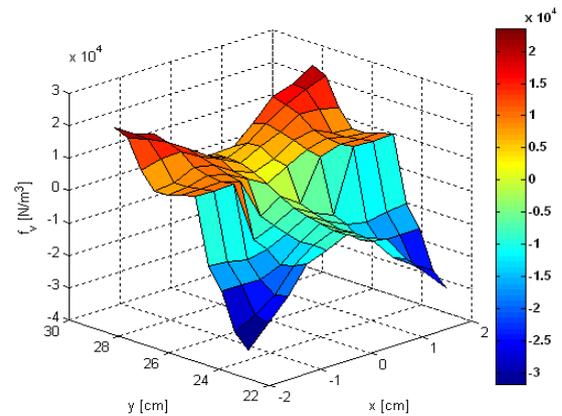
a)



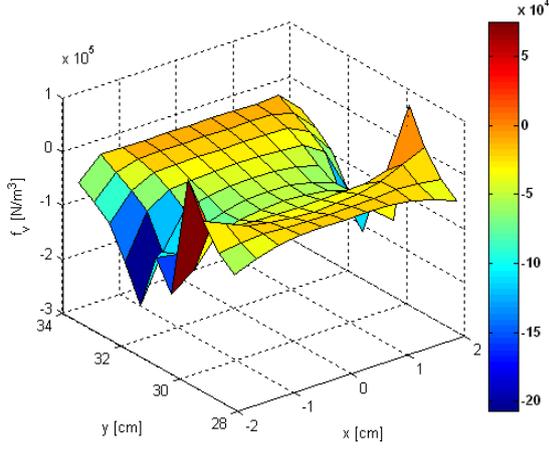
a)



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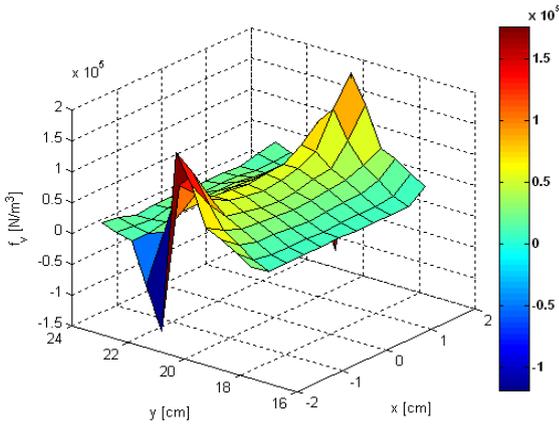


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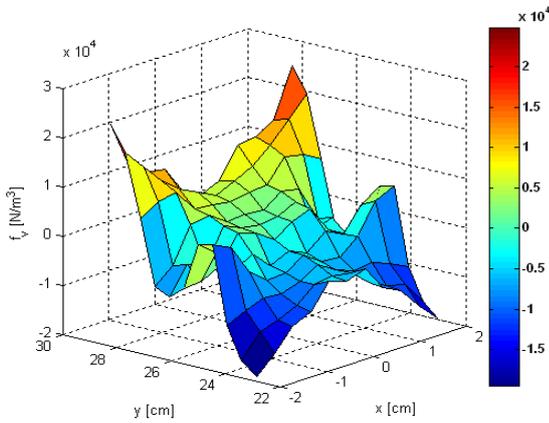


c)

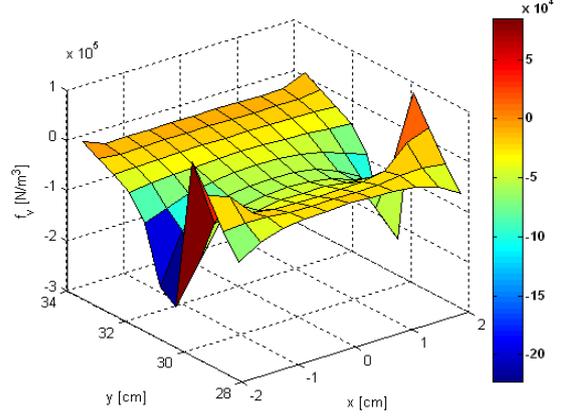
Fig. 10. The specific volume force distribution in a perpendicular plane on the poles surface passing through $z = 0\text{cm}$, for: a. the ferrofluid sample having the base placed in the nonuniform field; b. the ferrofluid sample placed in the uniform field; c. the ferrofluid sample having the free surface placed in the nonuniform field.



a)



b)



c)

Fig. 11. The specific volume force distribution in a perpendicular plane on the poles surface passing through $z = 1.2\text{cm}$, for: a. the ferrofluid sample having the base placed in the nonuniform field; b. the ferrofluid sample placed in the uniform field; c. the ferrofluid sample having the free surface placed in the nonuniform field.

When the magnetic liquid is placed with the base in the nonuniform field, the volume forces have positive values in almost all points of these planes (Fig. 8a, Fig. 9a, Fig. 10a and Fig. 11a). These forces have higher values, the closer to the poles surface they are ($x = 1.8$), and a uniform distribution after z axes. So, they have such an orientation that their pressure will compensate the pressure of the surface forces in magnetic liquid. As a result of the combined action of both pressures produced by the two specific forces, in the presence of magnetostriction and of the end effect, the magnetic liquid will rise between the electromagnet poles. The rise should be higher at poles surface.

For the magnetic liquid placed in the uniform field, the volume forces have both positive and negative values (Fig. 8b, Fig. 9b, Fig. 10b and Fig. 11b). Due to the fact that only the pressure due to the positive values compensates the pressure of the specific surface forces, then the magnetic liquid rise is much less than in the previous case.

When the magnetic liquid is placed with the free surface in the nonuniform field, the volume forces have negative values in almost all points of the planes (Fig. 8c, Fig. 9c, Fig. 10c and Fig. 11c). So, due to this orientation, their pressure amplifies the pressure due to the surface forces. The volume forces have higher values in the end effect area and closed to the poles surface. As a result of the combined action of both pressures produced by the two specific forces, the magnetic liquid will be compressed between the electromagnet poles. This effect has more intensity the closer to the poles surface it is.

VII. EXPERIMENTAL ANALYSIS

In order to verify the conclusions from the analytical study, an experimental analysis was made. A magnetic liquid sample, having the same size and shape as in the numerical analysis was placed between the poles of the Weiss electromagnet, having the coils powered by a DC current of $I = 10.05\text{A}$. The rise of the magnetic liquid was measured and analyzed having as reference the middle point of the free surface at the start of the experiment. Fig. 12a, b, c shows the rise of the magnetic liquid for the three cases analyzed above:



Fig. 12. The magnetic liquid rise, for: a. the ferrofluid sample having the base placed in the nonuniform field; b. the ferrofluid sample placed in the uniform field; c. the ferrofluid sample having the free surface placed in the nonuniform field.

Doing a qualitatively analysis of the effects observed by experiments, we can establish the following conclusions:

- Due to the end effect, the volume forces exert a higher pressure in the magnetic liquid, these forces being oriented towards the more intensive field area. They compensate the pressure due to the surface forces, pushing the liquid between the poles and causing the higher rise of the ferrofluid sample (Fig.12 a)).
- When the ferrofluid is placed in the uniform field (Fig.12 b)), a part of the volume pressure compensates the pressure due to the surface forces while the other part amplifies this effect. This thing causes a small rise of the magnetic liquid in the middle point of the free surface, but a higher rise close to the poles surface.
- When the ferrofluid sample is placed with the free surface in the nonuniform field, the volume forces pressure amplify the effect of the surface forces pressure causing a small deformation of the liquid free surface (Fig.12 c)).

VIII. CONCLUSIONS

Although the magnetostrictive terms that appear in the specific force expressions do not have a contribution on the total force exerted by the magnetic field, they are important in the force localization, and, as a consequence, in understanding of the actual physical mechanism of the occurred effects.

The main role for the magnetic field rise in Quincke's effect case is due to the volume forces which act in the end effect being oriented towards the more intensive field area. This thing can be proved analytical if the magnetostrictive term is not neglected in the magnetic forces expressions. Its absence in the forces expression leads to a superficial localization of the forces which exert traction on the points of the free surface.

Following this explanations, the mechanism of other specific related effects could be reconsidered, for example: the raising of the magnetic liquid around a vertical conductor with a current flow, or getting vertical liquid "bridges" (including in the microgravity), etc.

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